Compito di Matematica Generale del 21/9/2018 $\boxed{\text{CMG1}}$

1) $f(x)=x^2-2\log x$. C.E.: $x>0$. $\lim_{x\to 0^+} f(x)=+\infty$; $\lim_{x\to+\infty} f(x)=+\infty$.

$f'(x)=2x-\frac{2}{x}=2\left(\frac{x^2-1}{x}\right)\geq 0$ per $x^2-1\geq 0\Rightarrow$

$\Rightarrow x\leq -1 \cup x\geq 1$

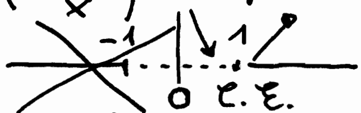
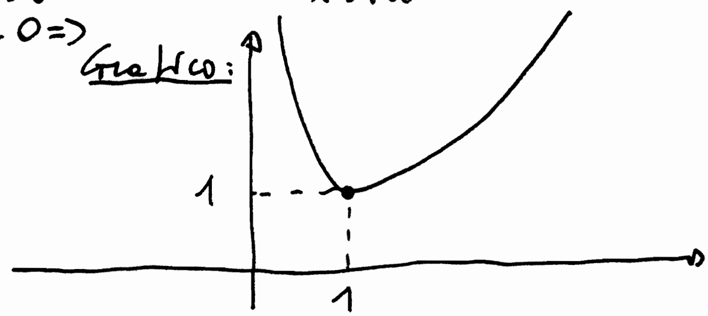
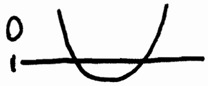


Grafico:



$x=1$ punto di minimo; $f(1)=1>0$
quindi $f(x)>0\ \forall x\in$ C.E.

$f''(x)=2+\frac{2}{x^2}>0\ \forall x$:



2) $\lim_{x\to 0}\frac{1-\cos 2x}{1-\cos x}=\lim_{x\to 0}\frac{1-\cos 2x}{4x^2}\cdot 4\cdot\frac{x^2}{1-\cos x}=\frac{1}{2}\cdot 4\cdot 2=4$.

$\lim_{x\to+\infty}\frac{x-3^{1-x}}{x+\operatorname{sen}x}=\lim_{x\to+\infty}\frac{x}{x}=1$. $\left(3^{1-x}\to 0;\ \operatorname{sen}x=o(x)\right)$.

3) $f(x)=\frac{1}{1-\log x}$. C.E.: $\begin{cases}x>0\\1-\log x\neq 0\end{cases}\Rightarrow\begin{cases}x>0\\\log x=1\end{cases}\Rightarrow\begin{cases}x>0\\x\neq e\end{cases}$; C.E.: $]0;e[\ \cup\ ]e;+\infty[$.

$\lim_{x\to 0^+} f(x)=\left(\frac{1}{1-(\to-\infty)}\right)=0^+$: Discontinuità di III specie solo da destra.

$\lim_{x\to e^-} f(x)=\left(\frac{1}{\to 0^+}\right)=+\infty$; $\lim_{x\to e^+} f(x)=\left(\frac{1}{\to 0^-}\right)=-\infty$: Discontinuità di II specie.

4) $f(x)=3x^3-9x^2+3x+5$; $f'(x)=9x^2-18x+3=3\Rightarrow 9x(x-2)=0\Rightarrow x=0$ ó $x=2$.

Se $x=0$: $f(0)=5$; $f'(0)=3\Rightarrow$ Eq. retta tangente: $y-5=3(x-0)\Rightarrow y=3x+5$.

Se $x=2$: $f(2)=-1$; $f'(2)=3\Rightarrow$ Eq. retta tangente: $y+1=3(x-2)\Rightarrow y=3x-7$.

La retta $y=3x+5$ è tangente al grafico della funzione in $x=0$.

5) $\int_0^1 e^x-e^{1-x}\,dx=\left(e^x+e^{1-x}\right|_0^1=(e+1)-(1+e)=0$.

6) $f(x)=\log(1-3x)$; $g(x)=2x-3$. $F(x)=f(g(x))=f(2x-3)=\log(1-3(2x-3))=\log(10-6x)$.

C.E.: $10-6x>0\Rightarrow 6x<10\Rightarrow x<\frac{5}{3}$: C.E.: $]-\infty;\frac{5}{3}[$.

