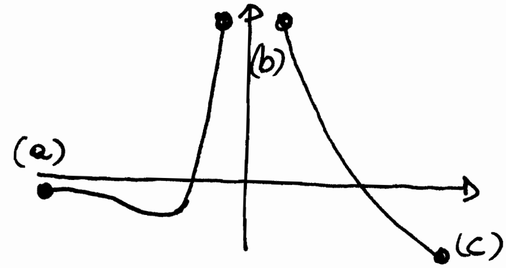
Prova Intermedia di Matematica Generale del 5/11/2018 $\boxed{MGA1}$

1) $\lim\limits_{x\to-\infty} f(x)=0^- : \forall\varepsilon>0\ \exists\,\delta(\varepsilon): x<\delta(\varepsilon) \Rightarrow -\varepsilon<f(x)<0$

$\lim\limits_{x\to 0} f(x)=+\infty : \forall\varepsilon\ \exists\,\delta(\varepsilon): 0<|x|<\delta(\varepsilon) \Rightarrow f(x)>\varepsilon$

$\lim\limits_{x\to+\infty} f(x)=-\infty : \forall\varepsilon\ \exists\,\delta(\varepsilon): x>\delta(\varepsilon) \Rightarrow f(x)<\varepsilon$



2) $$\lim_{x\to 0}\frac{\log(1+\operatorname{sen}^2x)}{3^{x^2}-\cos 3x}=\lim_{x\to 0}\frac{\log(1+\operatorname{sen}^2x)}{\operatorname{sen}^2x}\cdot\frac{\operatorname{sen}^2x}{x^2}\cdot\frac{1}{\frac{3^{x^2}-1}{x^2}+\frac{1-\cos 3x}{9x^2}\cdot 9}=1\cdot 1\cdot\frac{1}{\log 3+\frac{9}{2}}=\frac{2}{\log 9+9}.$$

$$\lim_{x\to+\infty}\left(\frac{5+x^2}{3+x^2}\right)^{1+2x^2}=\lim_{x\to+\infty}\left(\frac{3+x^2}{3+x^2}+\frac{2}{3+x^2}\right)^{1+2x^2}=\lim_{x\to+\infty}\left[\left(1+\frac{2}{3+x^2}\right)^{3+x^2}\right]^{\frac{1+2x^2}{3+x^2}}=(e^2)^2=e^4.$$

3) $$f(x)=\frac{x-1}{x+2}\Rightarrow f(g(x))=\frac{g(x)-1}{g(x)+2}=3x-1\Rightarrow g(x)-1=3xg(x)-g(x)+6x-2\Rightarrow g(x)=\frac{6x-1}{2-3x}\Rightarrow$$

$$g(\log_2 x)=\frac{6\log_2 x-1}{2-3\log_2 x}=y\Rightarrow 6\log_2 x-1=2y-3y\log_2 x\Rightarrow\log_2 x=\frac{2y+1}{3y+6}\Rightarrow x=2^{\frac{2y+1}{3y+6}}\Rightarrow$$

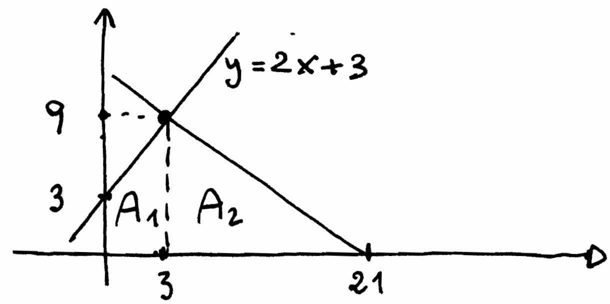
$$\Rightarrow F^{-1}(x)=2^{\frac{2x+1}{3x+6}}$$

4) Da $y=2x+3$ per $x=3$ si ha $y=9$.

Equazione perpendicolare: $y-9=-\frac{1}{2}(x-3)\Rightarrow$

$\Rightarrow y=-\frac{1}{2}x+\frac{21}{2}=0$ per $x=21$.

Area $=A_1+A_2=\frac{1}{2}\cdot 3\cdot(3+9)+\frac{1}{2}\cdot 18\cdot 9=18+81=99$.



