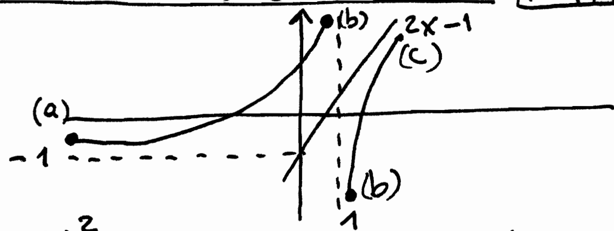
Prova Intermedia di Matematica Generale del 5/11/2018 MGA2

1) Se $x \to -\infty$ $\lim f(x) = -1^{+}$
Discontinuità II specie in $x=1$
A destra asintoto obliquo $y = 2x-1$



2) $\lim\limits_{x\to 0} \dfrac{(1+\operatorname{tg}x^2)^2-\cos x}{3x^2} = \dfrac{1}{3}\left(\lim\limits_{x\to 0} \dfrac{(1+\operatorname{tg}x^2)^2-1}{\operatorname{tg}x^2}\cdot\dfrac{\operatorname{tg}x^2}{x^2} + \dfrac{1-\cos x}{x^2}\right) = \dfrac{1}{3}\left(2\cdot 1 + \dfrac{1}{2}\right) = \dfrac{5}{6}.$

$\lim\limits_{x\to +\infty}\left(\dfrac{1+x^2}{1+2x^2}\right)^{\frac{x-1}{1-2x}} = \left(\to \dfrac{1}{2}\right)^{\left(\to -\frac{1}{2}\right)} = 2^{\frac{1}{2}} = \sqrt{2}.$

3) $f^{-1}(x) = 3^{x-1} = y \Rightarrow x-1 = \log_3 y \Rightarrow x = \log_3 y + 1 \Rightarrow f(x) = \log_3 x + 1.$

$g^{-1}(x) = \dfrac{1-x}{x+1} = y \Rightarrow 1-x = xy+y \Rightarrow x(y+1) = 1-y \Rightarrow x = \dfrac{1-y}{y+1} \Rightarrow g(x) = \dfrac{1-x}{x+1} = g^{-1}(x).$

$F(x) = g(f(x)) = g(\log_3 x + 1) = \dfrac{1-\log_3 x - 1}{\log_3 x + 1 + 1} = -\dfrac{\log_3 x}{\log_3 x + 2} = y \Rightarrow -\log_3 x = y\cdot\log_3 x + 2y \Rightarrow$

$\Rightarrow \log_3 x = \dfrac{-2y}{y+1} \Rightarrow x = 3^{-\frac{2y}{y+1}} \Rightarrow F^{-1}(x) = 3^{-\frac{2x}{x+1}}.$

4)

A	B	$P_1: (A \Rightarrow B)$	$P_2:$ (non A e B)	$P_3:$ (A o B)	$P: [P_1 \Leftrightarrow (P_2 \Rightarrow P_3)]$		
1	1	1	0	1	1	1	1
1	0	0	0	1	0	0	1
0	1	1	1	1	1	1	1
0	0	1	0	0	1	1	1
						*	

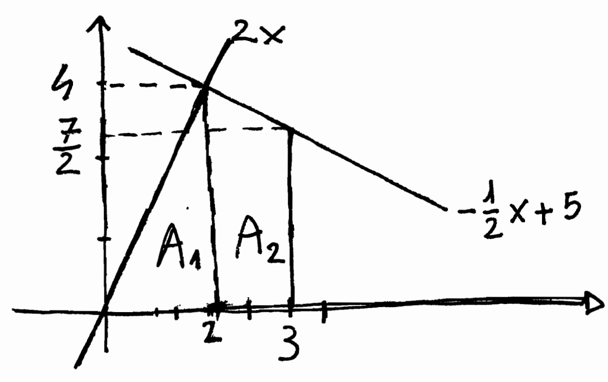
5) Se $y = 2x$ per $x=2$ si ha $y=4$.

Equazione perpendicolare: $y-4 = -\frac{1}{2}(x-2) \Rightarrow$

$\Rightarrow y = -\frac{1}{2}x + 5$. Se $x=3$ si ha $y = \frac{7}{2}$.

