Compito di Matematica Generale del 8/1/2020 $\boxed{\text{MGA1}}$

1) $f(x) = x + \log(1-x)$. C.E.: $x < 1$. $\lim\limits_{x\to-\infty} f(x) = -\infty$; $\lim\limits_{x\to 1^-} f(x) = -\infty$.

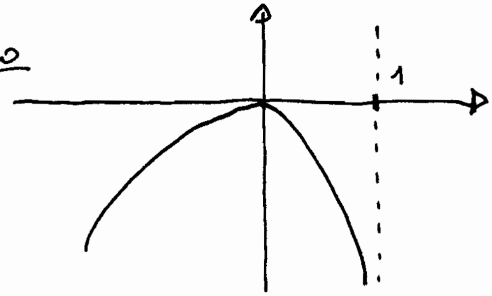
$f'(x) = 1 - \frac{1}{1-x} = \frac{-x}{1-x} \geq 0$ per $x \leq 0$

$f(0) = 0 \Rightarrow f(x) \leq 0 \ \forall x \in C.E.$

$f''(x) = -\left(-\frac{1}{(1-x)^2}\cdot(-1)\right) = -\frac{1}{(1-x)^2} < 0 \ \forall x \in C.E.$

funzione sempre concava.

Grafico



2) $\lim\limits_{x\to 0} \frac{\text{sen}^2 3x}{1-\cos 2x} = \lim\limits_{x\to 0} \frac{\text{sen}^2 3x}{(3x)^2} \cdot \frac{9x^2}{4x^2} \cdot \frac{4x^2}{1-\cos 2x} = 1 \cdot \frac{9}{4} \cdot 2 = \frac{9}{2}$.

$\lim\limits_{x\to+\infty} \left(1 + \frac{2}{3x-1}\right)^{1-x} = \lim\limits_{x\to+\infty} \left[\left(1+\frac{2}{3x-1}\right)^{3x-1}\right]^{\frac{1-x}{3x-1}} = \left(e^2\right)^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{e^2}}$.

3) $\lim\limits_{x\to 0} \frac{2^{kx}-1}{\log(1+2x)} = \lim\limits_{x\to 0} \frac{2^{kx}-1}{kx} \cdot \frac{k}{2} \cdot \frac{2x}{\log(1+2x)} = \frac{k}{2} \cdot \log 2 = 3 \Rightarrow k = \frac{6}{\log 2} = 6 \cdot \log_2 e$.

4) $f(x) = 2^{x+3} = y \Rightarrow x+3 = \log_2 y \Rightarrow x = \log_2 y - 3$; $f^{-1}(x) = \log_2 x - 3$;

$g(x) = \frac{x-2}{2x+1} = y \Rightarrow x - 2 = 2xy + y \Rightarrow x(1-2y) = y+2 \Rightarrow x = \frac{y+2}{1-2y}$; $g^{-1}(x) = \frac{x+2}{1-2x}$.

$f^{-1}(g^{-1}(x)) = f^{-1}\left(\frac{x+2}{1-2x}\right) = \log_2\left(\frac{x+2}{1-2x}\right) - 3$; $g^{-1}(f^{-1}(x)) = g^{-1}(\log_2 x - 3) = \frac{\log_2 x - 1}{7 - 2\log_2 x}$.

5) Se $f'(x) = x - e^{3x} \Rightarrow f(x) = \int x - e^{3x}\,dx + k = \frac{x^2}{2} - \frac{1}{3}e^{3x} + k$.

Se $f(0) = 1 \Rightarrow \frac{0}{2} - \frac{1}{3}e^0 + k = k - \frac{1}{3} = 1 \Rightarrow k = \frac{4}{3}$. Quindi $f(x) = \frac{x^2}{2} - \frac{1}{3}e^{3x} + \frac{4}{3}$.

6) $f(x;y) = X \cdot Y = (xy; 2-3y)\cdot(x+4; xy) = x^2y + 4xy + 2xy - 3xy^2 = x^2y + 6xy - 3xy^2$.