5)

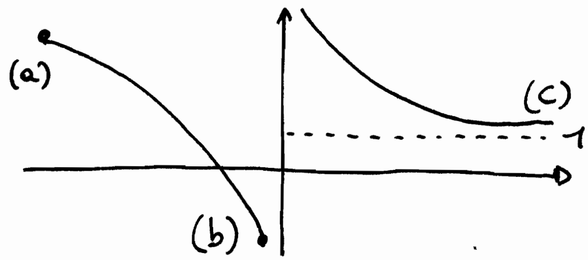
A	B	$P_1: (A \Leftrightarrow B)$	$P_2: (A$ o non $B)$	$P: P_1 \Rightarrow P_2$	non P
1	1	1	1	1	0
1	0	0	1	1	0
0	1	0	0	1	0
0	0	1	1	1	0

Prova Intermedia di Matematica Generale del 5/11/2018 $\boxed{\text{MGB1}}$

1) $\lim_{x\to-\infty} f(x)=+\infty : \forall\varepsilon\ \exists\delta(\varepsilon): x<\delta(\varepsilon)\Rightarrow f(x)>\varepsilon$

$\lim_{x\to0^-} f(x)=-\infty : \forall\varepsilon\ \exists\delta(\varepsilon): -\delta(\varepsilon)<x<0\Rightarrow f(x)<\varepsilon$

$\lim_{x\to+\infty} f(x)=1^+ : \forall\varepsilon>0\ \exists\delta(\varepsilon): x>\delta(\varepsilon)\Rightarrow 1<f(x)<1+\varepsilon$



2) $$\lim_{x\to0}\frac{\sqrt[3]{1+x^2}-\cos 2x}{\operatorname{arctg}^2 x}=\lim_{x\to0}\left[\frac{(1+x^2)^{\frac13}-1}{x^2}+\frac{1-\cos 2x}{4x^2}\cdot 4\right]\cdot\frac{x^2}{\operatorname{arctg}^2 x}=\left(\frac13+\frac12\cdot4\right)\cdot1=\frac73.$$

$$\lim_{x\to+\infty}\left(\frac{3+x^2}{2+x^2}\right)^{1+3x^2}=\lim_{x\to+\infty}\left(\frac{2+x^2}{2+x^2}+\frac{1}{2+x^2}\right)^{1+3x^2}=\lim_{x\to+\infty}\left[\left(1+\frac{1}{2+x^2}\right)^{2+x^2}\right]^{\frac{1+3x^2}{2+x^2}}=e^3.$$

3) Se $f(x)=\frac{x+3}{x-2}\Rightarrow f(g(x))=\frac{g(x)+3}{g(x)-2}=2x+1\Rightarrow g(x)+3=2x\,g(x)+g(x)-4x-2\Rightarrow g(x)=\frac{4x+5}{2x}\Rightarrow$

$g(2^x)=\frac{4\cdot2^x+5}{2\cdot2^x}=y\Rightarrow 4\cdot2^x+5=2y\cdot2^x\Rightarrow 2^x=\frac{5}{2y-4}\Rightarrow x=\log_2\frac{5}{2y-4}\Rightarrow F^{-1}(x)=\log_2\frac{5}{2x-4}.$

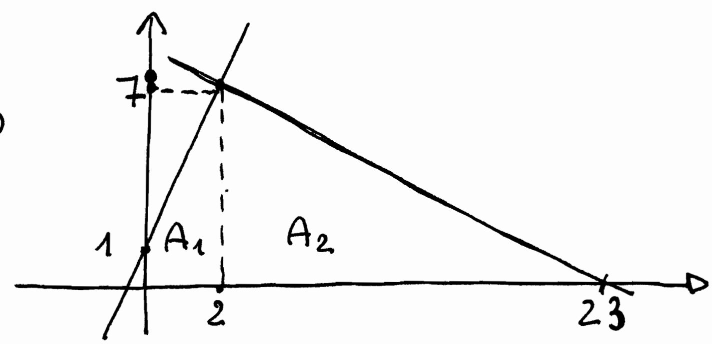
4) Da $y=3x+1$ per $x=2$ si ha $y=7$.

Equazione perpendicolare: $y-7=-\frac13(x-2)\Rightarrow$

$\Rightarrow y=-\frac13x+\frac{23}{3}=0$ per $x=23$.

