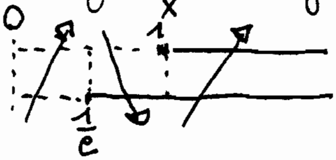
Compito di Matematica Generale del 3/9/2019 $\boxed{CMG1}$

1) $f(x)=x^2\cdot\log^2 x$. C.E.: $x>0$. $\lim_{x\to 0^+} f(x)=0^+$; $\lim_{x\to+\infty} f(x)=+\infty$; $f(x)\geq 0\ \forall x>0$.

$f'(x)=2x\log^2 x+x^2\cdot 2\log x\cdot\frac{1}{x}=2x\log x(\log x+1)\geq 0$ per $\qquad f(1)=0$.

$\log x\geq 0: x\geq 1$

$\log x+1\geq 0: x\geq\frac{1}{e}$



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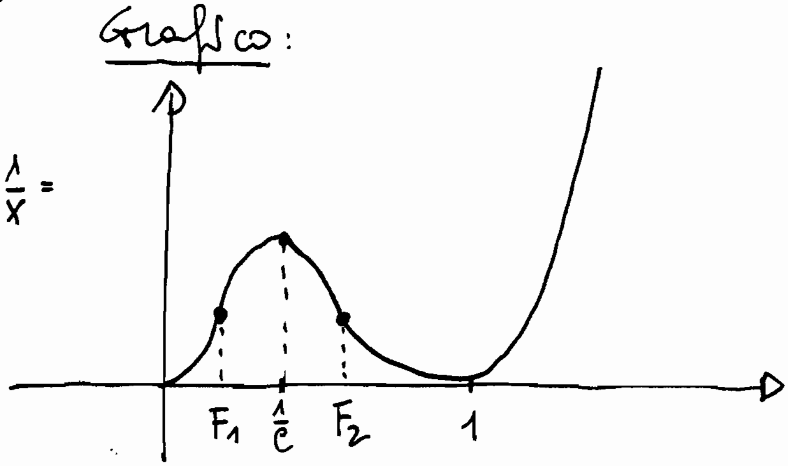
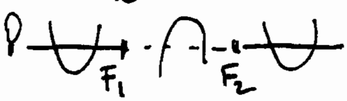


$f''(x)=2\log x(\log x+1)+2x\cdot\frac{1}{x}(\log x+1)+2x\log x\cdot\frac{1}{x}=$

$f''(x)=2(\log^2 x+3\log x+1)\geq 0$ per

$\log x\leq\frac{-3-\sqrt{5}}{2}$ e per $\log x\geq\frac{-3+\sqrt{5}}{2}$



2) $\lim_{x\to 0}\frac{\text{sen}\,3x-\text{arcsen}\,2x}{x}=\lim_{x\to 0}\frac{\text{sen}\,3x}{3x}\cdot 3-\frac{\text{arcsen}\,2x}{2x}\cdot 2=1\cdot 3-1\cdot 2=3-2=1$.

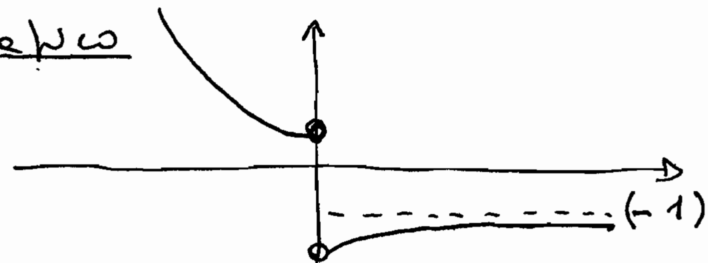
$\lim_{x\to+\infty}(x-\log^2 x)^{x-\text{sen}\,x}=(\to+\infty)^{(\to+\infty)}=+\infty$.

3) Disc. I. specie in $x=0$;

$\lim_{x\to-\infty} f(x)=+\infty$ e $\lim_{x\to+\infty} f(x)=-1$

Decrescente in $]-\infty;0[$; Crescente in $]0;+\infty[$

Grafico



4) $f^{-1}(x)=e^{x-2}=y\Rightarrow x-2=\log y\Rightarrow x=\log y+2\Rightarrow f(x)=\log x+2\Rightarrow f(f(x))=\log(\log x+2)+2$.

5) $\int_1^e\frac{\log x}{x}dx\Rightarrow\int\log x\cdot\frac{1}{x}dx=\int\log x\,d(\log x)=\frac{1}{2}\log^2 x\Rightarrow\left(\frac{1}{2}\log^2 x\right|_1^e=\frac{1}{2}(1-0)=\frac{1}{2}$.

6) $f(x)=x^3-3x^2+5x+1$. Coefficente angolare retta per $(1;2)$ e $(2;4)$: $m=\frac{4-2}{2-1}=2$.

