

# Compito di Analisi Matematica del 21/9/2018

CAM1

$$\text{IM1}) z = e^{1+\pi i} + e^{\pi i - i^2} = e^{1+\pi i} + e^{1+\pi i} = 2e^{1+\pi i} = 2e \cdot e^{\pi i} = 2e(\cos \pi + i \sin \pi) = -2e.$$

$$\sqrt[3]{z} = \sqrt[3]{2e} \cdot \left( \cos\left(\frac{\pi}{3} + K \cdot \frac{2\pi}{3}\right) + i \sin\left(\frac{\pi}{3} + K \cdot \frac{2\pi}{3}\right) \right); 0 \leq K \leq 2.$$

$$K=0: \sqrt[3]{2e} \cdot \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) = \sqrt[3]{2e} \cdot \left( \frac{1}{2} + i \frac{\sqrt{3}}{2} \right);$$

$$K=1: \sqrt[3]{2e} \cdot \left( \cos \pi + i \sin \pi \right) = \sqrt[3]{2e} \cdot (-1) = -\sqrt[3]{2e};$$

$$K=2: \sqrt[3]{2e} \cdot \left( \cos \frac{5}{3}\pi + i \sin \frac{5}{3}\pi \right) = \sqrt[3]{2e} \cdot \left( \frac{1}{2} - i \frac{\sqrt{3}}{2} \right).$$

$$\text{IM2}) f(x; y) = x \cdot |x-y|. \quad \mathbb{C}. \mathbb{E}: \mathbb{R}^2: \quad f(0; 0) = 0.$$

$$\lim_{(x,y) \rightarrow (0;0)} x \cdot |x-y| = 0 \cdot |0-0| = 0 : \quad f \in C(0;0).$$

$$\frac{\partial f}{\partial x}(0;0) = \lim_{h \rightarrow 0} \frac{h \cdot |h-0| - 0}{h} = 0; \quad \frac{\partial f}{\partial y}(0;0) = \lim_{h \rightarrow 0} \frac{0 \cdot |0-h| - 0}{h} = 0. \quad \text{Per la differentiabilità:}$$

$$\lim_{(x,y) \rightarrow (0;0)} \frac{x \cdot |x-y| - 0 - (0;0) \cdot (x; y)}{\sqrt{x^2+y^2}} \Rightarrow \lim_{\rho \rightarrow 0} \frac{\rho \cdot \cos \vartheta |\cos \vartheta - \sin \vartheta| - 0}{\rho} =$$

$$= \lim_{\rho \rightarrow 0} \rho \cos \vartheta |\cos \vartheta - \sin \vartheta| = 0 \quad \text{in modo uniforme in quanto } |\rho \cos \vartheta| |\cos \vartheta - \sin \vartheta| < 2\rho.$$

$$\text{IM3}) f(x; y; z) = x e^{x-y^2-z^2} = 2. \quad f(2; 1; -1) = 2 e^{2-1-1} = 2.$$

$$\nabla f(x; y; z) = \left( (x+1) e^{x-y^2-z^2}; -2xy e^{x-y^2-z^2}; -2xz e^{x-y^2-z^2} \right);$$

$$\nabla f(2; 1; -1) = (3; -4; 4). \quad \text{Da } f_z' = 4 \neq 0 \Rightarrow \text{Esiste } F: (x; y) \rightarrow z(x; y).$$

$$\frac{\partial z}{\partial x}(2; 1) = -\frac{3}{4}; \quad \frac{\partial z}{\partial y}(2; 1) = -\frac{4}{4} = 1.$$

$$\text{Equazione piano tangente in } (2; 1): \quad z - (-1) = -\frac{3}{4}(x-2) + 1(y-1) \Rightarrow z = -\frac{3}{4}x + y - \frac{1}{2}.$$

$$\text{IM4}) f(x; y) = x \cdot e^{y^2-x^2}: \quad \text{funzione differenziabile } \quad \forall (x; y) \in \mathbb{R}^2.$$

$$\nabla f(x; y) = ((1-2x^2) e^{y^2-x^2}; 2xy e^{y^2-x^2}). \quad \nabla f(1; 1) = (-1; 2).$$

$$\mathcal{D}_v f(1; 1) = (-1; 2) \cdot \left( \frac{1}{\sqrt{2}}; \frac{1}{\sqrt{2}} \right) = \frac{1}{\sqrt{2}}. \quad H f(x; y) = \begin{pmatrix} (4x^3 - 6x) e^{y^2-x^2} & 2y(1-2x^2) e^{y^2-x^2} \\ 2y(1-2x^2) e^{y^2-x^2} & 2x(1+2y^2) e^{y^2-x^2} \end{pmatrix};$$

$$H(1; 1) = \begin{pmatrix} -2 & -2 \\ -2 & 6 \end{pmatrix}. \quad \mathcal{D}_{v; v}^2 f(1; 1) = \left\| \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \right\| \cdot \left\| \begin{pmatrix} -2 & -2 \\ -2 & 6 \end{pmatrix} \cdot \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \right\| = \left\| \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \right\| \cdot \left\| -\frac{4}{\sqrt{2}} \right\| = 2 - 2 = 0.$$

$$\text{II M1}) \begin{cases} \text{Max/min } f(x,y) = x^2 + y^2 \\ \text{s.v.: } \begin{cases} 1-x-y \leq 0 \\ x^2 + y^2 - 1 \leq 0 \end{cases} \end{cases}$$