Condizioni I Ordine: $\nabla f = (0;0) \Rightarrow \begin{cases} f'_x = 2xy + 6y - 3y^2 = y(2x+6-3y) = 0 \\ f'_y = x^2 + 6x - 6xy = x(x+6-6y) = 0 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases} \cup \begin{cases} x = 0 \\ y = 2 \end{cases} \cup \begin{cases} x = -6 \\ y = 0 \end{cases} \cup \begin{cases} x = 6y - 6 \\ 12y - 12 + 6 - 3y = 0 \end{cases} \Rightarrow \begin{cases} x = 6y-6 \\ 9y = 6 \end{cases} \Rightarrow \begin{cases} x = -2 \\ y = \frac{2}{3} \end{cases}$.

MGA2

$H(x;y)=\begin{Vmatrix} 2y & 2x+6-6y \\ 2x+6-6y & -6x \end{Vmatrix}$. $H(0;0)=\begin{Vmatrix} 0 & 6 \\ 6 & 0 \end{Vmatrix}$: $|H_2|=-36<0 \Rightarrow (0;0)$ Punto di Sella ;

$H(0;2)=\begin{Vmatrix} 4 & -6 \\ -6 & 0 \end{Vmatrix}$: $|H_2|=-36<0 \Rightarrow (0;2)$ Punto di Sella ;

$H(-6;0)=\begin{Vmatrix} 0 & -6 \\ -6 & 36 \end{Vmatrix}$: $|H_2|=-36<0 \Rightarrow (-6;0)$ Punto di Sella;

$H(-2;\frac{2}{3})=\begin{Vmatrix} \frac{4}{3} & -2 \\ -2 & 12 \end{Vmatrix}$: $\begin{cases} |H_1|=\frac{4}{3}>0;\ |H_1|=12>0 \\ |H_2|=\frac{4}{3}\cdot 12-4=16-4>0 \end{cases} \Rightarrow (-2;\frac{2}{3})$ Punto di minimo.

7) $f(x)=e^{2x}-4e^x \Rightarrow f'(x)=2e^{2x}-4e^x$; $g(x)=6x-1 \Rightarrow g'(x)=6$.

Per avere tangenti parallele: $f'(x_0)=g'(x_0) \Rightarrow 2e^{2x}-4e^x=6 \Rightarrow e^{2x}-2e^x-3=0 \Rightarrow$

$\Rightarrow e^x=1\pm\sqrt{1+3}=1\pm 2=\left\langle\begin{array}{l}-1\\3\end{array}\right.$. Quindi $e^x=3 \Rightarrow x_0=\log 3$.

8) $A\cdot X=\begin{Vmatrix} 1 & 2 & 1 \\ -1 & 2 & 1 \\ 2 & 2 & -1 \end{Vmatrix}\cdot\begin{Vmatrix} k \\ 1 \\ k \end{Vmatrix}=\begin{Vmatrix} k+2+k \\ -k+2+k \\ 2k+2-k \end{Vmatrix}=\begin{Vmatrix} 2k+2 \\ 2 \\ k+2 \end{Vmatrix}$. $(2k+2;2;k+2)//(1;1;1) \Rightarrow$

$\Rightarrow \frac{2k+2}{1}=\frac{2}{1}=\frac{k+2}{1}$: Vera se $k=0$. $A\cdot X \perp Y \Rightarrow (2k+2;2;k+2)(1;1;1)=3k+6=0 \Rightarrow k=-2$.

9) $f(x)=x^3-kx^2+3$: funzione continua e derivabile $\forall x\in\mathbb{R}$. Deve essere:

$f(-1)=f(2) \Rightarrow -1-k+\cancel{3}=8-4k+\cancel{3} \Rightarrow 3k=9 \Rightarrow k=3 \Rightarrow f(x)=x^3-3x^2+3$.

Quindi $f'(x)=3x^2-6x=3x(x-2)=0 \Rightarrow f'(x)=0$ per $x=0$ ed anche per $x=2$.

10)

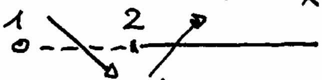
A B	(A e B)	(A ⇒ (A e B))	(non A)	(non A o B)	B ⇔ (non A o B)
1 1	1	1	0	1	1
1 0	0	0	0	0	1
0 1	0	1	1	1	1
0 0	0	1	1	1	0
		(P_1)			(P_2)

Per avere una Tautologia occorre la proposizione P_1 o P_2.

Compito di Matematica Generale del 8/1/2020 $\boxed{MGB1}$

1) $f(x) = x - \log(x-1)$. C.E.: $x > 1$. $\lim_{x\to 1^+} f(x) = +\infty$; $\lim_{x\to +\infty} f(x) = +\infty$.