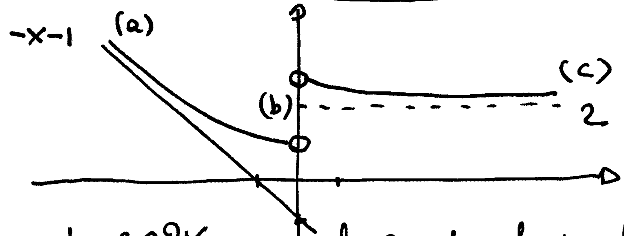
Area $= A_1 + A_2 = \frac{1}{2}\cdot 2\cdot 4 + \frac{1}{2}\cdot 1\cdot\left(4+\frac{7}{2}\right) =$

$= 4 + \frac{1}{2}\cdot\frac{15}{2} = 4 + \frac{15}{4} = \frac{31}{4}.$



Prova Intermedia di Matematica Generale del 5/11/2018 $\boxed{\text{MGB2}}$

1) A sinistra asintoto obliquo da sopra $y=-x-1$
Discontinuità I specie in $x=0$
Sulla destra asintoto orizzontale $y=2$



2) $$\lim_{x\to 0}\frac{3^{\sin^2 x}-\cos 2x}{x^2}=\lim_{x\to 0}\frac{3^{\sin^2 x}-1}{\sin^2 x}\cdot\frac{\sin^2 x}{x^2}+\frac{1-\cos 2x}{4x^2}\cdot 4=\log 3\cdot 1+\frac{1}{2}\cdot 4=\log 3+2.$$

$$\lim_{x\to+\infty}\left(\frac{3+2x^2}{1+x^2}\right)^{\frac{x^2}{1-2x}}=(\to 2)^{(\to -\infty)}=0^+.$$

3) $$f^{-1}(x)=\log_3(x-1)=y\Rightarrow x-1=3^y\Rightarrow x=3^y+1\Rightarrow f(x)=3^x+1.$$

$$g^{-1}(x)=\frac{x+2}{x-1}=y\Rightarrow x+2=xy-y\Rightarrow x(y-1)=y+2\Rightarrow x=\frac{y+2}{y-1}\Rightarrow g(x)=\frac{x+2}{x-1}=g^{-1}(x).$$

$$F(x)=f(g(x))=f\left(\frac{x+2}{x-1}\right)=3^{\frac{x+2}{x-1}}+1=y\Rightarrow 3^{\frac{x+2}{x-1}}=y-1\Rightarrow\frac{x+2}{x-1}=\log_3(y-1)\Rightarrow$$

$$\Rightarrow x+2=x\cdot\log_3(y-1)-\log_3(y-1)\Rightarrow x\left(\log_3(y-1)-1\right)=\log_3(y-1)+2\Rightarrow x=\frac{\log_3(y-1)+2}{\log_3(y-1)-1}\Rightarrow$$

$$\Rightarrow F^{-1}(x)=\frac{\log_3(x-1)+2}{\log_3(x-1)-1}=g^{-1}(f^{-1}(x)).$$

4)

A	B	$P_1:(B\Leftrightarrow A)$	$P_2:(A \text{ e } B)$	$P_3:(A \text{ o non } B)$	$P:[(P_1 \text{ e } P_2)$	$\Rightarrow$	$P_3]$
1	1	1	1	1	1	1	1
1	0	0	0	1	0	1	1
0	1	0	0	0	0	1	0
0	0	1	0	1	0	1	1

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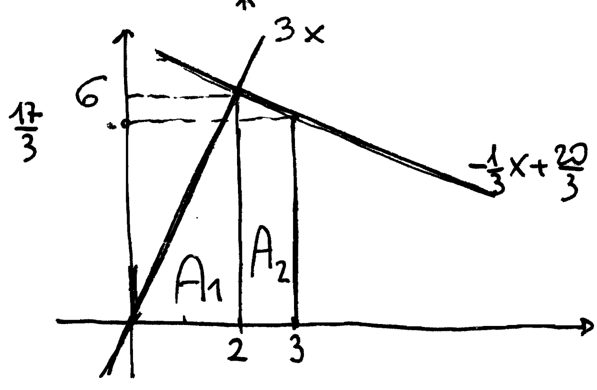
5) Se $y=3x$ per $x=2$ si ha $y=6$.

