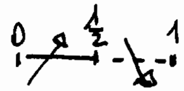
Compito di Matematica Generale del 5/7/2019 $\boxed{CMG1}$

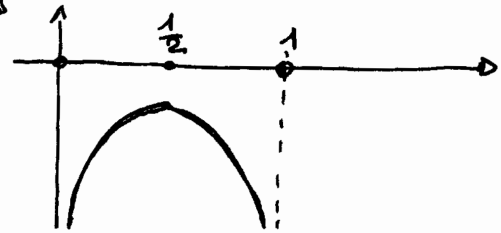
1) $f(x)=\log(x-x^2)$. C.E.: $x-x^2>0 \Rightarrow x(1-x)>0 \Rightarrow 0<x<1$.

$\lim_{x\to 0^+} f(x) = \lim_{x\to 1^-} f(x) = -\infty$. $f(x)>0$ per $x-x^2>1 \Rightarrow x^2-x+1<0$: Mai.

$f'(x)=\dfrac{1-2x}{x-x^2}\geq 0$ per $1-2x\geq 0 \Rightarrow x\leq \frac{1}{2}$:



Grafico



$f''(x)=\dfrac{-2(x-x^2)-(1-2x)^2}{(x-x^2)^2}=\dfrac{-2x^2+2x-1}{(x-x^2)^2}\geq 0$

per $2x^2-2x+1\leq 0$: Mai.

$f(\frac{1}{2})=\log\frac{1}{4}=-\log 4$.

2) $\displaystyle\lim_{x\to 0}\frac{1-\cos x}{\sqrt{x^2+1}-1}=\lim_{x\to 0}\frac{1-\cos x}{x^2}\cdot\frac{x^2}{\sqrt{1+x^2}-1}=\frac{1}{2}\cdot\lim_{t\to 0}\frac{t}{(t+1)^{\frac{1}{2}}-1}=\frac{1}{2}\cdot\frac{1}{\frac{1}{2}}=1.$

$\displaystyle\lim_{x\to+\infty}(x-\log x)^{\log x-x}=(\to+\infty)^{(\to-\infty)}=0^+.$

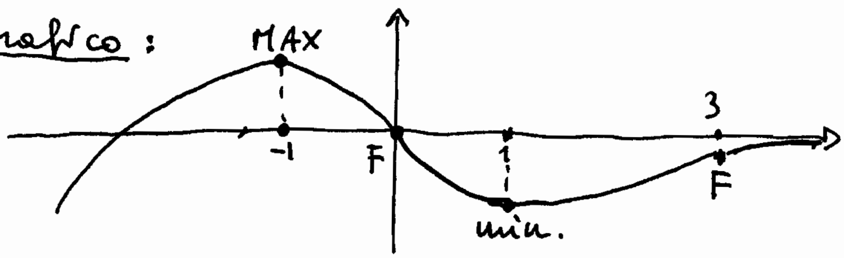
3) $f(x)$ è sempre continua

$\lim_{x\to-\infty} f(x)=-\infty$ e $\lim_{x\to+\infty} f(x)=0$

$f(x)$ è decrescente in $[-1;1]$

$f(x)$ è convessa in $[0;3]$.

Grafico:



4) $f(x)=e^{3x-2} \Rightarrow f(g(x))=e^{3g(x)-2}=3x-5 \Rightarrow 3g(x)-2=\log(3x-5) \Rightarrow$

$\Rightarrow g(x)=\frac{1}{3}\left(\log(3x-5)+2\right)=y \Rightarrow \log(3x-5)+2=3y \Rightarrow \log(3x-5)=3y-2 \Rightarrow$

$\Rightarrow 3x-5=e^{3y-2} \Rightarrow x=\frac{1}{3}\left(e^{3y-2}+5\right)$. $g^{-1}(x)=\frac{1}{3}\left(e^{3x-2}+5\right)$.

