

$$\text{IM1}) z = e^{1+\frac{3}{7}\pi i} \cdot e^{-1+\frac{5}{7}\pi i} = e^{(1-1)+\frac{35}{7}\pi i} = e^{5\pi i} = \cos 5\pi + i \sin 5\pi = \\ = \cos \pi + i \sin \pi = -1. \text{ Calcoliamo quindi } \sqrt[3]{-1} = \sqrt[3]{1} \cdot \left(\cos\left(\frac{\pi}{3} + k \frac{2\pi}{3}\right) + i \sin\left(\frac{\pi}{3} + k \frac{2\pi}{3}\right)\right).$$

$$\text{Per } K=0: \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} = \frac{1}{2} + i \frac{\sqrt{3}}{2}; \text{ Per } K=1: \cos \pi + i \sin \pi = -1; \text{ Per } K=2: \cos \frac{5}{3} + i \sin \frac{5}{3} = \frac{1}{2} - i \frac{\sqrt{3}}{2}.$$

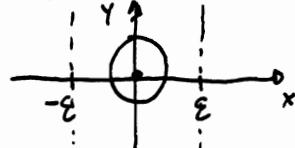
$$\text{IM2}) f(x,y) = \begin{cases} \frac{x^3 \operatorname{sen} y}{x^2+y^2} & : (x,y) \neq (0,0) \\ 0 & : (x,y) = (0,0) \end{cases}. \text{ Valutiamo la continuità in } (0,0):$$

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 \operatorname{sen} y}{x^2+y^2} = 0 \Rightarrow \left| \frac{x^3 \operatorname{sen} y}{x^2+y^2} - 0 \right| = \left| \frac{x^2}{x^2+y^2} \right| \cdot |x \cdot \operatorname{sen} y| \leq 1 \cdot |x| \cdot 1 < \varepsilon \text{ per } -\varepsilon < x < \varepsilon.$$

Quindi la funzione è continua in  $(0,0)$ .

$$\frac{\partial f}{\partial x}(0,0) = \lim_{h \rightarrow 0} \frac{h^3 \cdot 0}{h^2 \cdot h} = 0 = \frac{\partial f}{\partial y}(0,0) = \lim_{h \rightarrow 0} \frac{0 \cdot \operatorname{sen} h}{h^2 \cdot h}.$$

Per valutare la differentiabilità:



$$\lim_{(x,y) \rightarrow (0,0)} \left( \frac{x^3 \cdot \operatorname{sen} y}{x^2+y^2} - 0 - (0,0)(x-0, y-0) \right) \cdot \frac{1}{\sqrt{x^2+y^2}} \Rightarrow \lim_{\rho \rightarrow 0} \frac{\rho^3 \cdot \cos^3 \vartheta \cdot \operatorname{sen}(\rho \operatorname{sen} \vartheta)}{\rho^2 \cdot \rho} \Rightarrow$$

$$\Rightarrow \lim_{\rho \rightarrow 0} \cos^3 \vartheta \cdot \operatorname{sen}(\rho \operatorname{sen} \vartheta) = \cos^3 \vartheta \cdot 0 = 0. |\cos^3 \vartheta \cdot \operatorname{sen}(\rho \operatorname{sen} \vartheta)| \leq 1 \cdot |\rho \operatorname{sen} \vartheta| \leq \rho < \varepsilon.$$

Quindi la funzione è differentiabile in  $(0,0)$  e quindi in  $\mathbb{R}^2$ .

$$\text{IM3}) f(x,y,z) = x e^{y-z} - y e^{z-x} + z e^{x-y} = 0; f(0,0,0) = 0 - 0 + 0 = 0.$$

$$\nabla f(x,y,z) = (e^{y-z} + y e^{z-x} + z e^{x-y}; x e^{y-z} - e^{z-x} - z e^{x-y}; -x e^{y-z} - y e^{z-x} + e^{x-y});$$

$$\nabla f(0,0,0) = (1+0+0; 0-1-0; -0-0+1) = (1;-1;1). \text{ Dato che } f'_z = 1 \neq 0 \text{ esiste}$$

la funzione implicita  $(x,y) \rightarrow z(x,y)$ .  $\frac{\partial z}{\partial x}(0,0) = -\frac{1}{1} = -1; \frac{\partial z}{\partial y}(0,0) = -\frac{1}{1} = 1$ .