CMG2

$F'(x) = (-6)\cdot\frac{1}{10-6x} < 0 \;\forall x \in C.E.\,.\; \lim_{x\to-\infty} F(x) = +\infty \,;\; \lim_{x\to \frac{5}{3}^-} F(x) = -\infty.$

$F(x): \left]-\infty ; \frac{5}{3}\right[\to \mathbb{R} \,;\; F^{-1}(x): \mathbb{R} \to \left]-\infty ; \frac{5}{3}\right[.$

$\log(10-6x) = y \Rightarrow 10-6x = e^y \Rightarrow 6x = 10 - e^y \Rightarrow x = \frac{1}{6}(10-e^y) : F^{-1}(x) = \frac{5}{3} - \frac{1}{6}e^x.$

7) $f(x) = e^{3x} - \cos 2x \,.\; e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} \Rightarrow e^{3x} = 1 + 3x + \frac{9}{2}x^2 + \frac{9}{2}x^3 + o(x^3).$

$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24} \Rightarrow \cos 2x = 1 - 2x^2 + \frac{2}{3}x^4 .\; f(x) = \left(1+3x+\frac{9}{2}x^2+\frac{9}{2}x^3\right) - \left(1 - 2x^2\right) \Rightarrow$

$\Rightarrow f(x) = 3x + \frac{13}{2}x^2 + \frac{9}{2}x^3 + o(x^3).$

8) $A\cdot B\cdot X = \begin{Vmatrix} 1 & 2 & 3 \\ 0 & 1 & k \end{Vmatrix} \cdot \begin{Vmatrix} 1 & k \\ k & 1 \\ 0 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} 1 \\ 1 \end{Vmatrix} = \begin{Vmatrix} 1+2k+0 & k+2+3 \\ 0+k+0 & 0+1+k \end{Vmatrix} \cdot \begin{Vmatrix} 1 \\ 1 \end{Vmatrix} = \begin{Vmatrix} 1+2k & k+5 \\ k & 1+k \end{Vmatrix} \cdot \begin{Vmatrix} 1 \\ 1 \end{Vmatrix} =$

$= \begin{Vmatrix} 1+2k+k+5 \\ k+1+k \end{Vmatrix} = \begin{Vmatrix} 3k+6 \\ 2k+1 \end{Vmatrix} ;\; (A\cdot B\cdot X) \perp Y : (3k+6 ; 2k+1)\cdot(1;-1) = 3k+6-2k-1 = k+5 = 0 \Rightarrow k = -5.$

9)

A B C	(B e C)	(A J (B e C))	P_1	(A o B)	non (A δ B)	(A o C)	non (A δ C)	P_2
1 1 1	1	1	0	1	0	1	0	0
1 1 0	0	1	0	1	0	1	0	0
1 0 1	0	1	0	1	0	1	0	0
1 0 0	0	1	0	1	0	1	0	0
0 1 1	1	1	0	1	0	1	0	0
0 1 0	0	0	1	1	0	0	1	1
0 0 1	0	0	1	0	1	1	0	1
0 0 0	0	0	1	0	1	0	1	1

Le due proposizioni SONO logicamente equivalenti.

10) $f(x;y) = x^3 - y^2 - 3x + 6y .\; \nabla f(x;y) = (0;0).$

$\begin{cases} f'_x = 3x^2 - 3 = 3(x^2-1) = 0 \\ f'_y = 6 - 2y = 2(3-y) = 0 \end{cases} \Rightarrow \begin{cases} x = \pm 1 \\ y = 3 \end{cases} \Rightarrow P_1 = (1;3) ;\; P_2 = (-1;3).\; H(x;y) = \begin{Vmatrix} 6x & 0 \\ 0 & -2 \end{Vmatrix}.$

$H(1;3) = \begin{Vmatrix} 6 & 0 \\ 0 & -2 \end{Vmatrix} : \begin{cases} 6 > 0 \\ -2 < 0 \end{cases} \Rightarrow$ Punto di Sella;

$H(-1;3) = \begin{Vmatrix} -6 & 0 \\ 0 & -2 \end{Vmatrix} : \begin{cases} -6 < 0 ;\; -2 < 0 \\ 12 - 0 > 0 \end{cases} \Rightarrow$ Punto di Massimo.