TASKS of MATHEMATICS
for Economic Applications AA. 2018/19

Intermediate Test December 2018

I M 1) Find the square roots of the complex number $z = e^{1+3\pi i}$.

I M 2) Given the matrix $\mathbb{A} = \begin{Vmatrix} 1 & 0 & 1 \\ 0 & k & 0 \\ 1 & 0 & 1 \end{Vmatrix}$, find the values for the parameter k such that the matrix admits multiple eigenvalues, and then find the corresponding eigenvectors.

I M 3) In the linear map $f : \mathbb{R}^3 \to \mathbb{R}^4$, $f(\mathbb{X}) = \mathbb{A} \cdot \mathbb{X}$ with $\mathbb{A} = \begin{Vmatrix} 1 & x_1 & y_1 \\ 2 & x_2 & y_2 \\ -1 & x_3 & y_3 \\ 3 & x_4 & y_4 \end{Vmatrix}$ it is :

$\begin{cases} f(1,1,1) = (3,6,-3,9) \\ f(1,-1,1) = (3,4,-7,5) \end{cases}$. Find the dimentions of the Image and of the Kernel of such linear map.

I M 4) Given the vectors $\mathbb{X}_1 = (6,2,2)$, $\mathbb{X}_2 = (4,8,4)$ and $\mathbb{X}_3 = (2,k,-6)$, find the value of the parameter k for which the three vectors are linearly dependent and then a correspon-ding expression for the linear combination between them.

I M 5) Given the matrices $\mathbb{A} = \begin{Vmatrix} 1 & 2 \\ 2 & -1 \end{Vmatrix}$ and $\mathbb{P} = \begin{Vmatrix} 3 & 2 \\ 1 & 1 \end{Vmatrix}$ determine a matrix $\mathbb{B}$ similar to the matrix $\mathbb{A}$.

I Winter Exam Session 2019

I M 1) If the complex number z has a cubic root with modulus equal to 3 and the argument equal to $\frac{\pi}{12}$, determine z and the other two cubic roots of that number.

I M 2) Given the matrix $\mathbb{A} = \begin{Vmatrix} 1 & -1 \\ 1 & k \end{Vmatrix}$, study, on varying the parameter k, its eigenvalues, determining when they are real and when they are complex, when they are simple and when they are multiple.

I M 3) Given the vectors $\mathbb{X}_1 = (1,1,1)$, $\mathbb{X}_2 = (1,-1,0)$ and $\mathbb{X}_3 = (2,0,-1)$, after ha-ving verified that they form a basis for $\mathbb{R}^3$, find the coordinates in this base of the vector $\mathbb{Y} = (1,3,2)$. What is the meaning of the result found ?

I M 4) Given the matrix $\mathbb{A} = \begin{Vmatrix} 1 & -1 & 0 & 1 & -1 \\ 1 & 1 & 1 & 0 & 1 \\ 2 & m & 1 & 1 & k \end{Vmatrix}$, check, on varying the parameters m and k, dim(Ker) and dim(Imm) of the linear map $f : \mathbb{R}^5 \to \mathbb{R}^3$, $f(\mathbb{X}) = \mathbb{A} \cdot \mathbb{X}$, and determi-ne, when dim(Imm) is minimal, if the vector $\mathbb{Y} = (2,-1,2)$ belongs or not to Imm($\mathbb{A}$).

II M 1) Given the equation $f(x,y,z) = x \sin y - y \cos z + x^2 z = 0$, satisfied at the point $P = (1,0,0)$, verify if an implicit function can be defined with such equation and then calcu-late the gradient of such function at the proper point.

II M 2) Solve the problem: $\begin{cases} \text{Max/min } f(x,y) = x + y \\ \text{u.c. } x^2 \le y \le 1 - x^2 \end{cases}$.

II M 3) Given the function $f(x,y) = x\, e^{y-x}$ and the unit vector $v = (\cos\alpha, \sin\alpha)$, find the values for the parameter α for which $D_v f(1,1) = 0$.

II M 4) Given the Hessian matrix $\mathbb{H} = \begin{Vmatrix} k & 0 \\ 0 & k-1 \end{Vmatrix}$ calculated at a certain stationary point, check, on varying the parameter k, the nature of the stationary point.

II Winter Exam Session 2019

I M 1) Calculate the sum of the six roots of the equation $x^6 + x^2 = 0$.

