Compito di Matematica Generale del 10/6/2018 $\boxed{\text{CMG 1}}$

1) $f(x)=\dfrac{e^x}{e^x+1}$. C.E.: $\mathbb{R}$; $\lim\limits_{x\to-\infty} f(x)=0^+$; $\lim\limits_{x\to+\infty} f(x)=1^-$. $f(x)>0\ \forall x\in\mathbb{R}$.

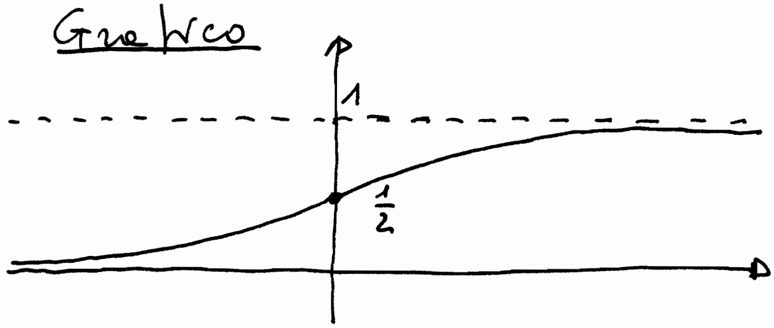
$f'(x)=\dfrac{e^x(e^x+1)-e^x\cdot e^x}{(e^x+1)^2}=\dfrac{e^x}{(e^x+1)^2}>0\ \forall x\in\mathbb{R}$: funzione strettamente crescente.

$$f''(x)=\frac{e^x\cdot(e^x+1)^2-e^x\cdot 2\cdot(e^x+1)\cdot e^x}{(e^x+1)^4}=$$

$$=\frac{e^x(e^x+1)(e^x+1-2e^x)}{(e^x+1)^4}=\frac{e^x(1-e^x)}{(e^x+1)^3}\geq 0$$

per $1-e^x>0 \Rightarrow e^x<1 \Rightarrow x<0$. $f(0)=\frac{1}{2}$.

Grafico



2) $\lim\limits_{x\to 0}\dfrac{\operatorname{sen}^2 3x}{1-\cos x}=\lim\limits_{x\to 0}\dfrac{\operatorname{sen}3x}{3x}\cdot\dfrac{\operatorname{sen}3x}{3x}\cdot 9\cdot\dfrac{x^2}{1-\cos x}=1\cdot 1\cdot 9\cdot 2=18$.

$\lim\limits_{x\to-\infty}\dfrac{\log(1-x)-2x}{\sqrt{1-x}+3x}=\lim\limits_{x\to-\infty}\dfrac{-2x}{3x}=-\dfrac{2}{3}$. $\left(\log(1-x)=o(2x);\ \sqrt{1-x}=o(3x)\right)$.

3) Da Esercizio n. 1: $f(x)$ è invertibile in $\mathbb{R}$, con $f:\mathbb{R}\to]0;1[\Rightarrow f^{-1}:]0;1[\to\mathbb{R}$.

$\dfrac{e^x}{e^x+1}=y \Rightarrow e^x=e^x\cdot y+y \Rightarrow e^x(1-y)=y \Rightarrow e^x=\dfrac{y}{1-y} \Rightarrow x=\log\dfrac{y}{1-y}$. $f^{-1}(x)=\log\dfrac{x}{1-x}$.

4) $f(x)=x^5\cdot e^{-x}$. $f\in\mathcal{C}^1(\mathbb{R})$. $f'(x)=5x^4e^{-x}+x^5(-e^{-x})=x^4e^{-x}\cdot(5-x)\geq 0 \Rightarrow$

$\Rightarrow 5-x\geq 0 \Rightarrow x\leq 5$. In $x=5$ punto di Max. $f'(0)=0 \Rightarrow x=0$ Punto di flesso.

5)

A	B	C	D	(A e B)	non C	(non C o D)	(A e B) ⇒ (non C o D)
1	1	1	1	1	0	1	1
1	0	1	1	0	0	1	1
0	1	1	1	0	0	1	1
0	0	1	1	0	0	1	1

6) $\displaystyle\int_0^1 x-e^{kx}\,dx=\left(\frac{1}{2}x^2-\frac{1}{k}\cdot e^{kx}\right|_0^1=\left(\frac{1}{2}-\frac{1}{k}e^k\right)-\left(0-\frac{1}{k}\right)=\frac{1}{2}-\frac{1}{k}e^k+\frac{1}{k}=\frac{1}{2}\Rightarrow$

$\Rightarrow \dfrac{1}{k}\left(1-e^k\right)=0 \Rightarrow 1-e^k=0 \Rightarrow k=0$. Soluzione non accettabile.