$f'(x) = 1 - \frac{1}{x-1} = \frac{x-2}{x-1} \geq 0$ per $x \geq 2$

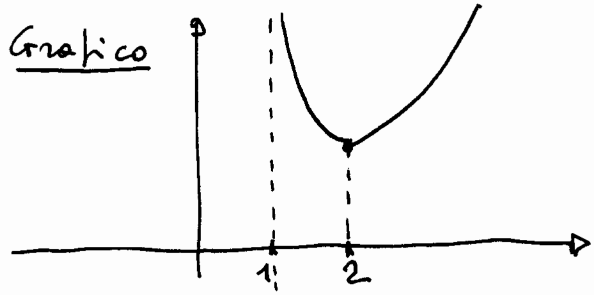


$f(2) = 2 \Rightarrow f(x) > 0 \ \forall x \in$ C.E.

$f''(x) = -\left(-\frac{1}{(x-1)^2}\right) = \frac{1}{(x-1)^2} > 0 \ \forall x \in$ C.E.

funzione sempre convessa.

Grafico



2) $\lim_{x\to 0} \frac{\operatorname{sen}^2 2x}{1-\cos 3x} = \lim_{x\to 0} \frac{\operatorname{sen}^2 2x}{(2x)^2} \cdot \frac{4x^2}{9x^2} \cdot \frac{9x^2}{1-\cos 3x} = 1 \cdot \frac{4}{9} \cdot 2 = \frac{8}{9}$.

$\lim_{x\to +\infty} \left(1 - \frac{3}{2x-1}\right)^{x-2} = \lim_{x\to +\infty} \left[\left(1 + \frac{(-3)}{2x-1}\right)^{2x-1}\right]^{\frac{x-2}{2x-1}} = (e^{-3})^{\frac{1}{2}} = \frac{1}{\sqrt{e^3}} = \frac{1}{e\sqrt{e}}$.

3) $\lim_{x\to 0} \frac{\log(1-x)}{3^{kx}-1} = \lim_{x\to 0} \frac{\log(1+(-x))}{-x} \cdot \frac{-x}{kx} \cdot \frac{kx}{3^{kx}-1} = 1 \cdot \left(-\frac{1}{k}\right) \cdot \frac{1}{\log 3} = 2 \Rightarrow k = -\frac{1}{2} \cdot \log_3 e$.

4) $f(x) = \frac{2x-1}{x+2} = y \Rightarrow 2x - 1 = xy + 2y \Rightarrow x(2-y) = 2y+1 \Rightarrow x = \frac{2y+1}{2-y}$; $f^{-1}(x) = \frac{2x+1}{2-x}$;

$g(x) = 3^{x-2} = y \Rightarrow x - 2 = \log_3 y \Rightarrow x = \log_3 y + 2$; $g^{-1}(x) = \log_3 x + 2$.

$f^{-1}(g^{-1}(x)) = f^{-1}(\log_3 x + 2) = \frac{2\log_3 x + 5}{-\log_3 x}$; $g^{-1}(f^{-1}(x)) = g^{-1}\left(\frac{2x+1}{2-x}\right) = \log_3 \frac{2x+1}{2-x} + 2$.

5) Se $f'(x) = e^{2x} - x \Rightarrow f(x) = \int e^{2x} - x \, dx + k = \frac{1}{2}e^{2x} - \frac{x^2}{2} + k$.

Se $f(0) = 2 \Rightarrow \frac{1}{2}e^0 - \frac{0}{2} + k = k + \frac{1}{2} = 2 \Rightarrow k = \frac{3}{2}$. Quindi $f(x) = \frac{1}{2}e^{2x} - \frac{x^2}{2} + \frac{3}{2}$.

6) $f(x;y) = X \cdot Y = (3x-2; xy) \cdot (xy; y+4) = 3x^2y - 2xy + xy^2 + 4xy = 3x^2y + 2xy + xy^2$.