I M 2) Given the matrix $\mathbb{A} = \begin{Vmatrix} 1 & 4 & 3 \\ 2 & 3 & 1 \\ 0 & 0 & k \end{Vmatrix}$ find the values of the parameter k for which the matrix admits multiple eigenvalues and then check if, for these values, the given matrix is diagonalizable.

I M 3) Given the linear system $\begin{cases} x_1 - x_2 - x_3 + x_4 = 1 \\ 2x_1 + x_2 + 2x_3 + x_4 = -1 \\ 3x_1 + x_3 + hx_4 = m \end{cases}$ determine the values of h and m for which the system admits ∞^2 solutions and then find such solutions.

I M 4) Starting from the vector $\mathbb{X}_1 = (1, 0, -1)$, determine an orthonormal basis for $\mathbb{R}^3$.

II M 1) Given the equation $f(x,y) = x\,e^{x+y} + y\,e^{x-y} = 0$ satisfied at the point $P = (0,0)$, verify that it is possible to define an implicit function $x \to y(x)$ and then calculate the derivative of such function at the proper point.

II M 2) Solve the problem: $\begin{cases} \text{Max/min } f(x,y) = xy^2 - x^2 \\ \text{u.c. } x^2 + y^2 \leq 1 \end{cases}$.

II M 3) Given the function $f(x,y) = x\,e^{y-x} - y\,e^{x-y}$ and the unit vector $v = (\cos\alpha, \sin\alpha)$, find the values for the parameter α for which $D_v f(0,0) = D_v f(1,1)$.

II M 4) Given the function $f(x,y,z) = xy^2 - x^2 - y^2 - z^2$ analyze the nature of its stationary points.

I Additional Exam Session 2019

I M 1) If $z = (1-i)^3$, calculate $\sqrt{z}$.

I M 2) Given the matrix $\mathbb{A} = \begin{Vmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{Vmatrix}$ find an orthogonal matrix which diagonalizes $\mathbb{A}$.

I M 3) Given the linear system $\begin{cases} x_1 + x_2 + x_3 + x_4 = 1 \\ x_1 + x_2 + x_3 + 2x_4 = 1 \\ 3x_1 + 3x_2 + 3x_3 + hx_4 = m \end{cases}$ determine the values of h and m for which the system admits solutions and then calculate the number of such solutions.

I M 4) If the vector $\mathbb{X}$ has coordinates $(1, -1)$ in the basis $\mathbb{V} = \{(2,1); (1,1)\}$, determine its coordinates in the basis $\mathbb{W} = \{(3,1); (2,1)\}$.

II M 1) Given the equation $f(x,y,z) = x\,e^y + y\,e^z - xz = 0$, satisfied at the point $P = (1,0,1)$, verify that it is possible to define an implicit function $(x,y) \to z(x,y)$ and then calculate the derivatives of such function at the proper point.

II M 2) Solve the problem: $\begin{cases} \text{Max/min } f(x,y,z) = x^2 + y^2 + z^2 \\ \text{u.c.: } \begin{cases} x - y + z = 1 \\ x + y - z = 1 \end{cases} \end{cases}$.

II M 3) Given the function $f(x,y) = x^2y - xy^2 + y$ and the unit vectors $v = (\cos\alpha, \sin\alpha)$ and $w = (1,0)$, find the values for the parameter α for which $D_v f(1,1) = D_w f(1,1)$.

II M 4) Given the function $f(x,y)=(x-2y)^2+(y-2x)^2$ analyze the nature of its stationary points.

I Summer Exam Session 2019

I M 1) If $z=e^{1-3\pi i}$, calculate $\sqrt[3]{z}$.

I M 2) The matrix $\mathbb{A}=\begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix}$ admits the eigenvector $(1,1)$ corresponding to the eigenvalue $\lambda=0$ and the eigenvector $(1,-1)$ corresponding to the eigenvalue $\lambda=1$. Find the matrix $\mathbb{A}$.

I M 3) Check if the matrix $\mathbb{A}=\begin{Vmatrix} 3 & 2 & 1 \\ 1 & 4 & 4 \\ 1 & 2 & 5 \end{Vmatrix}$ is a diagonalizable one.

I M 4) Given the linear map $f:\mathbb{R}^3\to\mathbb{R}^2, f(\mathbb{X})=\mathbb{A}\cdot\mathbb{X}$ with $\mathbb{A}=\begin{Vmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \end{Vmatrix}$ find a basis for the Kernel of such map and then find all the vectors having $(1,1)$ as their image.

II M 1) Given the function $f(x,y)=x^2-k\,xy+y^2$, v the unit vector of $(1,1)$ and w the unit vector of $(1,2)$, find the value for the parameter k for which $D_v f(1,1)=D_w f(1,1)$ and then calculate $D^2_{v,w}f(1,1)$.