Quindi non esiste alcun valore di k che soddisfi l'uguaglianza.

CMG2

7) $f(x;y;z) = x\log y - y e^{z-x} - \cos(1+x-2z)$.

$\nabla f(x;y;z) = (f'_x; f'_y; f'_z) = \left(\log y + y e^{z-x} + \operatorname{sen}(1+x-2z); \frac{x}{y} - e^{z-x}; -y e^{z-x} - 2\operatorname{sen}(1+x-2z)\right)$.

$\nabla f(1;1;1) = (\log 1 + 1 e^0 + \operatorname{sen} 0; 1 - e^0; -1 e^0 - 2\operatorname{sen} 0) = (1; 0; -1)$.

8) $A \circ X = \begin{Vmatrix} 1 & k & 0 \\ 1 & -1 & 2 \end{Vmatrix} \circ \begin{Vmatrix} 1 \\ 1 \\ k \end{Vmatrix} = \begin{Vmatrix} 1 + k + 0 \\ 1 - 1 + 2k \end{Vmatrix} = \begin{Vmatrix} 1+k \\ 2k \end{Vmatrix} \Rightarrow A \circ X = (1+k; 2k)$.

a) $A \circ X \parallel (3;4) \Rightarrow \frac{1+k}{3} = \frac{2k}{4} \Rightarrow 4+4k = 6k \Rightarrow 2k = 4 \Rightarrow k = 2$.

b) $A \circ X \perp (3;4) \Rightarrow (1+k;2k)\cdot(3;4) = 0 \Rightarrow 3+3k+8k = 0 \Rightarrow 11k = -3 \Rightarrow k = -\frac{3}{11}$.

c) $\|A \circ X\| = 1 \Rightarrow \sqrt{(1+k)^2 + (2k)^2} = 1 \Rightarrow 1+k^2+2k+4k^2 = 5k^2+2k+1 = 1 \Rightarrow$

$\Rightarrow 5k^2 + 2k = k(5k+2) = 0 < \begin{matrix} k = 0 \\ k = -\frac{2}{5} \end{matrix}$.

9) Equazione retta per $(1;3)$ e $(2;5)$: $m = \frac{5-3}{2-1} = 2 \Rightarrow y - 3 = 2(x-1) \Rightarrow$

$\Rightarrow y = 2x + 1$. Da $f(x) = e^x + k \Rightarrow f'(x) = e^x \Rightarrow$ (per la tangenta): $e^x = 2 \Rightarrow x = \log 2$.

$f(\log 2) = e^{\log 2} + k = y(\log 2) = 2\log 2 + 1 \Rightarrow 2 + k = \log 4 + 1 \Rightarrow k = \log 4 - 1$.

10) $f(x) = \frac{x}{x+2}$; $g(x) = \frac{x+1}{x-1}$.

$f(x) \sim g(x)$: $\lim_{x \to x_0} \frac{f(x)}{g(x)} = 1 \Rightarrow \lim_{x \to x_0} \frac{x}{x+2} \cdot \frac{x-1}{x+1} = \lim_{x \to x_0} \frac{x^2 - x}{x^2+3x+2} = 1$ vera per

$x \to -\infty$; $x \to +\infty$ e se $x^2 - x = x^2 + 3x + 2 \Rightarrow 4x + 2 = 0 \Rightarrow x \to -\frac{1}{2}$.

$f(x) = o(g(x))$: $\lim_{x \to x_0} \frac{f(x)}{g(x)} = 0 \Rightarrow \lim_{x \to x_0} \frac{x^2 - x}{x^2+3x+2} = 0$: vera per $x^2 - x \to 0 \Rightarrow x \to 0$ e $x \to 1$.

$g(x) = o(f(x))$: $\lim_{x \to x_0} \frac{g(x)}{f(x)} = 0 \Rightarrow \lim_{x \to x_0} \frac{x^2+3x+2}{x^2-x} = \lim_{x \to x_0} \frac{(x+1)(x+2)}{x^2-x} = 0$: vera per

$(x+1)(x+2) \to 0 \Rightarrow x \to -1$ e $x \to -2$.