Compito di Analisi Matematica del 15/01/2019 $\boxed{\text{CAM 1}}$

IM1) $z = e^{1+3\pi i} = e\cdot e^{3\pi i} = e\left(\cos 3\pi + i \operatorname{sen} 3\pi\right) = e\left(\cos\pi + i\operatorname{sen}\pi\right) = -e.$

$\sqrt[3]{z} = \sqrt[3]{e}\cdot\left(\cos\left(\frac{\pi}{3}+k\cdot\frac{2\pi}{3}\right) + i\operatorname{sen}\left(\frac{\pi}{3}+k\cdot\frac{2\pi}{3}\right)\right)$; $0\le k\le 2.$

<u>Per $k=0$</u>: $\sqrt[3]{e}\cdot\left(\cos\frac{\pi}{3}+i\operatorname{sen}\frac{\pi}{3}\right) = \sqrt[3]{e}\cdot\left(\frac{1}{2}+i\cdot\frac{\sqrt{3}}{2}\right)$; <u>Per $k=1$</u>: $\sqrt[3]{e}\cdot\left(\cos\pi+i\operatorname{sen}\pi\right) = -\sqrt[3]{e}$;

<u>Per $k=2$</u>: $\sqrt[3]{e}\cdot\left(\cos\frac{5}{3}\pi+i\operatorname{sen}\frac{5}{3}\pi\right) = \sqrt[3]{e}\cdot\left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right).$

IM2) $f(x;y) = x\cdot\sqrt{x^2+y^2}$. $\lim\limits_{(x;y)\to(0;0)} f(x;y) = 0 = f(0;0) \Rightarrow f(x;y)\in\mathcal{C}(0;0).$

$\frac{\partial f}{\partial x}(0;0) = \lim\limits_{h\to0}\frac{h\cdot\sqrt{h^2}-0}{h} = \lim\limits_{h\to0}|h| = 0$; $\frac{\partial f}{\partial y}(0;0) = \lim\limits_{h\to0}\frac{0\cdot\sqrt{h^2}-0}{h} = 0.$ Quindi:

$$\lim_{(x;y)\to(0;0)}\frac{f(x;y)-f(0;0)-\nabla f(0;0)\cdot(x-0;y-0)}{\sqrt{x^2+y^2}} = \lim_{(x;y)\to(0;0)}\frac{x\cdot\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}} = \lim_{(x;y)\to(0;0)} x = 0.$$

La funzione è differenziabile in $(0;0)$.

IM3) $f(x;y;z) = x\operatorname{sen}y - y\cos z + x^2 z = 0.$ $f(1;0;0) = 1\cdot0 - 0\cdot0 + 1\cdot 0 = 0.$

$\nabla f(x;y;z) = \left(\operatorname{sen}y+2xz;\ x\cos y-\cos z;\ y\operatorname{sen}z+x^2\right)$. $\nabla f(1;0;0) = (0;0;1).$

Quindi avendo solo $f'_z\neq0$ possiamo definire solo $(x;y)\to z$ con

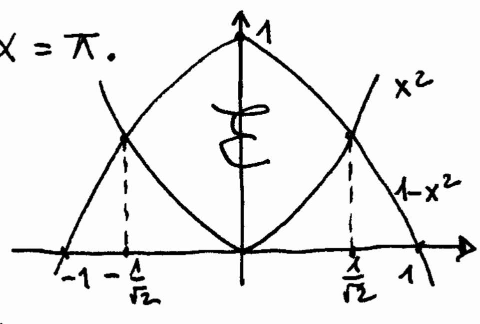
$z'_x = -\frac{0}{1} = z'_y = -\frac{0}{1} = 0$. La funzione $(x;y)\to z$ ha in $(1;0)$ un punto stazionario.