II M 2) Given the system $\begin{cases} f(x,y,z)=x^2+y^2+z^2-3xyz=0 \\ g(x,y,z)=e^{x-y}+e^{y-z}-2\,e^{z-x}=0 \end{cases}$, satisfied at the point $P=(1,1,1)$, verify that it is possible to define an implicit function $x\to(y(x),z(x))$ and then calculate the derivatives of such function at $x=1$.

II M 3) Solve the problem: $\begin{cases} \text{Max/min } f(x,y)=x^2+y \\ \text{u.c.: } \begin{cases} x^2+y^2\le 1 \\ y\ge 0 \end{cases} \end{cases}$.

II M 4) Given the function $f(x,y)=x^2-xy-x+y^2$ analyze the nature of its stationary points.

II Summer Exam Session 2019

I M 1) If $z_1=e^{-1+3\pi i}$ and $z_2=e\left(\cos\frac{3\pi}{4}+i\,\text{sen}\,\frac{3\pi}{4}\right)$, calculate $\sqrt{z_1\cdot z_2}$.

I M 2) Given the matrix $\mathbb{A}=\begin{Vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{Vmatrix}$ find an orthogonal matrix which diagonalizes $\mathbb{A}$.

I M 3) The matrix $\mathbb{A}=\begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix}$ admits the eigenvectors $(1,1)$ and $(1,-1)$ both corresponding to the eigenvalue $\lambda=1$. Find the matrix $\mathbb{A}$.

I M 4) Given the vector subspace $\mathbb{W}$ of $\mathbb{R}^3$ having base $\{\mathbb{W}_1=(1,0,1);\mathbb{W}_2=(1,1,0)\}$, check the values for the parameter k for which the vector $\mathbb{X}=(1,1,k)$ belongs to $\mathbb{W}$.

II M 1) Given the function $f(x,y)=x\,e^y-y\,e^x$ and the unit vector $v=(\cos\alpha,\text{sen}\,\alpha)$, find the values of α for which $D^2_{v,v}f(1,1)=e$.

II M 2) Given the equation $f(x,y,z) = x\,e^{y-z} - y\,e^{x-z} = 0$, satisfied at $P = (0,0,0)$, verify that it is possible to define an implicit function $(x,z) \to y(x,z)$ and then calculate the derivatives of such function at $y = 0$.

II M 3) Solve the problem: $\begin{cases} \text{Max/min } f(x,y) = x\,y^2 \\ \text{u.c.: } \begin{cases} 3y + x - 3 \le 0 \\ 0 \le x \\ 0 \le y \end{cases} \end{cases}$.

II M 4) Given the function $f(x,y,z) = x^3 - 4x^2 + y^2 + z^2 + 5x - 2y$ analyze the nature of its stationary points.

I Autumn Exam Session 2019

I M 1) Determine all the roots of the equation $x^4 - 16 = 0$.

I M 2) Given the matrix $\mathbb{A} = \begin{Vmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & k \end{Vmatrix}$ determine the value of the parameter k for which the matrix admits the imaginary unit i as an eigenvalue. For this value of k, find all the eigenvalues of the matrix and check if it is diagonalizable or not.

I M 3) Given the linear system $\begin{cases} x_1 + 2x_2 - x_3 + x_4 = 1 \\ x_1 + x_2 + m\,x_3 + x_4 = 2 \\ 2x_1 + 3x_2 + 2x_3 + kx_4 = 3 \end{cases}$, check for the existence and number of its solutions on varying the parameters m and k.

I M 4) Given the two orthogonal vectors $\mathbb{X}_1 = (1,1,0)$ and $\mathbb{X}_2 = (1,-1,0)$, find a third vector $\mathbb{X}_3$ orthogonal to $\mathbb{X}_1$ and $\mathbb{X}_2$, so as to create a basis for $\mathbb{R}^3$. Then find the coordinates of the vector $\mathbb{Y} = (1,1,1)$ in this basis.

II M 1) Given $f(x,y) = x\,e^y - y\,e^x$, determine all the directions $v = (\cos\alpha, \sin\alpha)$ for which it results $\mathcal{D}_v f(0,0) = \mathcal{D}^2_{vv} f(0,0)$.

II M 2) Given the system $\begin{cases} f(x,y,z) = xyz - x + y - z = 0 \\ g(x,y,z) = e^{x-y} - e^{y-z} = 0 \end{cases}$ and the point $\mathrm{P}_0 = (1,1,1)$, determine at least one implicit function that can be defined with it and then calculate the derivatives of such function at the proper point.

