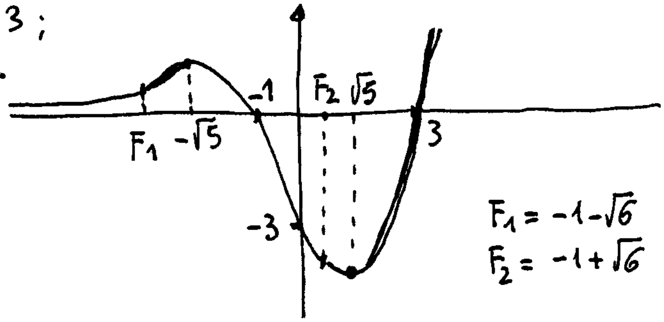
Compito di Matematica Generale del 15/1/2019 Compito A $\boxed{\text{MGA1}}$

1) $f(x)=(x^2-2x-3)e^x$. C.E.: $\mathbb{R}$. $\lim_{x\to-\infty} f(x)=(\to+\infty)\cdot(\to 0^+)=0^+$; $\lim_{x\to+\infty} f(x)=+\infty$.

$f(x)\geq 0$ per $x^2-2x-3=(x+1)(x-3)\geq 0$: $x\leq -1 \cup x\geq 3$; $f(0)=-3$.

(+) -1 (−) 3 (+)



$F_1=-1-\sqrt{6}$
$F_2=-1+\sqrt{6}$

$f'(x)=(2x-2+x^2-2x-3)e^x=(x^2-5)e^x\geq 0$
per $x\leq-\sqrt{5}\cup x\geq\sqrt{5}$ $-\sqrt{5}$ $\sqrt{5}$

$f''(x)=(2x+x^2-5)e^x\geq 0$ per $x^2+2x-5\geq 0$
per $x\leq -1-\sqrt{6}\cup x\geq -1+\sqrt{6}$ $-1-\sqrt{6}$ $-1+\sqrt{6}$

2) $\lim_{x\to 0}(1-3x)^{\frac{1}{2x}}=(e^{-3})^{\frac{1}{2}}=e^{-\frac{3}{2}}=\frac{1}{\sqrt{e^3}}$ (da $\lim_{t\to 0}(1+\alpha t)^{\frac{\beta}{t}}=e^{\alpha\beta}$)

$\lim_{x\to+\infty}\left(\frac{1+3x+x^2}{2x+1}\right)^{\frac{1+x^2}{1-x}}=(\to+\infty)^{(\to-\infty)}=0^+$.

3) $\lim_{x\to 0}\frac{x-\sin x}{x^\alpha}=\lim_{x\to 0}\frac{x-\left(x-\frac{x^3}{3!}+o(x^3)\right)}{x^\alpha}=\lim_{x\to 0}\frac{\frac{1}{6}x^3+o(x^3)}{x^\alpha}=\frac{1}{6}$ se $\alpha=3$.

4) Per avere una tangente perpendicolare a $y=\frac{1}{2}x+2$ dovrà essere $m=f'(x_0)=-2\Rightarrow$
$\Rightarrow f'(x)=-2\cdot e^{3-2x}=-2\Rightarrow e^{3-2x}=1\Rightarrow 3-2x=0\Rightarrow x_0=\frac{3}{2}$.

5) $\int_0^k e^{3x}-e^{3x-1}\,dx=\left(\frac{1}{3}e^{3x}-\frac{1}{3}e^{3x-1}\right|_0^k=\frac{1}{3}\left[\left(e^{3k}-e^{3k-1}\right)-\left(e^0-e^{-1}\right)\right]=2\Rightarrow$
$\Rightarrow e^{3k}(1-e^{-1})-(1-e^{-1})=6\Rightarrow(e^{3k}-1)(1-e^{-1})=6\Rightarrow e^{3k}=\frac{6}{1-e^{-1}}+1=\frac{6e}{e-1}+1\Rightarrow$
$\Rightarrow e^{3k}=\frac{7e-1}{e-1}\Rightarrow 3k=\log\frac{7e-1}{e-1}\Rightarrow k=\frac{1}{3}\log\frac{7e-1}{e-1}=\log\sqrt[3]{\frac{7e-1}{e-1}}$.