Area $=A_1+A_2=\frac12\cdot2\cdot(1+7)+\frac12\cdot7\cdot21=$

$=8+\frac{147}{2}=\frac{16+147}{2}=\frac{163}{2}.$



5)

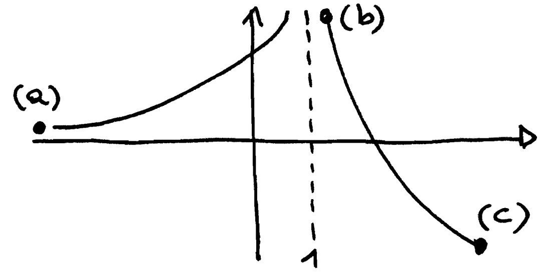
A	B	$P_1:(A\text{ e }B)$	$P_2:(A\Leftrightarrow \text{non}B)$	$P: P_2\Rightarrow P_1$	non P
1	1	1	0	1	0
1	0	0	1	0	1
0	1	0	1	0	1
0	0	0	0	1	0

Prova Intermedia di Matematica Generale del 5/11/2018 $\boxed{\text{MGC1}}$

1) $\lim_{x\to-\infty} f(x)=0:\ \forall\varepsilon>0\ \exists\delta(\varepsilon): x<\delta(\varepsilon)\Rightarrow|f(x)|<\varepsilon$

$\lim_{x\to1^+} f(x)=+\infty:\ \forall\varepsilon\ \exists\delta(\varepsilon): 1<x<1+\delta(\varepsilon)\Rightarrow f(x)>\varepsilon$

$\lim_{x\to+\infty} f(x)=-\infty:\ \forall\varepsilon\ \exists\delta(\varepsilon): x>\delta(\varepsilon)\Rightarrow f(x)<\varepsilon$



2) $$\lim_{x\to0}\frac{\log(1+\arctg^2x)}{2^{x^2}-\cos2x}=\lim_{x\to0}\frac{\log(1+\arctg^2x)}{\arctg^2x}\cdot\frac{\arctg^2x}{x^2}\cdot\frac{1}{\frac{2^{x^2}-1}{x^2}+\frac{1-\cos2x}{4x^2}\cdot4}=1\cdot1\cdot\frac{1}{\log2+\frac{4}{2}}=\frac{1}{\log2+2}.$$

$$\lim_{x\to+\infty}\left(\frac{4+x^2}{3+x^2}\right)^{1+4x^2}=\lim_{x\to+\infty}\left(\frac{3+x^2}{3+x^2}+\frac{1}{3+x^2}\right)^{1+4x^2}=\lim_{x\to+\infty}\left[\left(1+\frac{1}{3+x^2}\right)^{3+x^2}\right]^{\frac{1+4x^2}{3+x^2}}=e^4.$$

3) Se $f(x)=\frac{x+3}{x-1}\Rightarrow f(g(x))=\frac{g(x)+3}{g(x)-1}=2x-3\Rightarrow g(x)+3=2x\,g(x)-3g(x)-2x+3\Rightarrow g(x)=\frac{2x}{2x-4}=\frac{x}{x-2}\Rightarrow$

$g(\log_3x)=\frac{\log_3x}{\log_3x-2}=y\Rightarrow\log_3x=y\log_3x-2y\Rightarrow\log_3x=\frac{2y}{y-1}\Rightarrow x=3^{\frac{2y}{y-1}}\Rightarrow$

$\Rightarrow F^{-1}(x)=3^{\frac{2x}{x-1}}.$

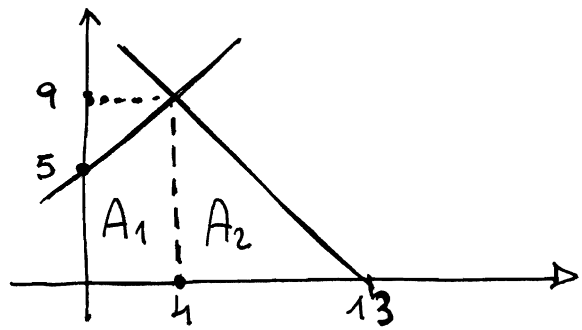
4) Da $y=x+5$ per $x=4$ si ha $y=9$.

Equazione perpendicolare: $y-9=-1(x-4)\Rightarrow$

$\Rightarrow y=-x+13=0$ per $x=13$.

