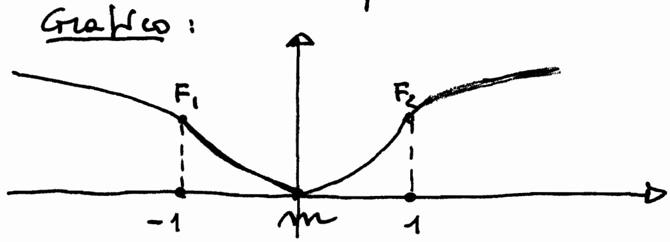
Compito di Matematica Generale del 23/3/2019 $\boxed{CMG1}$

1) $f(x) = \log(1+x^2)$. C.E.: $\mathbb{R}$. $\lim_{x\to\pm\infty} f(x) = +\infty$; $1+x^2 \geq 1 \Rightarrow f(x) \geq 0 \ \forall x \in \mathbb{R}$. $f(0) = 0$.

$f'(x) = \dfrac{2x}{1+x^2} \geq 0$ per $x \geq 0$: $\xrightarrow[m]{0}$

Grafico:



$f''(x) = 2 \cdot \dfrac{1(1+x^2) - x \cdot 2x}{(1+x^2)^2} = 2 \cdot \dfrac{1-x^2}{(1+x^2)^2} \geq 0$

per $1-x^2 \geq 0 \Rightarrow -1 \leq x \leq 1$ $\quad -1 \quad 1$

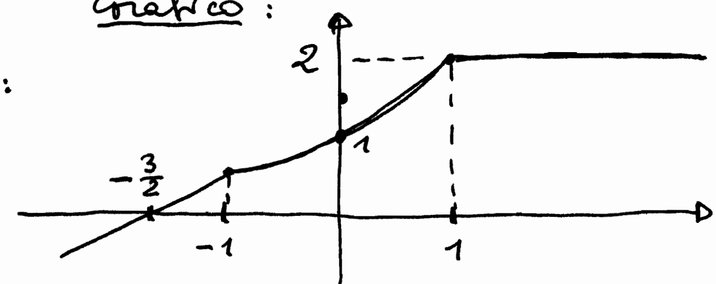
2) $\lim_{x\to 0} \dfrac{3^{-x}-1}{2x} = \dfrac{1}{2} \cdot \lim_{x\to 0} \dfrac{3^{-x}-1}{(-x)} \cdot (-1) = \dfrac{1}{2} \cdot \log 3 \cdot (-1) = -\dfrac{1}{2}\log 3 = \log \dfrac{1}{\sqrt{3}}$.

$\lim_{x\to+\infty} \dfrac{\log x - e^{-x}}{x + 2^{1-x}} = \lim_{x\to+\infty} \dfrac{\log x}{x} = 0 \quad \left(e^{-x} \to 0;\ 2^{1-x} \to 0;\ \log x = o(x)\right)$.

3) $f(x) = \begin{cases} x+k : x<-1 \\ 2^x : -1 \leq x \leq 1 \\ mx+2 : 1 < x \end{cases}$. Per la continuità:

$\begin{cases} -1+k = 2^{-1} \\ m\cdot 1 + 2 = 2^1 \end{cases} \Rightarrow \begin{cases} k = 1 + \frac{1}{2} = \frac{3}{2} \\ m = 2-2 = 0 \end{cases}$

Grafico:



4) Da $f(x) = x^2 - 3x + 5 \Rightarrow f(1) = 1-3+5 = 3$; $f'(x) = 2x-3$; $f'(1) = -1$.

Equazione retta tangente in $x=1$: $y - 3 = -1(x-1) \Rightarrow y = -x+4 \Rightarrow k = 4$.

5) $\displaystyle\int_1^e \frac{\log x}{x}\,dx = \frac{1}{2}\int_1^e 2\log x \cdot \frac{1}{x}\,dx = \frac{1}{2}\int_1^e 2\log x \, d(\log x) = \frac{1}{2}\left(\log^2 x \Big|_1^e = \frac{1}{2}(1-0) = \frac{1}{2}\right.$.

6) Da $f(x) = e^{3x-1} \Rightarrow F(x) = f(g(x)) = e^{3g(x)-1} = 1+x^2 \Rightarrow 3g(x) - 1 = \log(1+x^2) \Rightarrow$

$\Rightarrow 3g(x) = \log(1+x^2) + 1 \Rightarrow g(x) = \dfrac{1}{3}\left(\log(1+x^2) + 1\right)$.

7) $f(x) = e^x \cdot \operatorname{sen} x$; $f(0) = 1 \cdot 0 = 0$;

$f'(x) = e^x \cdot \operatorname{sen} x + e^x \cdot \cos x$; $f'(0) = 1\cdot 0 + 1 \cdot 1 = 1$. Quindi

$f(x) = f(0) + f'(0)\cdot x + o(x) \Rightarrow e^x \cdot \operatorname{sen} x = 0 + 1\cdot x + o(x) = x + o(x)$.

$\boxed{CMG2}$

8) $A \cdot B = C \Rightarrow \begin{Vmatrix} 1 & 1 & m \\ m & 1 & K \end{Vmatrix} \cdot \begin{Vmatrix} 2 & 0 \\ 0 & 1 \\ 1 & 1 \end{Vmatrix} = \begin{Vmatrix} 2+0+m & 0+1+m \\ 2m+0+K & 0+1+K \end{Vmatrix} = \begin{Vmatrix} 2+m & 1+m \\ 2m+K & 1+K \end{Vmatrix} \Rightarrow$

$\Rightarrow \begin{Vmatrix} 2+m & 1+m \\ 2m+K & 1+K \end{Vmatrix} = \begin{Vmatrix} 3 & 2 \\ 1 & 0 \end{Vmatrix} \Rightarrow \begin{cases} 2+m=3 \Rightarrow m=1 \text{ e } 1+m=2 \\ 1+K=0 \Rightarrow K=-1 \text{ e } 2m+K=2-1=1 \end{cases} \Rightarrow m=1 \text{ e } K=-1.$

9)

A	B	$A \Rightarrow B$	non $(A \Rightarrow B)$	A e (non $(A \Rightarrow B)$) : P	non B	$P \Rightarrow$ non B
1	1	1	0	0	0	1
1	0	0	1	1	1	1
0	1	1	0	0	0	1
0	0	1	0	0	1	1

La proposizione $P \Rightarrow$ non B risulta una tautologia.

10) $f(x;y;z) = x \cdot \operatorname{sen}\frac{y}{z} - x^{y-z}.$

$f'_x(x;y;z) = \operatorname{sen}\frac{y}{z} - (y-z)\cdot x^{y-z-1}$; $f'_x(1;0;1) = 0 + 1^{-2} = +1$;

$f'_y(x;y;z) = x \cdot \cos\frac{y}{z} \cdot \frac{1}{z} - x^{y-z} \cdot \log x$; $f'_y(1;0;1) = 1 - 1 \cdot 0 = 1$;

$f'_z(x;y;z) = x \cos\frac{y}{z} \cdot y \cdot \left(-\frac{1}{z^2}\right) - x^{y-z} \cdot \log x \cdot (-1)$; $f'_z(1;0;1) = 0 + 0 = 0.$

$\nabla f(1;0;1) = (1;1;0).$