IM4) $f(x;y) = x\,e^{y-x}$: funzione differenziabile $\forall(x;y)\in\mathbb{R}^2.$

$\nabla f(x;y) = \left((1-x)e^{y-x};\ x\,e^{y-x}\right)$; $\nabla f(1;1) = (0;1)$. $\mathcal{D}_v f(1;1) = \nabla f(1;1)\cdot v =$

$\mathcal{D}_v f(1;1) = (0;1)\cdot(\cos\alpha;\operatorname{sen}\alpha) = \operatorname{sen}\alpha = 0$ per $\alpha=0$ e $\alpha=\pi$.

<u>II</u>M1) $\begin{cases}\text{Max/min } f(x;y) = x+y\\ \text{s.v. } x^2\le y\le 1-x^2\end{cases} \Rightarrow \begin{cases}\text{Max/min } f(x;y)=x+y\\ \text{s.v.}\begin{cases}x^2-y\le0\\ x^2+y-1\le0\end{cases}\end{cases}$



Funzioni continue e differenziabili, $\mathcal{E}$ compatto, vincoli qualificati. Vale il Teorema di Weierstrass.

$\Lambda(x;y;\lambda_1;\lambda_2) = x+y-\lambda_1(x^2-y)-\lambda_2(x^2+y-1).$

<u>Caso $\lambda_1 = \lambda_2 = 0$</u>

$$\begin{cases} \Lambda'_x = 1 \neq 0 \\ \Lambda'_y = 1 \neq 0 \end{cases}$$ non ci sono soluzioni.

<u>Caso $\lambda_1 \neq 0; \lambda_2 = 0$</u>

$$\begin{cases} \Lambda'_x = 1 - 2\lambda_1 x = 0 \\ \Lambda'_y = 1 + \lambda_1 = 0 \\ y = x^2 \\ y \leq 1 - x^2 \end{cases} \Rightarrow \begin{cases} 1 + 2x = 0 \\ \lambda_1 = -1 \\ y = x^2 \\ y \leq 1 - x^2 \end{cases} \Rightarrow \begin{cases} x = -\frac{1}{2} \\ y = \frac{1}{4} \\ \lambda_1 = -1 < 0 \\ \frac{1}{4} \leq 1 - \frac{1}{4} : \text{vera} \end{cases}$$ <u>Min?</u>

<u>Caso $\lambda_1 = 0; \lambda_2 \neq 0$</u>

$$\begin{cases} \Lambda'_x = 1 - 2\lambda_2 x = 0 \\ \Lambda'_y = 1 - \lambda_2 = 0 \\ y = 1 - x^2 \\ y \geq x^2 \end{cases} \Rightarrow \begin{cases} 1 - 2x = 0 \\ \lambda_2 = 1 \\ y = 1 - x^2 \\ y \geq x^2 \end{cases} \Rightarrow \begin{cases} x = \frac{1}{2} \\ y = \frac{3}{4} \\ \lambda_2 = 1 > 0 \\ \frac{3}{4} \geq \frac{1}{4} : \text{vera} \end{cases}$$ <u>Max?</u>

<u>Caso $\lambda_1 \neq 0; \lambda_2 \neq 0$</u>

$$\begin{cases} \Lambda'_x = 1 - 2\lambda_1 x - 2\lambda_2 x = 0 \\ \Lambda'_y = 1 + \lambda_1 - \lambda_2 = 0 \\ \begin{cases} y = x^2 \\ y = 1 - x^2 \end{cases} \Rightarrow \begin{cases} x = \frac{1}{\sqrt{2}} \\ y = \frac{1}{2} \end{cases} \cup \begin{cases} x = -\frac{1}{\sqrt{2}} \\ y = \frac{1}{2} \end{cases} \end{cases}.$$