6) $f(x)=e^{3x}-3e^{2x}+2e^x$; $f'(x)=3e^{3x}-6e^{2x}+2e^x=e^x(3e^{2x}-6e^x+2)\geq 0$.
Se $e^x=t\Rightarrow 3t^2-6t+2\geq 0$; $t=\frac{3\pm\sqrt{9-6}}{3}=\frac{3\pm\sqrt{3}}{3}=1\pm\frac{\sqrt{3}}{3}$. Quindi $f'(x)\geq 0$
per $e^x\leq 1-\frac{\sqrt{3}}{3}\cup e^x\geq 1+\frac{\sqrt{3}}{3}\Rightarrow x\leq\log\left(1-\frac{\sqrt{3}}{3}\right)\cup x\geq\log\left(1+\frac{\sqrt{3}}{3}\right)$.

M 0 m
$\log\left(1-\frac{\sqrt{3}}{3}\right)$ $\log\left(1+\frac{\sqrt{3}}{3}\right)$

In $x=\log\left(1-\frac{\sqrt{3}}{3}\right)$ punto di Massimo; in $x=\log\left(1+\frac{\sqrt{3}}{3}\right)$ punto di minimo.

MGA2

7) $f(x;y) = y^2 - x^3y + 3xy$. $\nabla f(x;y) = (0;0) \Rightarrow$

$$\Rightarrow \begin{cases} f'_x = -3x^2y + 3y = 3y(1-x^2) = 0 \\ f'_y = 2y - x^3 + 3x = 0 \end{cases} \Rightarrow \begin{cases} y = 0 \\ 3x - x^3 = x(3-x^2) = 0 \end{cases} \Rightarrow \begin{cases} y=0 \\ x=0 \end{cases} \cup \begin{cases} y=0 \\ x=\sqrt{3} \end{cases} \cup \begin{cases} y=0 \\ x=-\sqrt{3} \end{cases}$$

$$\cup \begin{cases} 1-x^2=0 \\ 2y-x^3+3x=0 \end{cases} \Rightarrow \begin{cases} x=1 \\ 2y=-2 \end{cases} \Rightarrow \begin{cases} x=1 \\ y=-1 \end{cases} \cup \begin{cases} x=-1 \\ 2y=2 \end{cases} \Rightarrow \begin{cases} x=-1 \\ y=1 \end{cases}.$$

$(0;0); (\sqrt{3};0); (-\sqrt{3};0); (-1;1); (1;-1)$. 5 punti stazionari.

$H = \begin{Vmatrix} -6xy & 3-3x^2 \\ 3-3x^2 & 2 \end{Vmatrix}$. $H(0;0) = \begin{Vmatrix} 0 & 3 \\ 3 & 2 \end{Vmatrix}$: $|H_2| = -9 < 0$: Sella; $H(\sqrt{3};0) = \begin{Vmatrix} 0 & -6 \\ -6 & 2 \end{Vmatrix}$: $|H_2| = -36 < 0$: Sella;

$H(-\sqrt{3};0) = \begin{Vmatrix} 0 & -6 \\ -6 & 2 \end{Vmatrix}$: $|H_2| = -36 < 0$: sella; $H(-1;1) = \begin{Vmatrix} 6 & 0 \\ 0 & 2 \end{Vmatrix}$: $\begin{cases} |H_1| > 0 \\ |H_2| > 0 \end{cases}$: Minimo; $H(1;-1) = \begin{Vmatrix} 6 & 0 \\ 0 & 2 \end{Vmatrix}$: $\begin{cases} |H_1| > 0 \\ |H_2| > 0 \end{cases}$: Minimo

8) $f(x;y;z) = xe^{y-z} + x\,\text{sen}(x-2z) - (x+2)^{y+1}$. $\nabla f(x;y;z) = \left(\frac{\partial f}{\partial x}; \frac{\partial f}{\partial y}; \frac{\partial f}{\partial z}\right)$.