$$\text{Equazione piano tangente in } (0,0): z-0 = -1(x-0) + 1(y-0) \Rightarrow z = -x + y.$$

$$\text{IM4}) f(x,y) = x e^y + y e^x; v = (\cos \alpha; \sin \alpha). f(x,y) \text{ differenziabile 2 volte in } \mathbb{R}^2.$$

$$\nabla f(x,y) = (e^y + y e^x; x e^y + e^x); \nabla f(0,0) = (1;1); \mathcal{D}_v f(0,0) = \nabla f(0,0) \cdot v =$$

$$= (1;1) \cdot (\cos \alpha; \sin \alpha) = \cos \alpha + \sin \alpha = 0 \text{ per } \sin \alpha = -\cos \alpha \Rightarrow \alpha = \frac{3}{4}\pi \text{ e } \alpha = \frac{7}{4}\pi.$$

$$H(x; y) = \begin{vmatrix} y e^x & e^y + e^x \\ e^y + e^x & x e^y \end{vmatrix}; H(0; 0) = \begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix}. \mathcal{D}_{v, v}^2 f(0; 0) = v \cdot H(0; 0) \cdot v^T =$$

$$= \|\cos \alpha; \sin \alpha\| \cdot \begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix} \cdot \begin{vmatrix} \cos \alpha & \\ \sin \alpha & \end{vmatrix} = \|\cos \alpha \sin \alpha\| \cdot \begin{vmatrix} 2 \sin \alpha & \\ 2 \cos \alpha & \end{vmatrix} = 2 \sin \alpha \cos \alpha + 2 \sin \alpha \cos \alpha = 2 \sin 2\alpha.$$

$$\text{II M1)} \begin{cases} \text{Max/min } f(x; y) = x^2 - y^2 \\ \text{s.v. } x^2 + 4y^2 \leq 4 \end{cases}$$

$\mathbb{E}$  insieme compatto;  $f(x; y)$  continua; riunibili  
qualificati  $\Rightarrow$  esistono Max e min.

$$\Lambda(x; y; \lambda) = x^2 - y^2 - \lambda(x^2 + 4y^2 - 4)$$

$$\underline{\text{Caso } \lambda = 0}: \begin{cases} \Lambda'_x = 2x = 0 \\ \Lambda'_y = -2y = 0 \\ x^2 + 4y^2 \leq 4 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = 0 \\ 0 \leq 4 \end{cases}; H(x; y) = H(0; 0) = \begin{vmatrix} 2 & 0 \\ 0 & -2 \end{vmatrix}: \text{Punto di Sella.}$$

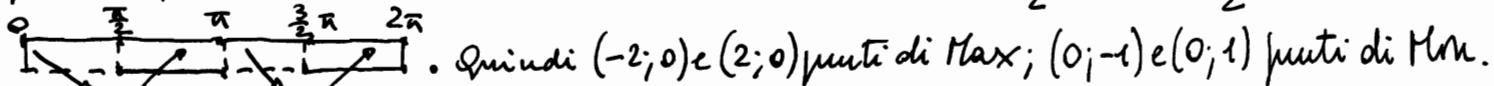
$$\underline{\text{Caso } \lambda \neq 0}: \begin{cases} \Lambda'_x = 2x - 2\lambda x = 2x(1-\lambda) = 0 \\ \Lambda'_y = -2y - 8\lambda y = -2y(1+4\lambda) = 0 \\ x^2 + 4y^2 = 4 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = 0 \\ 0+0=4 \end{cases} \cup \begin{cases} x = 0 \\ y = \pm 1 \\ 0+\pm 1=4 \end{cases} \cup \begin{cases} \lambda = 1 \\ y = 0 \\ x = \pm 2 \end{cases} \cup \begin{cases} \lambda = -\frac{1}{4} \\ y = \pm 1 \\ x = \pm 2 \end{cases}$$

già visto      min?      Max?      hyp.

$$f(0; \pm 1) = -1: \text{Min}; f(2; 0) = 4: \text{Max}.$$

$$\text{Se } \begin{cases} x = 2 \cos t \\ y = \sin t \end{cases} \Rightarrow f(t) = 4 \cos^2 t - \sin^2 t = 5 \cos^2 t - 1; f'(t) = 10 \cos t (-\sin t) = -5 \sin 2t.$$

$$f'(t) \geq 0 \text{ per } \sin 2t \leq 0 \Rightarrow \pi \leq 2t \leq 2\pi \cup 3\pi \leq 2t \leq 4\pi \Rightarrow \frac{\pi}{2} \leq t \leq \pi \cup \frac{3}{2}\pi \leq t \leq 2\pi.$$