Quindi $f'(x)=3x^2-6x+5=2\Rightarrow 3x^2-6x+3=3(x^2-2x+1)=0\Rightarrow$

$\Rightarrow 3\cdot(x-1)^2=0\Rightarrow x=1$. $f(1)=1-3+5+1=4$. Eq. retta tangente: $y-4=2(x-1)\Rightarrow y=2x+2$.

CMG2

7) $f(x) = e^{x-x^2} \Rightarrow f'(x) = (1-2x)\cdot e^{x-x^2} \Rightarrow$

$\Rightarrow f''(x) = (-2)e^{x-x^2} + (1-2x)(1-2x)\,e^{x-x^2} = \left((1-2x)^2-2\right)\cdot e^{x-x^2} =$

$= f''(x) = (4x^2-4x-1)\,e^{x-x^2} \geq 0$ per $4x^2-4x-1 \geq 0$: $x = \dfrac{2\pm\sqrt{4+4}}{4} =$

$= \dfrac{2\pm\sqrt{8}}{4} = \dfrac{2\pm2\sqrt{2}}{4} = \dfrac{1\pm\sqrt{2}}{2} \Rightarrow f''(x)\geq 0$ per $x \leq \dfrac{1-\sqrt{2}}{2}$ e per $x \geq \dfrac{1+\sqrt{2}}{2}$.

$f(x)$ è convessa in $\left]-\infty; \dfrac{1-\sqrt{2}}{2}\right]$ e in $\left[\dfrac{1+\sqrt{2}}{2}; +\infty\right[$.

8) $f(x;y) = x^3 - 3xy + y^2$. $\nabla f(x;y) = (0;0) \Rightarrow$

$$\Rightarrow \begin{cases} f'_x = 3x^2-3y = 0 \\ f'_y = 2y-3x = 0 \end{cases} \Rightarrow \begin{cases} 3\left(x^2-\frac{3}{2}x\right) = 3\cdot x\cdot\left(x-\frac{3}{2}\right) = 0 \\ y = \frac{3}{2}x \end{cases} \Rightarrow \begin{cases} x=0 \\ y=0 \end{cases} \cup \begin{cases} x = \frac{3}{2} \\ y = \frac{9}{4} \end{cases}.$$

$$H(x;y) = \left\|\begin{matrix} 6x & -3 \\ -3 & 2 \end{matrix}\right\| ; \ H(0;0) = \left\|\begin{matrix} 0 & -3 \\ -3 & 2 \end{matrix}\right\| : |H_2| = -9 < 0 : \text{Sella}.\ H\left(\tfrac{3}{2};\tfrac{9}{4}\right) = \left\|\begin{matrix} 9 & -3 \\ -3 & 2 \end{matrix}\right\| \Rightarrow \begin{cases} 9>0;\ 2>0 \\ 18-9>0 \end{cases}$$

quindi $\left(\frac{3}{2};\frac{9}{4}\right)$ è un punto di minimo.

9) $$A\cdot B = C \Rightarrow \left\|\begin{matrix} 1 & m & k \\ m & k & -1 \end{matrix}\right\| \cdot \left\|\begin{matrix} 1 & 2 \\ 2 & 1 \\ 1 & 1 \end{matrix}\right\| = \left\|\begin{matrix} 1+2m+k & 2+m+k \\ m+2k-1 & 2m+k-1 \end{matrix}\right\| = \left\|\begin{matrix} 3 & 3 \\ 0 & 1 \end{matrix}\right\| \Rightarrow$$

$$\begin{cases} 1+2m+k = 3 \\ 2+m+k = 3 \end{cases} \Rightarrow \begin{cases} 2m+k = 2 \\ m+k = 1 \end{cases} \Rightarrow \begin{cases} 2m+1-m = m+1 = 2 \\ k = 1-m \end{cases} \Rightarrow \begin{cases} m = 1 \\ k = 0 \end{cases}$$ che soddisfano anche le altre due condizioni.

10)

	A	B	C	$A \Rightarrow C$	non B	non B $\Rightarrow$ A	$(A\Rightarrow C)$ e (non B $\Rightarrow$ A)	$C \Rightarrow A$	
1)	1	1	1	1	0	1	1	1	*
2)	1	1	0	0	0	1	0	1	
3)	1	0	1	1	1	1	1	1	*
4)	1	0	0	0	1	1	0	1	
5)	~~0~~	~~1~~	~~1~~	~~1~~	~~0~~	~~1~~	~~1~~	~~0~~	
6)	0	1	0	1	0	1	1	1	*
7)	~~0~~	~~0~~	~~1~~	~~1~~	~~1~~	~~0~~	~~0~~	~~0~~	
8)	0	0	0	1	1	0	0	1	

La proposizione P_1 risulta vera nei casi 1), 3) e 6).