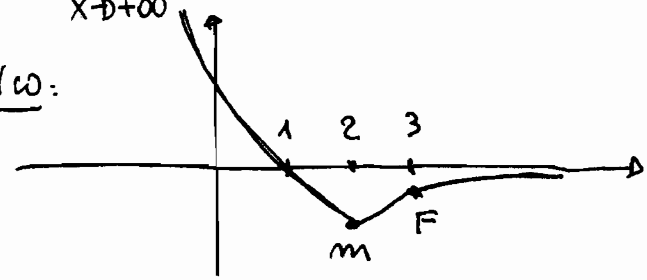
Compito di Matematica Generale del 12/10/2019 $\boxed{CMG1}$

1) $f(x) = (1-x)\,e^{-x}$. C.E.: $\mathbb{R}$. $\lim\limits_{x\to-\infty} f(x) = +\infty$; $\lim\limits_{x\to+\infty} f(x) = 0^-$. $f(x)\geq 0$ per $x \leq 1$.

$f'(x) = -e^{-x} - (1-x)\,e^{-x} = e^{-x}(x-2) \geq 0$ per $x \geq 2$

$f''(x) = -e^{-x}(x-2) + e^{-x} = e^{-x}(3-x) \geq 0$ per $x \leq 3$

Grafico:



2) $\lim\limits_{x\to 0} \dfrac{\operatorname{sen} 3x - \operatorname{sen} 2x + \operatorname{sen} x}{6x} = \lim\limits_{x\to 0} \dfrac{1}{6}\left(\dfrac{\operatorname{sen} 3x}{3x}\cdot 3 - \dfrac{\operatorname{sen} 2x}{2x}\cdot 2 + \dfrac{\operatorname{sen} x}{x}\right) = \dfrac{1}{6}(3-2+1) = \dfrac{1}{3}$.

$\lim\limits_{x\to+\infty} \left(\dfrac{1+\log x}{3x+1}\right)^{1-x} = (\to 0^+)^{(\to -\infty)} = +\infty$ $\big(\log x = o(3x)\big)$.

3) $f(x) = 2^{x+1}$. $f(g(x)) = 2^{g(x)+1} = x-1 \Rightarrow g(x)+1 = \log_2 (x-1) \Rightarrow g(x) = \log_2 (x-1) - 1 \Rightarrow$

$\Rightarrow \log_2 (x-1) - 1 = y \Rightarrow y+1 = \log_2 (x-1) \Rightarrow x-1 = 2^{y+1} \Rightarrow x = 2^{y+1} + 1$.

Quindi $g^{-1}(x) = 2^{x+1} + 1$.

4) $f(x) = x^2 - 3x + 2 = (x-1)(x-2)$; $g(x) = x^2 - 4x + 4 = (x-2)^2$.

$f(x) \sim g(x)$ se $\lim\limits_{x\to x_0} \dfrac{f(x)}{g(x)} = 1 \Rightarrow \lim\limits_{x\to x_0} \dfrac{(x-1)(x-2)}{(x-2)^2} = (x\neq 2) \lim\limits_{x\to x_0} \dfrac{x-1}{x-2} = 1$ impossibile se $x_0 \in \mathbb{R}$

$\lim\limits_{x\to\pm\infty} \dfrac{f(x)}{g(x)} = 1$ quindi $f(x)\sim g(x)$ se $x \to -\infty$ e se $x\to+\infty$;

$f(x) = o(g(x))$ se $\lim\limits_{x\to x_0} \dfrac{f(x)}{g(x)} = 0 \Rightarrow (x\neq 2) \lim\limits_{x\to x_0} \dfrac{x-1}{x-2} = 0$ se $x\to 1$;

$g(x) = o(f(x))$ se $\lim\limits_{x\to x_0} \dfrac{g(x)}{f(x)} = 0 \Rightarrow (x\neq 2) \lim\limits_{x\to x_0} \dfrac{x-2}{x-1} = 0$ se $x\to 2$.