Funzioni differentiabili,  $\Sigma$  limitato e chiuso, voci qualiificate.

$$L(x,y) = x^2 + y^2 - \lambda_1(1-x-y) - \lambda_2(x^2 + y^2 - 1)$$

Caso  $\lambda_1 = \lambda_2 = 0$

$$\begin{cases} \lambda'_x = 2x = 0 \\ \lambda'_y = 2y = 0 \\ y \geq x-1 \\ y \leq 1-x^2 \end{cases} \Rightarrow \begin{cases} x=0 \\ y=0 \\ 0 \geq -1 \\ 0 \leq 1 \end{cases} \Rightarrow (0,0) \notin \Sigma.$$

Caso  $\lambda_1 \neq 0; \lambda_2 = 0$

$$\begin{cases} \lambda'_x = 2x + \lambda_1 = 0 \\ \lambda'_y = 2y + \lambda_1 = 0 \\ y = 1-x \\ y \leq 1-x^2 \end{cases} \Rightarrow \begin{cases} \lambda_1 = -2x \\ \lambda_1 = -2y \\ y = 1-x \\ y \leq 1-x^2 \end{cases} \Rightarrow \begin{cases} y = x \\ 2x = 1 \\ \lambda_1 = -2x \\ y \leq 1-x^2 \end{cases} \Rightarrow \begin{cases} x = \frac{1}{2} \\ y = \frac{1}{2} \\ \lambda_1 = -1 \\ \frac{1}{2} \leq \frac{3}{4} : \text{vire} \end{cases} \text{Min?}$$

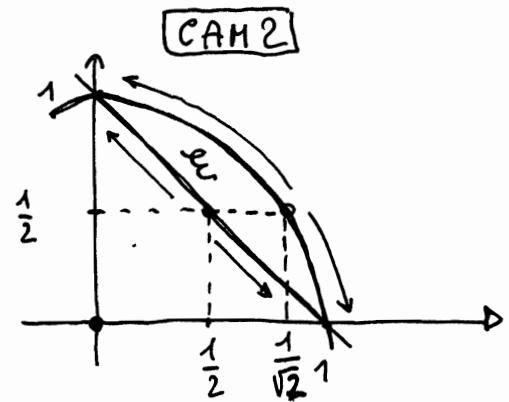
Caso  $\lambda_1 = 0; \lambda_2 \neq 0$

$$\begin{cases} \lambda'_x = 2x - 2\lambda_2 x = 2x(1-\lambda_2) = 0 \\ \lambda'_y = 2y - \lambda_2 = 0 \\ y = 1-x \\ y \geq 1-x \end{cases} \Rightarrow \begin{cases} x=0 \\ y=1 \\ \lambda_2=2 \end{cases} \text{Max?} \cup \begin{cases} \lambda_2=1 \\ y=\frac{1}{2} \\ x^2=\frac{1}{2} \\ y \geq 1-x \end{cases} \Rightarrow \begin{cases} x=\frac{1}{\sqrt{2}} \\ y=\frac{1}{2} \\ \frac{1}{2} \geq 1-\frac{1}{\sqrt{2}} \end{cases} \text{vire Max?}$$

Caso  $\lambda_1 \neq 0; \lambda_2 \neq 0$

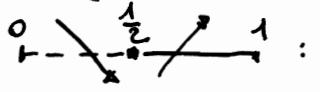
$$\begin{cases} \lambda'_x = 2x + \lambda_1 - 2\lambda_2 x = 0 \\ \lambda'_y = 2y + \lambda_1 - \lambda_2 = 0 \\ y = 1-x \\ y = 1-x^2 \end{cases} \Rightarrow \begin{cases} x=1 \\ y=0 \\ \lambda_1 - 2\lambda_2 = 0 \\ \lambda_1 - \lambda_2 = 0 \end{cases} \Rightarrow \begin{cases} x=1 \\ y=0 \\ \lambda_2 = 2 \\ \lambda_1 = \lambda_2 = 2 \end{cases} \Rightarrow \begin{cases} x=1 \\ y=0 \\ \lambda_1 = 2 \\ \lambda_2 = 2 \end{cases} \text{Max?}$$

$$\cup \begin{cases} x=0 \\ y=1 \\ \lambda_1 = 0 \\ 2 + \lambda_1 - \lambda_2 = 0 \end{cases} \Rightarrow \begin{cases} x=0 \\ y=1 \\ \lambda_1 = 0 \\ \lambda_2 = 2 \end{cases} \text{Max?}$$