Condizioni I Ordine: $\nabla f = (0;0) \Rightarrow \begin{cases} f'_x = 6xy + 2y + y^2 = y(6x + 2 + y) = 0 \\ f'_y = 3x^2 + 2x + 2xy = x(3x + 2 + 2y) = 0 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases} \cup \begin{cases} x = 0 \\ y = -2 \end{cases} \cup \begin{cases} x = -\frac{2}{3} \\ y = 0 \end{cases} \cup \begin{cases} y = -6x - 2 \\ 3x + 2 - 12x - 4 = 0 \end{cases} \Rightarrow \begin{cases} y = -6x - 2 \\ 9x = -2 \end{cases} \Rightarrow \begin{cases} x = -\frac{2}{9} \\ y = -\frac{2}{3} \end{cases}$.

MGB2

$H(x;y)=\begin{Vmatrix} 6y & 6x+2+2y \\ 6x+2+2y & 2x \end{Vmatrix}$. $H(0;0)=\begin{Vmatrix} 0 & 2 \\ 2 & 0 \end{Vmatrix}$: $|H_2|=-4<0 \Rightarrow (0;0)$ Punto di Sella;

$H(0;-2)=\begin{Vmatrix} -12 & -2 \\ -2 & 0 \end{Vmatrix}$: $|H_2|=-4<0 \Rightarrow (0;-2)$ Punto di Sella;

$H(-\frac{2}{3};0)=\begin{Vmatrix} 0 & -2 \\ -2 & -\frac{4}{3} \end{Vmatrix}$: $|H_2|=-4<0 \Rightarrow (-\frac{2}{3};0)$ Punto di Sella;

$H(-\frac{2}{9};-\frac{2}{3})=\begin{Vmatrix} -4 & -\frac{2}{3} \\ -\frac{2}{3} & -\frac{4}{9} \end{Vmatrix}$: $\begin{cases} |H_1|=-4<0; |H_1|=-\frac{4}{9}<0 \\ |H_2|=\frac{16}{9}-\frac{4}{9}>0 \end{cases} \Rightarrow (-\frac{2}{9};-\frac{2}{3})$ Punto di Massimo.

7) $f(x)=e^{2x}+2e^x \Rightarrow f'(x)=2e^{2x}+2e^x$; $g(x)=4x-1 \Rightarrow g'(x)=4$.

Per avere tangenti parallele: $f'(x_0)=g'(x_0) \Rightarrow 2e^{2x}+2e^x=4 \Rightarrow e^{2x}+e^x-2=0 \Rightarrow$

$\Rightarrow e^x=\frac{-1\pm\sqrt{1+8}}{2}=\frac{-1\pm 3}{2}=\begin{cases}-2\\1\end{cases}$. Quindi $e^x=1 \Rightarrow x_0=0$.

8) $A\cdot X=\begin{Vmatrix} 1 & 1 & 2 \\ 1 & -1 & 2 \\ -1 & 2 & 2 \end{Vmatrix}\cdot\begin{Vmatrix} K \\ K \\ 1 \end{Vmatrix}=\begin{Vmatrix} K+K+2 \\ K-K+2 \\ -K+2K+2 \end{Vmatrix}=\begin{Vmatrix} 2K+2 \\ 2 \\ K+2 \end{Vmatrix}$. $(2K+2;2;K+2)//(1;1;1) \Rightarrow$

$\Rightarrow \frac{2K+2}{1}=\frac{2}{1}=\frac{K+2}{1}$: Vera se $K=0$. $A\cdot X \perp Y \Rightarrow (2K+2;2;K+2)\cdot(1;1;1)=3K+6=0 \Rightarrow K=-2$.

9) $f(x)=x^3+Kx^2+1$: funzione continua e derivabile $\forall x\in\mathbb{R}$. Deve poi essere:

$f(-2)=f(1) \Rightarrow -8+4K+\cancel{1}=1+K+\cancel{1} \Rightarrow 3K=9 \Rightarrow K=3 \Rightarrow f(x)=x^3+3x^2+1$.

Quindi $f'(x)=3x^2+6x=3x(x+2)=0 \Rightarrow f'(x)=0$ per $x=0$ ed anche per $x=-2$.

10)

A	B	(non B)	(A δ non B)	((A δ non B) ⇒ A)	(A e B)	(B ⇔ (A e B))
1	1	0	1	1	1	1
1	0	1	1	1	0	1
0	1	0	0	1	0	0
0	0	1	1	0	0	1
				P_1		P_2

Per avere una tautologia occorre la proposizione P_1 δ P_2.