Equazione perpendicolare: $y-6=-\frac{1}{3}(x-2)\Rightarrow$

$\Rightarrow y=-\frac{1}{3}x+\frac{20}{3}$. Se $x=3$ si ha $y=\frac{17}{3}$

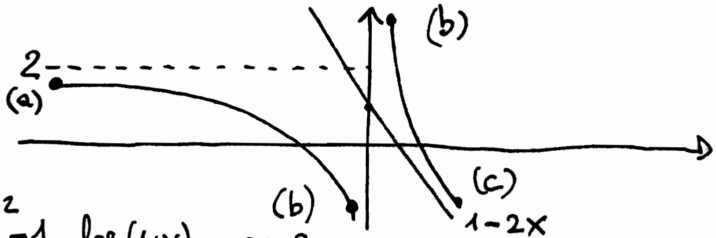


$$\text{Area}=A_1+A_2=\frac{1}{2}\cdot 2\cdot 6+\frac{1}{2}\cdot 1\cdot\left(6+\frac{17}{3}\right)=6+\frac{35}{6}=$$

$$=\frac{71}{6}.$$

Prova Intermedia di Matematica Generale del 5/11/2018 $\boxed{MGC2}$

1) A sinistra asintoto orizzontale da sotto $y=2$
Asintoto verticale in $x=0$
A destra asintoto obliquo $y=1-2x$



2) $$\lim_{x\to 0}\frac{[1+\log(1+x)]^2-\operatorname{sen}3x-1}{x}=\lim_{x\to 0}\frac{[1+\log(1+x)]^2-1}{\log(1+x)}\cdot\frac{\log(1+x)}{x}-\frac{\operatorname{sen}3x}{3x}\cdot 3=2\cdot 1-3=-1.$$

$$\lim_{x\to+\infty}\left(\frac{1+3x^2}{1+2x^2}\right)^{\frac{x^2-1}{2x}}=\left(\to\frac{3}{2}\right)^{(\to+\infty)}=+\infty.$$

3) $$f^{-1}(x)=\frac{3-x}{2-x}=y\Rightarrow 3-x=2y-xy\Rightarrow x(y-1)=2y-3\Rightarrow x=\frac{2y-3}{y-1}\Rightarrow f(x)=\frac{2x-3}{x-1}.$$

$$g^{-1}(x)=2^{x+1}=y\Rightarrow x+1=\log_2 y\Rightarrow x=\log_2 y-1\Rightarrow g(x)=\log_2 x-1.$$

$$F(x)=g(f(x))=g\left(\frac{2x-3}{x-1}\right)=\log_2\frac{2x-3}{x-1}-1=y\Rightarrow\log_2\frac{2x-3}{x-1}=y+1\Rightarrow\frac{2x-3}{x-1}=2^{y+1}\Rightarrow$$

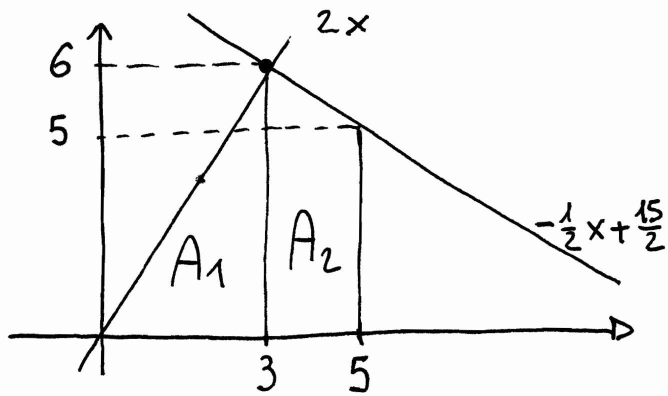
$$2x-3=x2^{y+1}-2^{y+1}\Rightarrow x(2^{y+1}-2)=2^{y+1}-3\Rightarrow x=\frac{2^{y+1}-3}{2^{y+1}-2}\Rightarrow F^{-1}(x)=\frac{2^{x+1}-3}{2^{x+1}-2}=f^{-1}(g^{-1}(x)).$$

4)