$$\begin{cases} x = \frac{1}{\sqrt{2}} \\ y = \frac{1}{2} \\ 1 - \sqrt{2}\lambda_1 - \sqrt{2}\lambda_2 = 0 \\ 1 + \lambda_1 - \lambda_2 = 0 \end{cases} \Rightarrow \begin{cases} x = \frac{1}{\sqrt{2}} \\ y = \frac{1}{2} \\ 1 - \sqrt{2}\lambda_1 - \sqrt{2} - \sqrt{2}\lambda_1 = 0 \\ \lambda_2 = 1 + \lambda_1 \end{cases} \Rightarrow \begin{cases} x = \frac{1}{\sqrt{2}} \\ y = \frac{1}{2} \\ \lambda_1 = \frac{1 - \sqrt{2}}{2\sqrt{2}} < 0 \\ \lambda_2 = \frac{\sqrt{2} + 1}{2\sqrt{2}} > 0 \end{cases}$$ <u>NULLA</u>

$$\begin{cases} x = -\frac{1}{\sqrt{2}} \\ y = \frac{1}{2} \\ 1 + \sqrt{2}\lambda_1 + \sqrt{2}\lambda_2 = 0 \\ 1 + \lambda_1 - \lambda_2 = 0 \end{cases} \Rightarrow \begin{cases} x = -\frac{1}{\sqrt{2}} \\ y = \frac{1}{2} \\ 1 + \sqrt{2}\lambda_1 + \sqrt{2} + \sqrt{2}\lambda_1 = 0 \\ \lambda_2 = 1 + \lambda_1 \end{cases} \Rightarrow \begin{cases} x = \frac{1}{\sqrt{2}} \\ y = \frac{1}{2} \\ \lambda_1 = -\frac{1 + \sqrt{2}}{2\sqrt{2}} < 0 \\ \lambda_2 = \frac{\sqrt{2} - 1}{2\sqrt{2}} > 0 \end{cases}.$$ <u>NULLA</u>

Per il Teorema di Weierstrass $\left(\frac{1}{2}; \frac{3}{4}\right)$ è il punto di Massimo $\left(f\left(\frac{1}{2}; \frac{3}{4}\right) = \frac{5}{4}\right)$ mentre $\left(-\frac{1}{2}; \frac{1}{4}\right)$ è il punto di minimo $\left(f\left(-\frac{1}{2}; \frac{1}{4}\right) = -\frac{1}{4}\right)$.

II M2) $H(P) = \begin{Vmatrix} K & 0 \\ 0 & K-1 \end{Vmatrix}$. $\begin{cases} |H_1| = K > 0 \text{ per } K > 0 \\ |H_1| = K - 1 > 0 \text{ per } K > 1 \end{cases}$;

$|H_2| = K(K-1) - 0 > 0$ per $\begin{cases} K > 0 \\ K - 1 > 0 \end{cases}$ $\quad 0 \quad 1 \quad + \quad - \quad + \quad$ $\Rightarrow |H_2| > 0$ per $K < 0 \cup K > 1$.

CAM3

$|H_1| > 0$ - - - - - - - - 0 —— 1 ——
$|H_1| > 0$ - - - - - - - - - - - - - - ——
$|H_2| > 0$ ———————— - - - ——

In $K<0$: $\begin{cases} |H_1| < 0 \\ |H_1| < 0 \\ |H_2| > 0 \end{cases}$: punto di Massimo.

in $0<K<1$: $|H_2|<0$: Punto di Sella ; in $1<K$: $\begin{cases} |H_1| > 0 \\ |H_1| > 0 \\ |H_2| > 0 \end{cases}$: punto di minimo.

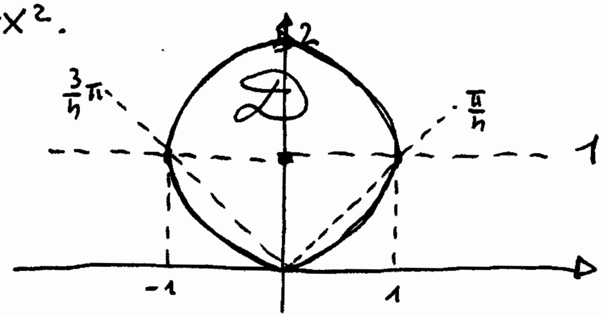
Per $K=0$ forma Semidefinita Negativa ; per $K=1$ forma semidefinita positiva.