5) $\displaystyle\int_1^2\frac{x-1}{x+1}\,dx=\int_1^2\frac{x+1-2}{x+1}\,dx=\int_1^2 1-\frac{2}{x+1}\,dx=\Big(x-2\log(x+1)\Big)\Big|_1^2=$

$=(2-2\log 3)-(1-2\log 2)=1+2(\log 2-\log 3)=1+2\log\frac{2}{3}=1+\log\frac{4}{9}=\log\frac{4}{9}e.$

6) $f(x)=x^3-3x^2-6x+1$. $f'(x)=3=m \Rightarrow 3x^2-6x-6=3 \Rightarrow$

$\Rightarrow 3x^2-6x-9=0 \Rightarrow x^2-2x-3=0 \Rightarrow x=1\pm\sqrt{1+3}=1\pm 2 \Rightarrow x_1=-1$ e $x_2=3$.

CMG 2

Equazione retta tangente in $x=-1$: $y-3=3(x+1) \Rightarrow y=3x+6$;

Equazione retta tangente in $x=3$: $y+17=3(x-3) \Rightarrow y=3x-26$.

7) $f(x)=e^{3x-3} \Rightarrow f(1)=e^{3-3}=1$;
$f'(x)=3e^{3x-3} \Rightarrow f'(1)=3\cdot e^{3-3}=3$;
$f''(x)=9e^{3x-3} \Rightarrow f''(1)=9\cdot e^{3-3}=9$.

$P_2(x;1)=1+3(x-1)+\frac{9}{2!}(x-1)^2$.

8) $f(x;y)=x^2y^2-2xy-y \Rightarrow \nabla f=\mathbb{O}: \begin{cases} f'_x=2xy^2-2y=2y(xy-1)=0 \\ f'_y=2x^2y-2x-1=0 \end{cases} \Rightarrow \begin{cases} y=0 \\ x=-\frac{1}{2} \end{cases}$

oppure $\begin{cases} xy=1 \\ 2x-2x-1=0 \end{cases}$: impossibile; $P_0:\left(-\frac{1}{2};0\right)$. $H(x;y)=\begin{Vmatrix} 2y^2 & 4xy-2 \\ 4xy-2 & 2x^2 \end{Vmatrix}$.

$H\left(-\frac{1}{2};0\right)=\begin{Vmatrix} 0 & -2 \\ -2 & \frac{1}{2} \end{Vmatrix} \Rightarrow |H_2|=-4<0:\left(-\frac{1}{2};0\right)$ Punto di Sella.

9) $A\cdot X=\begin{Vmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \end{Vmatrix}\cdot\begin{Vmatrix} m \\ 1 \\ k \end{Vmatrix}=\begin{Vmatrix} m+1 \\ k-1 \end{Vmatrix}$; $B\cdot X=\begin{Vmatrix} 0 & 2 & -1 \\ 1 & -1 & 0 \end{Vmatrix}\cdot\begin{Vmatrix} m \\ 1 \\ k \end{Vmatrix}=\begin{Vmatrix} 2-k \\ m-1 \end{Vmatrix}$

$A\cdot X \perp B\cdot X \Rightarrow (m+1;k-1)\cdot(2-k;m-1)=(m+1)(2-k)+(k-1)(m-1)=0 \Rightarrow$
$\Rightarrow 2m+2-\cancel{mk}-k+\cancel{mk}-m-k+1=m-2k+3=0 \Rightarrow m=2k-3$.

10)

	A	B	C	(BoC)	A⇒(BoC)	(BeC)	A⇒(BeC)
	1	1	1	1	1	1	1
*	1	1	0	1	~~1~~	~~0~~	~~0~~
*	1	0	1	1	~~1~~	~~0~~	~~0~~
	1	0	0	0	0	0	0
	0	1	1	1	1	1	1
	0	1	0	1	1	0	1
	0	0	1	1	1	0	1
	0	0	0	0	1	0	1

Risultano logicamente equivalenti eccettuato nei due casi con A vera e B e C una vera e una falsa.