II M 3) Solve the problem: $\begin{cases} \text{Max/min } f(x,y) = 2x - y^2 \\ \text{u.c. : } y^2 \le x \le 1 \end{cases}$.

II M 4) Given the function $f(x,y) = x^3 - 3xy + y^2$ analyze the nature of its stationary points.

II Autumn Exam Session 2019

I M 1) Calcolate $\sqrt{\dfrac{2(1-i)(2+3i)}{(5+i)(1+i)}}$.

I M 2) Find eigenvalues and corresponding eigenvectors of the matrix $\mathbb{A} = \begin{Vmatrix} 1 & 2 & 1 \\ 4 & 0 & 0 \\ 1 & 2 & 1 \end{Vmatrix}$.

I M 3) Consider the linear map $f : \mathbb{R}^4 \to \mathbb{R}^3$, $\mathbb{Y} = \mathbb{A} \cdot \mathbb{X}$ for which:
$f(x_1,x_2,x_3,x_4) = (x_1 + x_2 + x_3; x_1 + x_3 + x_4; x_1 + x_2 + x_4)$.
Determine a basis for the Kernel and a basis for the Image of this linear map.

I M 4) Since the vectors $\mathbb{X}_1 = (1, 2, -1)$, $\mathbb{X}_2 = (2, 0, 1)$ e $\mathbb{X}_3 = (x_1, x_2, x_3)$ form a basis for $\mathbb{R}^3$ and since in this basis the coordinates of the vector $\mathbb{Y} = (1, 3, 0)$ are $(2, -2, 1)$, determine $\mathbb{X}_3$.

II M 1) Given the equation $f(x, y) = x^3 y + xy^2 - 2y = 0$ and the point $\mathrm{P}_0 = (1, 1)$ satisfying it, determine first and second order derivatives of the implicit function $x \to y(x)$ definable with it.

II M 2) Given the function $f(x, y) = x\, y^3 - 3x^2 + 3x - 2y$ and the point $P_0 = (1, -2)$, compute $\mathcal{D}_v f(0, 0)$, where v represents the direction from the origin $(0, 0)$ to P_0.

II M 3) Solve the problem: $\begin{cases} \text{Max/min } f(x, y) = y\,(x - 1) \\ \text{u.c.: } \begin{cases} x^2 - y - 1 \le 0 \\ y \le 0 \end{cases} \end{cases}$.

II M 4) Given the function $f(x, y) = x^3 - kxy + y^2$ analyze the nature of its stationary points on varying the parameter k.

II Additional Exam Session 2019

I M 1) Calcolate $\sqrt{(1 + i)^4 (1 - i)^6}$.

I M 2) Find the eigenvalues of the matrix $\mathbb{A} = \begin{Vmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{Vmatrix}$.

I M 3) Given the linear map $f: \mathbb{R}^4 \to \mathbb{R}^3$, $\mathbb{Y} = \mathbb{A} \cdot \mathbb{X}$, with $\mathbb{A} = \begin{Vmatrix} 1 & 1 & 2 & 1 \\ 1 & -1 & m & 0 \\ 1 & -1 & 1 & k \end{Vmatrix}$, determine a basis for the Kernel and a basis for the Image of the linear map generated by $\mathbb{A}$, knowing that the Kernel and the Image have the same dimensions.

I M 4) Given the matrix $\mathbb{A} = \begin{Vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{Vmatrix}$ determine its inverse matrix.

II M 1) Given the equation $f(x, y) = x^3 y - xy^3 + x - y = 1$ and the point $\mathrm{P}_0 = (1, 0)$ satisfying it, determine first order derivative of a possible implicit function definable with it.

II M 2) Given the functions $f(x, y) = x^2 + y^2$ and $g(x, y) = x + xy$, the point $P_0 = (1, 1)$ and the unit vector $v = (\cos\alpha, \sin\alpha)$, find the values of α for which $\mathcal{D}_v f(P_0) = \mathcal{D}_v g(P_0)$.

II M 3) Determine maximum and minimum points for the function $f(x, y) = x^2 - y$ in the rectangle $\mathbb{Q} = \{(x, y) \in \mathbb{R}^2 \colon 0 \le x \le 2 ;\, -1 \le y \le 0\}$.

II M 4) Given the function $f(x, y) = x^3 - x^2 y^2 + y^2$ analyze the nature of its stationary points.