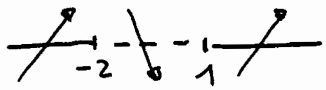
Compito di Matematica Generale del 19/9/2019 $\boxed{CMG1}$

1) $f(x)=2x^3+3x^2-12x+1$. C.E.: $\mathbb{R}$. $\lim_{x\to-\infty} f(x)=-\infty$; $\lim_{x\to+\infty} f(x)=+\infty$.

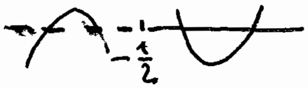
$f(0)=1$

$f'(x)=6x^2+6x-12=6(x^2+x-2)\geq 0$

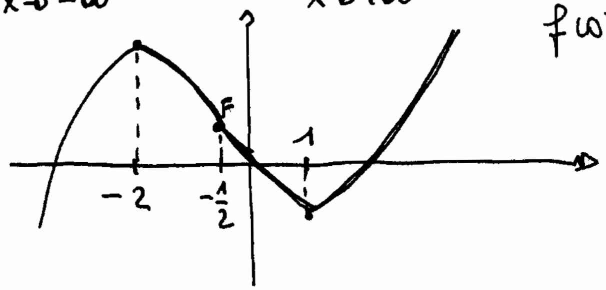
$x=\frac{-1\pm\sqrt{1+8}}{2}=\left\langle\begin{matrix}1\\-2\end{matrix}\right.$



$f''(x)=12x+6=6(2x+1)\geq 0$ per $x\geq-\frac{1}{2}$



$f(-2)=21$; $f(1)=-6$



2) $\lim_{x\to0}\frac{7^x-5^x}{6x}=\frac{1}{6}\lim_{x\to0}\frac{7^x-1}{x}-\frac{5^x-1}{x}=\frac{1}{6}(\log7-\log5)=\log\left(\frac{7}{5}\right)^{\frac{1}{6}}$.

$\lim_{x\to+\infty}\left(1-\frac{1}{3x}\right)^{2x}=\lim_{x\to+\infty}\left[\left(1+\frac{(-1)}{3x}\right)^{3x}\right]^{\frac{2}{3}}=\left(e^{-1}\right)^{\frac{2}{3}}=\frac{1}{\sqrt[3]{e^2}}$.

3) $f(x)=\frac{x-1}{x^2-3x+2}=\frac{x-1}{(x-1)(x-2)}$. C.E.: $\mathbb{R}\setminus\{1;2\}$.

$\lim_{x\to1}f(x)=\lim_{x\to1}\frac{1}{x-2}=-1$: $x=1$ Discontinuità di $\underline{III}^\circ$ specie;

$\lim_{x\to2^-}f(x)=\frac{(\to1)}{(\to1)(\to0^-)}=-\infty$; $\lim_{x\to2^+}f(x)=\frac{(\to1)}{(\to1)(\to0^+)}=+\infty$: $x=2$ Discontinuità di II specie infinita.

4) $f(x)=3^{x-1}$; $g(x)=2^{x+1}$. $f(g(x))=f(2^{x+1})=3^{2^{x+1}-1}=y$ $\Rightarrow$

$\Rightarrow 2^{x+1}-1=\log_3 y \Rightarrow 2^{x+1}=\log_3 y+1 \Rightarrow x+1=\log_2(\log_3 y+1)\Rightarrow x=\log_2(\log_3 y+1)-1$.

Inversa di $f(g(x))$: $y=\log_2(\log_3 x+1)-1$.

5) $\int_0^1 e^{3x}-e^{2x}\,dx=\left(\frac{1}{3}e^{3x}-\frac{1}{2}e^{2x}\right|_0^1=\left(\frac{1}{3}e^3-\frac{1}{2}e^2\right)-\left(\frac{1}{3}-\frac{1}{2}\right)=\frac{1}{6}(2e^3-3e^2+1)$.

6) $f(x)=\log 2x$; $f'(x)=\frac{1}{2x}\cdot2=\frac{1}{x}$. Coeff. angolare perpendicolare $m=-\frac{1}{-3}=\frac{1}{3}$.

$f'(x)=\frac{1}{3}\Rightarrow\frac{1}{x}=\frac{1}{3}\Rightarrow x=3$. $f(3)=\log6\Rightarrow$ Eq. retta tangente:

$y-\log6=\frac{1}{3}(x-3)\Rightarrow y=\frac{1}{3}x-1+\log6=\frac{1}{3}x+\log\frac{6}{e}$.

CMG2

7) $f(x) = x e^{x-1}$; $f(1) = 1$

$f'(x) = 1 \cdot e^{x-1} + x e^{x-1}$; $f'(1) = 1+1 = 2$;

$f''(x) = e^{x-1} + 1 \cdot e^{x-1} + x e^{x-1}$; $f''(1) = 1+1+1 = 3$;

$P_2(x;1) = 1 + 2 \cdot (x-1) + \frac{3}{2}(x-1)^2 \Rightarrow f(x) = 1 + 2(x-1) + \frac{3}{2}(x-1)^2 + o\left((x-1)^2\right)$.

8) $f(x;y;z) = x^{3y} - 3 \,\text{sen}\,(xz) + y^3 z^2$

$\nabla f(x;y;z) = \left(3y x^{3y-1} - 3z \cos(xz);\ x^{3y} \cdot 3 \cdot \log x + 3y^2 z^2;\ -3x \cos(xz) + 2y^3 z\right)$

$\nabla f(1;1;0) = \left(3 \cdot 1 \cdot 1 - 3 \cdot 0 \cdot \cos 0;\ 1 \cdot 3 \cdot 0 + 3 \cdot 1 \cdot 0;\ -3 \cos 0 + 2 \cdot 1 \cdot 0\right) = (3;0;-3)$.

9) $X \cdot A \cdot Y = \begin{Vmatrix} x & 1 \end{Vmatrix} \cdot \begin{Vmatrix} e^x & 2e^x \\ 1 & -x \end{Vmatrix} \cdot \begin{Vmatrix} 1 \\ -1 \end{Vmatrix} = \begin{Vmatrix} x & 1 \end{Vmatrix} \cdot \begin{Vmatrix} e^x - 2e^x \\ 1 + x \end{Vmatrix} = -x e^x + 1 + x = 1 \Rightarrow$

$\Rightarrow x - x e^x = x(1 - e^x) = 0 \Rightarrow \begin{cases} x = 0 \\ e^x = 1 \end{cases} \Rightarrow x = 0$.

10)

A	B	C	$(B \setminus C)$	$(B \setminus A)$	$[(B \setminus C) \setminus (B \setminus A)]$	$A \cap B$
1	1	1	0	0	0	1
1	1	0	1	0	1	1
1	0	1	0	0	0	0
1	0	0	0	0	0	0
0	1	1	0	1	0	0
0	1	0	1	1	0	0
0	0	1	0	0	0	0
0	0	0	0	0	0	0

Dalla $\underline{\text{II}}$ riga vediamo che $[(B \setminus C) \setminus (B \setminus A)] \subset A \cap B$.