5) $\displaystyle\int_1^e \frac{1}{2x} - \frac{1}{3x}\,dx = \left(\frac{1}{2}\log x - \frac{1}{3}\log x\right|_1^e = \left(\frac{1}{6}\log x\right|_1^e = \frac{1}{6}(1-0) = \frac{1}{6}$.

6) $f(x) = \log^2 x - 2\log x$; $f(1) = 0 - 0 = 0$; $f'(x) = 2\cdot\dfrac{1}{x}\log x - 2\cdot\dfrac{1}{x}$;

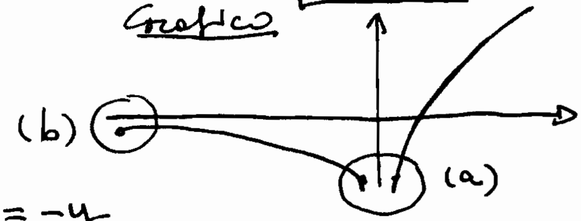
$f'(1) = 0 - 2 = -2$. Equazione retta tangente: $y - 0 = -2(x-1) \Rightarrow y = -2x + 2$.

CMG2

7) $\forall \varepsilon \quad \exists \delta(\varepsilon): 0<|x|<\delta \Rightarrow f(x) < \varepsilon : \lim_{x \to 0} f(x) = -\infty$

$\forall \varepsilon > 0 \; \exists \delta(\varepsilon): x < \delta \Rightarrow |f(x)| < \varepsilon : \lim_{x \to -\infty} f(x) = 0$

Grafico



8) $X \perp Y_1 : (x;y;z)\cdot(0;1;1) = y+z = 0 \Rightarrow z = -y$

$X \perp Y_2 : (x;y;z)\cdot(1;1;0) = x+y = 0 \Rightarrow x = -y$

Quindi $X = (-y;y;-y) \Rightarrow \|X\| = \sqrt{y^2+y^2+y^2} = \sqrt{3y^2} = \sqrt{27} \Rightarrow y^2 = 9 \Rightarrow y = \pm 3.$

Quindi $X = (3;-3;3)$ oppure $X = (-3;3;3)$.

9) $f(x;y) = 12x^2 + 4xy - 2x + y^2$. $\nabla f(x;y) = (0;0) \Rightarrow$

$$\Rightarrow \begin{cases} f'_x = 24x + 4y - 2 = 2(12x + 2y - 1) = 0 \\ f'_y = 4x + 2y = 0 \end{cases} \Rightarrow \begin{cases} 8x - 1 = 0 \\ y = -2x \end{cases} \Rightarrow \begin{cases} x = \frac{1}{8} \\ y = -\frac{1}{4} \end{cases}.$$

$$H(x;y) = \begin{Vmatrix} 24 & 4 \\ 4 & 2 \end{Vmatrix} = H\left(\tfrac{1}{8}; -\tfrac{1}{4}\right) \Rightarrow \begin{cases} |H_1| = 24 > 0 ; 2 > 0 \\ |H_2| = 48 - 16 > 0 \end{cases} \Rightarrow \left(\tfrac{1}{8}; -\tfrac{1}{4}\right) \text{ Punto di minimo.}$$

10)

A	B	C	$(B \sigma C)$	$A e (B \sigma C)$	$(A e B)$	$(A e B) \sigma C$	$[(A e (B \sigma C)] \Rightarrow [(A e B) \sigma C]$
1	1	1	1	1	1	1	1
1	1	0	1	1	1	1	1
1	0	1	1	1	0	1	1
1	0	0	0	0	0	0	1
0	1	1	1	0	0	1	1
0	1	0	1	0	0	0	1
0	0	1	1	0	0	1	1
0	0	0	0	0	0	0	1
				*		*	

La proposizione $[A e (B \sigma C)] \Rightarrow [(A e B) \sigma C]$ è una tautologia.