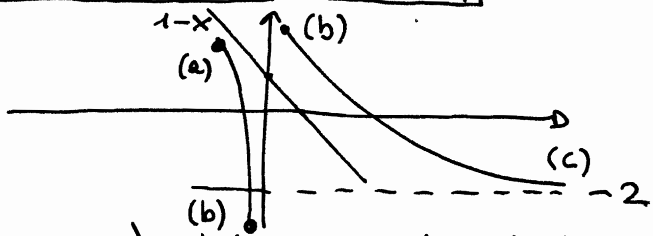
A	B	$P_1:(A \text{ o } B)$	$P_2:(A\Rightarrow B)$	$P_3:(\text{non} A \text{ e } B)$	$P:[(P_2\Rightarrow P_3)$	$\Rightarrow$	$P_1]$
1	1	1	1	0	0	1	1
1	0	1	0	0	1	1	1
0	1	1	1	1	1	1	1
0	0	0	1	0	0	1	0

*

5) Se $y=2x$ per $x=3$ si ha $y=6$.
Equazione perpendicolare: $y-6=-\frac{1}{2}(x-3)\Rightarrow$
$\Rightarrow y=-\frac{1}{2}x+\frac{15}{2}$. Se $x=5$ si ha $y=5$.
Area $=A_1+A_2=\frac{1}{2}\cdot 3\cdot 6+\frac{1}{2}\cdot 2\cdot(6+5)=$
$=9+11=20.$



Prova Intermedia di Matematica Generale del 5/11/2018 $\boxed{\text{MGD2}}$



1) A sinistra asintoto obliquo da sotto $y = 1-x$
Discontinuità di II specie in $x=0$
A destra asintoto orizzontale $y=-2$

2) $$\lim_{x\to 0}\frac{\cos 3x-\cos 2x}{5x^2}=\frac{1}{5}\left(\lim_{x\to 0}\frac{1-\cos 2x}{4x^2}\cdot 4-\frac{1-\cos 3x}{9x^2}\cdot 9\right)=\frac{1}{5}\left(\frac{1}{2}\cdot 4-\frac{1}{2}\cdot 9\right)=\frac{1}{5}\left(-\frac{5}{2}\right)=-\frac{1}{2}.$$

$$\lim_{x\to+\infty}\left(\frac{3+2x^2}{1+3x^2}\right)^{\frac{x}{1-2x}}=\left(\to\frac{2}{3}\right)^{\left(\to-\frac{1}{2}\right)}=\left(\frac{3}{2}\right)^{\frac{1}{2}}=\sqrt{\frac{3}{2}}.$$

3) $$f^{-1}(x)=\frac{2-x}{x-1}=y\Rightarrow 2-x=xy-y\Rightarrow x(y+1)=y+2\Rightarrow x=\frac{y+2}{y+1}\Rightarrow f(x)=\frac{x+2}{x+1}.$$

$$g^{-1}(x)=\log_2(x+2)=y\Rightarrow x+2=2^y\Rightarrow x=2^y-2\Rightarrow g(x)=2^x-2.$$

$$F(x)=f(g(x))=f(2^x-2)=\frac{2^x-2+2}{2^x-2+1}=\frac{2^x}{2^x-1}=y\Rightarrow 2^x=y\cdot 2^x-y\Rightarrow 2^x(y-1)=y\Rightarrow$$

$$\Rightarrow 2^x=\frac{y}{y-1}\Rightarrow x=\log_2\frac{y}{y-1}\Rightarrow F^{-1}(x)=\log_2\frac{x}{x-1}.$$

4)

A	B	$P_1:(A\Leftrightarrow B)$	$P_2:(A\Leftrightarrow \text{non } B)$	$P_3:(A \text{ o } B)$	$P:[(P_1 \text{ e } P_2)\Rightarrow P_3]$		
1	1	1	0	1	0	1	1
1	0	0	1	1	0	1	1
0	1	0	1	1	0	1	1
0	0	1	0	0	0	1	0

5) Se $y=3x$ per $x=3$ si ha $y=9$.
Equazione perpendicolare: $y-9=-\frac{1}{3}(x-3)\Rightarrow$
$\Rightarrow y=-\frac{1}{3}x+10$. Se $x=6$ si ha $y=8$.
Area $=A_1+A_2=\frac{1}{2}\cdot 3\cdot 9+\frac{1}{2}\cdot 3\cdot(9+8)=$
$=\frac{27}{2}+\frac{51}{2}=\frac{78}{2}=39.$