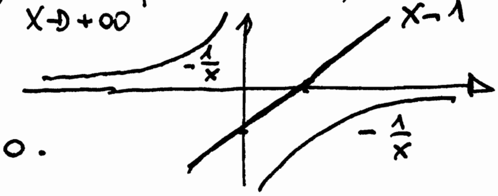
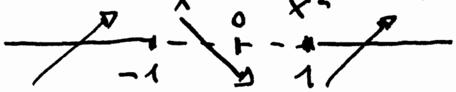
Compito di Matematica Generale del 6/9/2018 $\boxed{CMG1}$

1) $f(x) = x - 1 + \frac{1}{x}$. C.E.: $x \neq 0$. $\lim_{x\to -\infty} f(x) = -\infty$; $\lim_{x\to +\infty} f(x) = +\infty$;
$\lim_{x\to 0^-} f(x) = -\infty$; $\lim_{x\to 0^+} f(x) = +\infty$. $f(x) \geq 0$ per:
$x - 1 \geq -\frac{1}{x}$
vera per $x > 0$.



$f'(x) = 1 - \frac{1}{x^2} = \frac{x^2-1}{x^2} \geq 0$ per $x \leq -1 \cup x \geq 1$



$f''(x) = 2 \cdot \frac{1}{x^3} > 0$ per $x > 0$

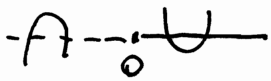
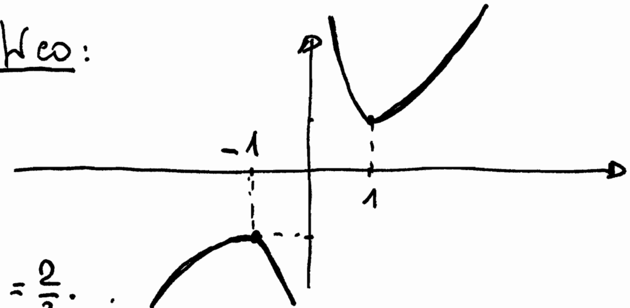


Grafico:



2) $\lim_{x\to 0} \frac{\log(1+2x)}{\log(1+3x)} = \lim_{x\to 0} \frac{\log(1+2x)}{2x} \cdot \frac{3x}{\log(1+3x)} \cdot \frac{2}{3} = 1 \cdot 1 \cdot \frac{2}{3} = \frac{2}{3}$.

$\lim_{x\to +\infty} \frac{e^{x-x^2} - 1}{x} = \left(\frac{e^{\to(-\infty)} - 1}{\to +\infty} : \frac{(\to 0) - 1}{\to +\infty} \right) = 0^-$.

3) $f(x) = x^2 - kx + 1$; $f(1) = 2 - k$; $f'(x) = 2x - k$; $f'(1) = 2 - k \Rightarrow 2 - k = 3 \Rightarrow k = -1$.

$f(x) = x^2 + x + 1$; $f(1) = 3 \Rightarrow y - 3 = 3(x-1) \Rightarrow y = 3x - 3 + 3 \Rightarrow y = 3x$.

4) $e^x = o(x) \Leftrightarrow \lim_{x\to x_0} \frac{e^x}{x} = 0$ vera se e solo se $x \to -\infty$.

5) $f(x) = e^{1-2x^3+9x^2-12x}$; $f'(x) = (-6x^2+18x-12)\cdot e^{1-2x^3+9x^2-12x} =$
$= (-6)(x^2-3x+2)\cdot e^{1-2x^3+9x^2-12x} \geq 0$ per $x^2 - 3x + 2 \leq 0$ vera per $1 \leq x \leq 2$.

Crescente in $1 < x < 2$; Decrescente in $x < 1$ e $x > 2$;
$x = 1$ punto di minimo; $x = 2$ punto di massimo.

6) $\int_0^{\pi} 3\sin x - 2\cos x \, dx = \left| -3\cos x - 2\sin x \right|_0^{\pi} = (-3(-1) - 2\cdot 0) - (-3(1) - 2\cdot 0) = 6$.

7) $A \cdot X = \begin{Vmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ k & 0 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} 1 \\ -1 \\ 1 \end{Vmatrix} = \begin{Vmatrix} 1+0+1 \\ 0-1+1 \\ k+0+1 \end{Vmatrix} = \begin{Vmatrix} 2 \\ 0 \\ k+1 \end{Vmatrix} \Rightarrow A \cdot X = (2; 0; k+1)$.

CMG2

a) $A \circ X \parallel Y_1$: $(2;0;K+1) \parallel (4;0;3) \Rightarrow \frac{2}{4} = \frac{K+1}{3} \Rightarrow 4K+4=6 \Rightarrow$
$\Rightarrow K = \frac{2}{4} = \frac{1}{2} \Rightarrow A \circ X = \left(2;0;\frac{3}{2}\right)$.

b) $A \circ X \perp Y_2$: $(2;0;K+1) \perp (1;2;-1) \Rightarrow (2;0;K+1) \cdot (1;2;-1) = 0 \Rightarrow$
$\Rightarrow 2+0-K-1 = 0 \Rightarrow K=1 \Rightarrow A \circ X = (2;0;2)$.

c) $\|A \circ X\| = \sqrt{5} \Rightarrow \sqrt{4+0+(K+1)^2} = \sqrt{4+K^2+2K+1} = \sqrt{K^2+2K+5} = \sqrt{5} \Rightarrow$
$\Rightarrow K^2+2K+5 = 5 \Rightarrow K^2+2K = K(K+2) = 0 \Rightarrow K=0$ oppure $K=-2 \Rightarrow$
$\Rightarrow A \circ X = (2;0;1)$ oppure $A \circ X = (2;0;-1)$.

8) Dato che $(A \Rightarrow B) \Leftrightarrow$ (non $B \Rightarrow$ non A) avremo che:

$(A \Rightarrow$ non $B) \Leftrightarrow$ (non (non B) $\Rightarrow$ non A) $\Leftrightarrow$ ($B \Rightarrow$ non A).

Quindi $\square$ va sostituito con ($\Rightarrow$).

9) $f(x;y) = \log x + \log y - x - y$. C.E.: $\{(x;y) \in \mathbb{R}^2 : x>0; y>0\}$.

$$\begin{cases} f'_x = \frac{1}{x} - 1 = 0 \\ f'_y = \frac{1}{y} - 1 = 0 \end{cases} \Rightarrow \begin{cases} x = 1 \\ y = 1 \end{cases}. \quad H(x;y) = \begin{Vmatrix} -\frac{1}{x^2} & 0 \\ 0 & -\frac{1}{y^2} \end{Vmatrix} \Rightarrow H(1;1) = \begin{Vmatrix} -1 & 0 \\ 0 & -1 \end{Vmatrix} : \begin{cases} f''_{xx} < 0;\ f''_{yy} < 0 \\ |H_2| = 1 - 0 > 0 \end{cases}$$

quindi $(1;1)$ è un punto di Massimo.

10) $f(x) = e^{3x} - 1$; $g(x) = 1 - 2x$; $h(x) = 1 - x^2$.

$F(x) = f(g(h(x))) = f(g(1-x^2)) = f(1-2(1-x^2)) = f(1-2+2x^2) = f(2x^2-1) =$
$= e^{3(2x^2-1)} - 1 = e^{6x^2-3} - 1$.

$F'(x) = (12x)\, e^{6x^2-3}$.