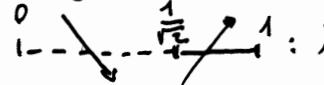


CAM 3

$$\text{Se } y = 1-x \Rightarrow f(x) = 2x^2 - 2x + 1 \Rightarrow f'(x) = 4x - 2 \geq 0 \text{ per } x \geq \frac{1}{2};$$

 : in  $x = \frac{1}{2} \Rightarrow y = \frac{1}{2}$  punto di minimo con  $f\left(\frac{1}{2}; \frac{1}{2}\right) = \frac{1}{2}$ .

$$\text{Se } y = 1-x^2 \Rightarrow f(x) = x^4 - x^2 + 1 \Rightarrow f'(x) = 4x^3 - 2x = 2x(2x^2 - 1) \geq 0 \text{ per } x \geq \frac{1}{\sqrt{2}};$$

 : in  $x = \frac{1}{\sqrt{2}} \Rightarrow y = \frac{1}{2}$  punto di minimo sul bordo ma  $\lambda_2 = 1 > 0$  che indica un possibile massimo per cui in  $(\frac{1}{\sqrt{2}}, \frac{1}{2})$  non c'è nulla.

Quindi Min in  $(\frac{1}{2}, \frac{1}{2})$  e Max in  $(1, 0)$  e  $(0, 1)$ , con  $f(1, 0) = f(0, 1) = 1$ .

$$\text{II M2)} \begin{cases} x' = x+y \\ y' = x-y+e^t \end{cases} \Rightarrow \begin{cases} x' - x - y = 0 \\ -x + y' + y = e^t \end{cases} \Rightarrow \begin{vmatrix} 0 & -1 \\ -1 & 0+1 \end{vmatrix} (x) = \begin{vmatrix} 0 & -1 \\ e^t & 0+1 \end{vmatrix} \Rightarrow (D^2 - 2)(x) = e^t.$$

$$\lambda^2 - 2 = 0 \text{ per } \lambda = \pm \sqrt{2} \Rightarrow x(t) = c_1 e^{\sqrt{2}t} + c_2 e^{-\sqrt{2}t}. \text{ Poi } x'_0(t) = a \cdot e^t \Rightarrow x'_0(t) = x''_0(t) = a e^t \Rightarrow a e^t - 2a e^t = e^t \Rightarrow a = -1 \Rightarrow x(t) = c_1 e^{\sqrt{2}t} + c_2 e^{-\sqrt{2}t} - e^t. \text{ Da } y = x' - x \text{ segue:}$$

$$y(t) = \sqrt{2} c_1 e^{\sqrt{2}t} - \sqrt{2} c_2 e^{-\sqrt{2}t} - e^t - c_1 e^{\sqrt{2}t} - c_2 e^{-\sqrt{2}t} + e^t \Rightarrow y(t) = (\sqrt{2}-1)c_1 e^{\sqrt{2}t} - (\sqrt{2}+1)e^{-\sqrt{2}t}.$$

$$\text{II M3)} \begin{cases} y'' = y \\ y(0) = 1 \\ y'(0) = -1 \end{cases} . y'' - y = 0 \Rightarrow \lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1 \Rightarrow y(x) = c_1 e^x + c_2 e^{-x}.$$

$$\begin{cases} y(0) = 1 \\ y'(0) = 1 \end{cases} \Rightarrow \begin{cases} c_1 + c_2 = 1 \\ c_1 - c_2 = -1 \end{cases} \Rightarrow \begin{cases} c_1 = 0 \\ c_2 = 1 \end{cases} \Rightarrow y(x) = e^{-x}.$$

$$\text{II M4)} \iint_D y \, dx \, dy \text{ con } D = \{(x, y) : x^2 + y^2 \leq 1; (1-x^2) \leq y\}$$

$$\begin{aligned} \int_0^1 \int_{(1-x)^2}^{\sqrt{1-x^2}} y \, dy \, dx &= \int_0^1 \left( \frac{y^2}{2} \Big|_{(1-x)^2}^{\sqrt{1-x^2}} \right) dx = \\ &= \int_0^1 \left( \frac{1-x^2}{2} - \frac{(1-x)^4}{2} \right) dx = \left( \frac{1}{2}x - \frac{x^3}{6} + \frac{(1-x)^5}{10} \right) \Big|_0^1 = \end{aligned}$$

$$= \left( \frac{1}{2} - \frac{1}{6} + 0 \right) - \left( 0 - 0 + \frac{1}{10} \right) = \frac{1}{3} - \frac{1}{10} = \frac{7}{30}.$$