Area $=A_1+A_2=\frac{1}{2}\cdot4\cdot(5+9)+\frac{1}{2}\cdot9\cdot9=$

$=2\cdot14+\frac{81}{2}=\frac{56+81}{2}=\frac{137}{2}.$



5)

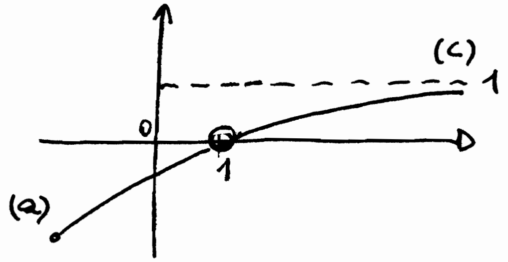
A	B	$P_1: (A \text{ o } B)$	$P_2: (A\Rightarrow \text{non } B)$	$P: P_1\Rightarrow P_2$	non P
1	1	1	0	0	1
1	0	1	1	1	0
0	1	1	1	1	0
0	0	0	1	1	0

Prova Intermedia di Matematica Generale del 5/11/2018 $\boxed{\text{MGD1}}$

1) $\lim\limits_{x\to-\infty} f(x)=-\infty : \forall\varepsilon\ \exists\delta(\varepsilon): x<\delta(\varepsilon) \Rightarrow f(x)<\varepsilon$

$\lim\limits_{x\to 1} f(x)=0 : \forall\varepsilon>0\ \exists\delta(\varepsilon): 0<|x-1|<\delta(\varepsilon) \Rightarrow |f(x)|<\varepsilon$

$\lim\limits_{x\to+\infty} f(x)=1^- : \forall\varepsilon>0\ \exists\delta(\varepsilon): x>\delta(\varepsilon) \Rightarrow 1-\varepsilon<f(x)<1$



2) $$\lim_{x\to0}\frac{\sqrt[4]{1+x^2}-\cos 3x}{\arcsin^2 x}=\lim_{x\to0}\left(\frac{(1+x^2)^{\frac14}-1}{x^2}+\frac{1-\cos 3x}{9x^2}\cdot 9\right)\cdot\frac{x^2}{\arcsin^2 x}=\left(\frac14+\frac12\cdot 9\right)\cdot 1=\frac{19}{4}.$$

$$\lim_{x\to+\infty}\left(\frac{4+x^2}{2+x^2}\right)^{1+5x^2}=\lim_{x\to+\infty}\left(\frac{2+x^2}{2+x^2}+\frac{2}{2+x^2}\right)^{1+5x^2}=\lim_{x\to+\infty}\left[\left(1+\frac{2}{2+x^2}\right)^{2+x^2}\right]^{\frac{1+5x^2}{2+x^2}}=\left(e^2\right)^5=e^{10}.$$

3) Se $f(x)=\frac{x-1}{x+3}\Rightarrow f(g(x))=\frac{g(x)-1}{g(x)+3}=3x-2\Rightarrow g(x)-1=3xg(x)-2g(x)+9x-6\Rightarrow g(x)=\frac{5-9x}{3x-3}\Rightarrow$

$g(3^x)=\frac{5-9\cdot3^x}{3\cdot3^x-3}=y\Rightarrow 5-9\cdot3^x=3y\cdot3^x-3y\Rightarrow 3^x=\frac{3y+5}{3y+9}\Rightarrow x=\log_3\left(\frac{3y+5}{3y+9}\right)\Rightarrow F^{-1}(x)=\log_3\left(\frac{3x+5}{3x+9}\right).$

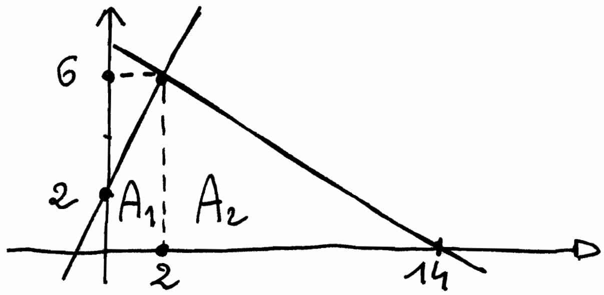
4) Da $y=2x+2$ per $x=2$ si ha $y=6$.

Equazione perpendicolare: $y-6=-\frac12(x-2)\Rightarrow$

$\Rightarrow y=-\frac12 x+7=0$ per $x=14$.

Area $=A_1+A_2=\frac12\cdot2\cdot(2+6)+\frac12\cdot6\cdot12=$

$=8+36=44.$



5)

A	B	$P_1: (A\Rightarrow B)$	$P_2: (\text{non}\, A \text{ e } B)$	$P: P_2\Rightarrow P_1$	non P
1	1	1	0	1	0
1	0	0	0	1	0
0	1	1	1	1	0
0	0	1	0	1	0