$f'_x = \left(e^{y-z} + \text{sen}(x-2z) + x\cos(x-2z) - (y+1)\cdot(x+2)^y\right)$; $f'_x(0;0;0) = 1+0+0-1 = 0$;

$f'_y = \left(xe^{y-z} + 0 - (x+2)^{y+1}\cdot \log(x+2)\right)$; $f'_y(0;0;0) = 0 - 2\cdot\log 2 = -\log 4$;

$f'_z = \left(-xe^{y-z} - 2x\cos(x-2z) + 0\right)$; $f'_z(0;0;0) = 0 - 0 = 0$. $\nabla f(0;0;0) = (0; \log\frac{1}{4}; 0)$.

9) $A \cdot X = \begin{Vmatrix} k & 2 & 1 \\ 2 & k & 2 \\ 1 & 1 & k \end{Vmatrix} \cdot \begin{Vmatrix} 2 \\ -1 \\ 1 \end{Vmatrix} = \begin{Vmatrix} 2k-2+1 \\ 4-k+2 \\ 2-1+k \end{Vmatrix} = \begin{Vmatrix} 2k-1 \\ 6-k \\ k+1 \end{Vmatrix}$.

$A \cdot X \,//\, (2;10;4) \Rightarrow \frac{2k-1}{2} = \frac{6-k}{10} = \frac{k+1}{4} \Rightarrow \begin{cases} 20k-10 = 12-2k \\ 24-4k = 10k+10 \end{cases} \Rightarrow \begin{cases} 22k = 22 \\ 14k = 14 \end{cases} \Rightarrow k = 1$;

$A \cdot X \perp (2;10;4) \Rightarrow (2k-1; 6-k; k+1)(2;10;4) = 4k - 2 + 60 - 10k + 4k + 4 = 62 - 2k = 0 \Rightarrow k = 31$.

10)

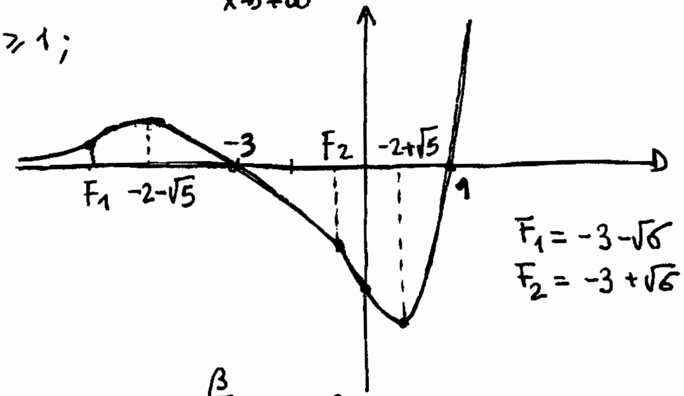
A	B	C	$(B \circ C)$	$(A \Rightarrow (B \circ C))$	$(A e C)$	$((A e C) \Rightarrow B)$	$[A \Rightarrow (B \circ C)] \Leftrightarrow [(A e C) \Rightarrow B]$
1	1	1	1	1	1	1	1
1	1	0	1	1	0	1	1
0	1	1	1	1	0	1	1
0	1	0	1	1	0	1	1

Compito di Matematica Generale del 15/1/2019 Compito B $\boxed{MGB1}$

1) $f(x)=(x^2+2x-3)\cdot e^x$. C.E.: $\mathbb{R}$. $\lim\limits_{x\to-\infty} f(x)=(\to+\infty)\cdot(\to 0^+)=0^+$; $\lim\limits_{x\to+\infty} f(x)=+\infty$.

$f(x)\geq 0$ per $x^2+2x-3=(x-1)(x+3)\geq 0$: $x\leq -3 \cup x\geq 1$; $f(0)=-3$.

(+) -3 (−) 1 (+)



$F_1=-3-\sqrt{6}$
$F_2=-3+\sqrt{6}$

$f'(x)=(2x+2+x^2+2x-3)e^x=(x^2+4x-1)e^x\geq 0$
per $x\leq -2-\sqrt{5} \cup x\geq -2+\sqrt{5}$ $\quad -2-\sqrt{5} \quad 0 \quad -2+\sqrt{5}$

$f''(x)=(2x+4+x^2+4x-1)e^x\geq 0$ per $x^2+6x+3\geq 0$
per $x\leq -3-\sqrt{6} \cup x\geq -3+\sqrt{6}$ $\quad -3-\sqrt{6} \quad -3+\sqrt{6} \quad 0$