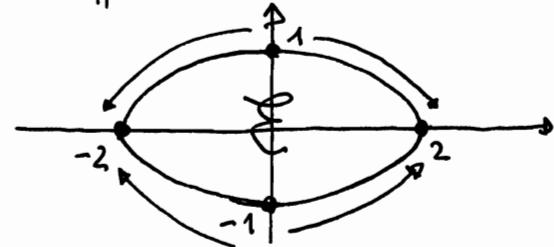


$$\text{II M2)} \begin{cases} x' = -x + y + 1 \\ y' = x - y + e^t \end{cases} \Rightarrow \begin{cases} x' + x - y = 1 \\ -x + y' + y = e^t \end{cases} \Rightarrow \begin{vmatrix} D+1 & -1 \\ -1 & D+1 \end{vmatrix} \cdot \begin{vmatrix} x \\ y \end{vmatrix} = \begin{vmatrix} 1 \\ e^t \end{vmatrix} \Rightarrow$$

$$\Rightarrow \begin{vmatrix} D+1 & -1 \\ -1 & D+1 \end{vmatrix} (x) = \begin{vmatrix} 1 & -1 \\ e^t & D+1 \end{vmatrix} \Rightarrow (D^2 + 2D + 1 - 1)(x) = (D^2 + 2D)(x) = 1 + e^t. \lambda^2 + 2\lambda = 0 \mu \Rightarrow x'' + 2x' = 1 + e^t.$$

$$\lambda = 0 \text{ e } \lambda = -2; \text{ Soluzione generale eq. omogenea: } x(t) = c_1 + c_2 e^{-2t}.$$

Dato che 1 è annichilato da  $\mathcal{D}$ , per trovare una soluzione particolare delle



CAM3

una omogenea bisogna ipotizzare  $x_0(t) = Q + bt + c e^t \Rightarrow$

$$x_0' = b + c e^t; x_0'' = c e^t \Rightarrow c e^t + 2b + 2c e^t = 1 + e^t \Rightarrow \begin{cases} 2b = 1 \\ 3c = 1 \end{cases} \Rightarrow \begin{cases} b = \frac{1}{2} \\ c = \frac{1}{3} \end{cases}$$

Quindi  $x(t) = c_1 + c_2 e^{-2t} + \frac{1}{2}t + \frac{1}{3}e^t$ . Da  $y = x' + x - 1 \Rightarrow$

$$\Rightarrow y(t) = -2c_2 e^{-2t} + \frac{1}{2} + \frac{1}{3}e^t + c_1 + c_2 e^{-2t} + \frac{1}{2}t + \frac{1}{3}e^t - 1 \Rightarrow$$

$$\Rightarrow y(t) = c_1 - c_2 e^{-2t} + \frac{1}{2}t - \frac{1}{2} + \frac{2}{3}e^t.$$

$$\text{II M3)} \begin{cases} y \cdot y' \cdot x = (1+x^2)(1+y^2) \\ y(1) = 1 \end{cases} \Rightarrow \frac{y}{1+y^2} \cdot y' = \frac{1+x^2}{x} \quad (x \neq 0) \Rightarrow \int \frac{y}{1+y^2} dy = \int \frac{1}{x} + x dx + K \Rightarrow$$

$$\Rightarrow \frac{1}{2} \log(1+y^2) = \log x + \frac{1}{2}x^2 + K \Rightarrow \log(1+y^2) = 2 \log x + x^2 + m \Rightarrow$$

$$\Rightarrow 1+y^2 = e^{2 \log x} \cdot e^{x^2} \cdot e^m \Rightarrow y^2 = x^2 \cdot e^{x^2} \cdot h - 1 \Rightarrow y = \pm \sqrt{x^2 e^{x^2} \cdot h - 1}.$$

$$\text{Se } y(1) = 1 > 0 \Rightarrow 1 = \sqrt{1 \cdot e \cdot h - 1} \Rightarrow e \cdot h = 2 \Rightarrow h = \frac{2}{e}.$$

Soluzione problema di Cauchy :  $y = \sqrt{2x^2 \cdot e^{x^2-1} - 1}$ .

$$\text{II M4)} \iint_D x^2 + y^2 dx dy =$$

$$= \int_{-1}^1 \int_{x^2-1}^{1-x^2} x^2 + y^2 dy dx = \int_{-1}^1 \left( x^2 y + \frac{y^3}{3} \right) \Big|_{x^2-1}^{1-x^2} dx =$$

$$= \int_{-1}^1 x^2(1-x^2) + \frac{(1-x^2)^3}{3} - x^2(x^2-1) - \frac{(x^2-1)^3}{3} dx = \int_{-1}^1 x^2(1-x^2-x^2+1) + \frac{1}{3}((1-x^2)^3 - (x^2-1)^3) dx =$$

$$= \int_{-1}^1 2x^2 - 2x^4 + \frac{1}{3}(1-3x^2+3x^4-x^6-3x^4+3x^2+1) dx = \int_{-1}^1 2(x^2-x^4+\frac{1}{3}-x^2+x^4-\frac{1}{3}x^6) dx =$$

$$= 2 \int_{-1}^1 \frac{1}{3} - \frac{1}{3}x^6 dx = \frac{2}{3} \left( x - \frac{x^7}{7} \right) \Big|_{-1}^{+1} = \frac{2}{3} \left[ (1 - \frac{1}{7}) - (-1 + \frac{1}{7}) \right] = \frac{2}{3} (2 - \frac{2}{7}) = \frac{8}{7}.$$