II M3) $\begin{cases} y'(1+x^2) = xy \\ y(0) = 1 \end{cases} \Rightarrow \frac{1}{y}\cdot y' = \frac{x}{1+x^2} \quad (y \neq 0) \Rightarrow \int \frac{1}{y}\,dy = \int \frac{x}{1+x^2}\,dx + K \Rightarrow$

$\Rightarrow \log y = \frac{1}{2}\log(1+x^2) + K \Rightarrow y = e^K \cdot e^{\frac{1}{2}\log(1+x^2)} = m\cdot(1+x^2)^{\frac{1}{2}} = m\cdot\sqrt{1+x^2}$.

$y(0) = 1 \Rightarrow 1 = m\sqrt{1} \Rightarrow m = 1 \Rightarrow y = \sqrt{1+x^2}$.

II M4) $\mathcal{D} = \{(x;y) : 1 \leq y \,;\, x^2+y^2 \leq 2y\}$



$x^2+y^2-2y=0$ Circonferenza di centro $(0;1)$ e $z=1$.

$x^2+y^2-2y=0 \Rightarrow \rho^2 - 2\rho\,\mathrm{sen}\,\vartheta = \rho(\rho - 2\,\mathrm{sen}\,\vartheta) = 0 \Rightarrow$

$\Rightarrow \rho = 2\,\mathrm{sen}\,\vartheta$. Calcolando l'integrale per sostituzione avremo:

$$\iint_D f(x;y)\,dx\,dy = \int_{\frac{\pi}{4}}^{\frac{3}{4}\pi}\int_{\frac{1}{\mathrm{sen}\,\vartheta}}^{2\,\mathrm{sen}\,\vartheta} \frac{\rho\cos\vartheta}{\cancel{\sqrt{\rho^2}}}\cdot\cancel{\rho}\,d\rho\,d\vartheta =$$

Se $y = 1 \Rightarrow \rho\,\mathrm{sen}\,\vartheta = 1 \Rightarrow \rho = \frac{1}{\mathrm{sen}\,\vartheta}$

$$= \int_{\frac{\pi}{4}}^{\frac{3}{4}\pi}\left(\frac{1}{2}\rho^2\cos\vartheta\Big|_{\frac{1}{\mathrm{sen}\,\vartheta}}^{2\,\mathrm{sen}\,\vartheta}\right)d\vartheta = \int_{\frac{\pi}{4}}^{\frac{3}{4}\pi}\frac{1}{2}\left(4\,\mathrm{sen}^2\vartheta\cos\vartheta - \frac{1}{\mathrm{sen}^2\vartheta}\cos\vartheta\right)d\vartheta =$$

$$= \frac{1}{2}\left(\int_{\frac{\pi}{4}}^{\frac{3}{4}\pi} 4\,\mathrm{sen}^2\vartheta\,d\,\mathrm{sen}\,\vartheta - \int_{\frac{\pi}{4}}^{\frac{3}{4}\pi}\frac{1}{\mathrm{sen}^2\vartheta}\,d\,\mathrm{sen}\,\vartheta\right) = \frac{1}{2}\left(\frac{4}{3}\mathrm{sen}^3\vartheta + \frac{1}{\mathrm{sen}\,\vartheta}\right|_{\frac{\pi}{4}}^{\frac{3}{4}\pi} =$$

$$= \frac{1}{2}\left(\left(\frac{4}{3}\cdot\frac{1}{2\sqrt{2}} + \sqrt{2}\right) - \left(\frac{4}{3}\cdot\frac{1}{2\sqrt{2}} + \sqrt{2}\right)\right) = 0.$$