2) $\lim\limits_{x\to 0}(1-2x)^{\frac{1}{3x}}=(e^{-2})^{\frac{1}{3}}=e^{-\frac{2}{3}}=\frac{1}{\sqrt[3]{e^2}}$ $\left(\text{da } \lim\limits_{t\to 0}(1+\alpha t)^{\frac{\beta}{t}}=e^{\alpha\beta}\right)$.

$$\lim_{x\to+\infty}\left(\frac{1-3x-x^2}{1-x}\right)^{\frac{1+x^2}{1+x}}=(\to+\infty)^{(\to+\infty)}=+\infty.$$

3) $\lim\limits_{x\to 0}\frac{x-\log(1+x)}{x^\alpha}=\lim\limits_{x\to 0}\frac{x-\left(x-\frac{x^2}{2}+o(x^2)\right)}{x^\alpha}=\lim\limits_{x\to 0}\frac{\frac{x^2}{2}+o(x^2)}{x^\alpha}=\frac{1}{2}$ se $\alpha=2$.

4) Per avere una tangente perpendicolare a $y=1-2x$ dovrà essere $m=f'(x_0)=\frac{1}{2}$ $\Rightarrow$
$\Rightarrow f'(x)=\frac{3}{3x-1}=\frac{1}{2} \Rightarrow 3x-1=6 \Rightarrow 3x=7 \Rightarrow x_0=\frac{7}{3}$.

5) $\int_0^k e^{2x+1}-e^{2x}\,dx=\left(\frac{1}{2}e^{2x+1}-\frac{1}{2}e^{2x}\right|_0^k=\frac{1}{2}\left[\left(e^{2k+1}-e^{2k}\right)-(e-1)\right]=3 \Rightarrow$
$\Rightarrow e^{2k}(e-1)-(e-1)=6 \Rightarrow (e^{2k}-1)(e-1)=6 \Rightarrow e^{2k}=\frac{6}{e-1}+1=\frac{6+e-1}{e-1}=\frac{5+e}{e-1} \Rightarrow$
$\Rightarrow 2k=\log\frac{5+e}{e-1} \Rightarrow k=\frac{1}{2}\log\frac{5+e}{e-1} \Rightarrow k=\log\sqrt{\frac{5+e}{e-1}}$.

6) $f(x)=e^{3x}-3e^{2x}+e^x$; $f'(x)=3e^{3x}-6e^{2x}+e^x=e^x(3e^{2x}-6e^x+1)\geq 0$. Se $e^x=t$:
$3t^2-6t+1\geq 0$; $t=\frac{3\pm\sqrt{9-3}}{3}=\frac{3\pm\sqrt{6}}{3}=1\pm\frac{\sqrt{6}}{3}$. Quindi $f'(x)\geq 0$ per
$e^x\leq 1-\frac{\sqrt{6}}{3} \cup e^x\geq 1+\frac{\sqrt{6}}{3} \Rightarrow x\leq\log\left(1-\frac{\sqrt{6}}{3}\right) \cup x\geq\log\left(1+\frac{\sqrt{6}}{3}\right)$. $\quad$ M $\quad \log\left(1-\frac{\sqrt{6}}{3}\right) \quad$ m $\quad \log\left(1+\frac{\sqrt{6}}{3}\right)$

In $x=\log\left(1-\frac{\sqrt{6}}{3}\right)$ punto di Massimo; in $x=\log\left(1+\frac{\sqrt{6}}{3}\right)$ punto di minimo.

MGB2

7) $f(x;y) = x^2 - y^3 x + 3xy$. $\nabla f(x;y) = (0;0) \Rightarrow$

$$\Rightarrow \begin{cases} f'_x = 2x - y^3 + 3y = 0 \\ f'_y = 3x - 3y^2 x = 3x(1-y^2) = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ 3y - y^3 = y(3-y^2) = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases} \cup \begin{cases} x = 0 \\ y = \sqrt{3} \end{cases} \cup \begin{cases} x = 0 \\ y = -\sqrt{3} \end{cases} \sigma$$

$$\cup \begin{cases} 1 - y^2 = 0 \\ 2x - y^3 + 3y = 0 \end{cases} \Rightarrow \begin{cases} 2x = -2 \\ y = 1 \end{cases} \Rightarrow \begin{cases} x = -1 \\ y = 1 \end{cases} \cup \begin{cases} 2x = 2 \\ y = -1 \end{cases} \Rightarrow \begin{cases} x = 1 \\ y = -1 \end{cases} . (0;0); (0;\sqrt{3}); (0;-\sqrt{3}); (1;-1); (-1;1).$$

$H = \begin{Vmatrix} 2 & 3-3y^2 \\ 3-3y^2 & -6xy \end{Vmatrix}$. $H(0;0) = \begin{Vmatrix} 2 & 3 \\ 3 & 0 \end{Vmatrix}$: $|H_2| = -9 < 0$: Sella. $H(0;\sqrt{3}) = \begin{Vmatrix} 2 & -6 \\ -6 & 0 \end{Vmatrix}$: $|H_2| = -36 < 0$: Sella.

$H(0;-\sqrt{3}) = \begin{Vmatrix} 2 & -6 \\ -6 & 0 \end{Vmatrix}$: $|H_2| = -36 < 0$: Sella. $H(1;-1) = \begin{Vmatrix} 2 & 0 \\ 0 & 6 \end{Vmatrix}$: $\begin{cases} |H_1| > 0 \\ |H_2| > 0 \end{cases}$: <u>Minimo</u>; $H(-1;1) = \begin{Vmatrix} 2 & 0 \\ 0 & 6 \end{Vmatrix}$: $\begin{cases} |H_1| > 0 \\ |H_2| > 0 \end{cases}$: <u>Minimo</u>

8) $f(x;y;z) = x e^{y-x} + y \operatorname{sen}(x-2z) - (y+1)^{z+2}$. $\nabla f(x;y;z) = \left(\frac{\partial f}{\partial x}; \frac{\partial f}{\partial y}; \frac{\partial f}{\partial z} \right)$.

$f'_x = \left(e^{y-x} - x e^{y-x} + y \cos(x-2z) + 0 \right)$; $f'_x(0;0;0) = 1 - 0 + 0 = 1$;

$f'_y = \left(x e^{y-x} + \operatorname{sen}(x-2z) - (z+2)(y+1)^{z+1} \right)$; $f'_y(0;0;0) = 0 + 0 - 2 \cdot 1 = -2$;

$f'_z = \left(0 + y \cdot (-2) \cos(x-2z) - (y+1)^{z+2} \cdot \log(y+1) \right)$; $f'_z(0;0;0) = 0 - \log 1 = 0$. $\nabla f(0;0;0) = (1;-2;0)$.

9) $A \cdot X = \begin{Vmatrix} k & 1 & 1 \\ 2 & k & 1 \\ 2 & 1 & k \end{Vmatrix} \cdot \begin{Vmatrix} 1 \\ 2 \\ -1 \end{Vmatrix} = \begin{Vmatrix} k+2-1 \\ 2+2k-1 \\ 2+2-k \end{Vmatrix} = \begin{Vmatrix} k+1 \\ 2k+1 \\ 4-k \end{Vmatrix}$.

$A \cdot X \,/\!/\, (6;10;4) \Rightarrow \frac{k+1}{6} = \frac{2k+1}{10} = \frac{4-k}{4} \Rightarrow \begin{cases} 10k + 10 = 12k + 6 \\ 8k + 4 = 40 - 10k \end{cases} \Rightarrow \begin{cases} 2k = 4 \\ 18k = 36 \end{cases} \Rightarrow k = 2$;

$A \cdot X \perp (6;10;4) \Rightarrow (k+1; 2k+1; 4-k) \cdot (6;10;4) = 6k + 6 + 20k + 10 + 16 - 4k = 22k + 32 = 0 \Rightarrow k = -\frac{16}{11}$.

10)

A	B	C	(B o C)	((B o C) ⇒ A)	(A e C)	(B ⇒ (A e C))	[(B o C) ⇒ A] ⇔ [B ⇒ (A e C)]
1	0	1	1	1	1	1	1
1	0	0	0	1	0	1	1
0	0	1	1	0	0	1	0
0	0	0	0	1	0	1	1