Compito di Matematica Generale del 3/2/2020 $\boxed{\text{MGA 1}}$

1) $f(x)=(x-1)\cdot e^{2-x}$. C.E.: $\mathbb{R}$. $\lim_{x\to-\infty} f(x)=-\infty$; $\lim_{x\to+\infty} f(x)=0^+$. $f(x)\geq 0$ per $x\geq 1$. $f(0)=-e^2$.

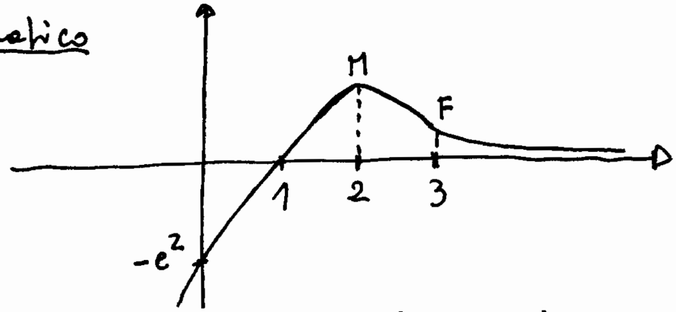
$f'(x)=1\cdot e^{2-x}-(x-1)e^{2-x}=(2-x)e^{2-x}\geq 0$ per

$2-x\geq 0 \Rightarrow x\leq 2$

$f''(x)=-e^{2-x}-(2-x)e^{2-x}=(x-3)e^{2-x}\geq 0$ per

$x-3\geq 0: x\geq 3$

Grafico



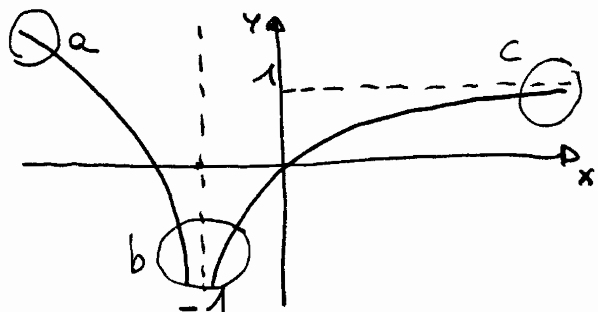
2) $\lim_{x\to 0}\frac{1-\cos x}{\operatorname{sen}3x\cdot(e^x-1)}=\lim_{x\to 0}\frac{1-\cos x}{x^2}\cdot\frac{3x}{\operatorname{sen}3x}\cdot\frac{1}{3}\cdot\frac{x}{e^x-1}=\frac{1}{2}\cdot 1\cdot\frac{1}{3}\cdot 1=\frac{1}{6}$.

$\lim_{x\to+\infty}\frac{e^x-x}{2^x-3^x}=\lim_{x\to+\infty}\frac{e^x}{-3^x}=\lim_{x\to+\infty}-\left(\frac{e}{3}\right)^x=0^-$ $\left(x=o(e^x);\ 2^x=o(3^x);\ \frac{e}{3}<1\right)$.

3) a) $\forall\varepsilon\ \exists\delta(\varepsilon): x<\delta(\varepsilon)\Rightarrow f(x)>\varepsilon : \lim_{x\to-\infty} f(x)=+\infty$;

b) $\forall\varepsilon\ \exists\delta(\varepsilon): 0<|x+1|<\delta(\varepsilon)\Rightarrow f(x)<\varepsilon : \lim_{x\to-1} f(x)=-\infty$;

c) $\forall\varepsilon>0\ \exists\delta(\varepsilon): x>\delta(\varepsilon)\Rightarrow |f(x)-1|<\varepsilon : \lim_{x\to+\infty} f(x)=1$.



4) $f(x)=3^{1-x}$; $f(g^{-1}(x))=3^{1-g^{-1}(x)}=2x+1 \Rightarrow 1-g^{-1}(x)=\log_3(2x+1)\Rightarrow$

$\Rightarrow g^{-1}(x)=1-\log_3(2x+1)=y\Rightarrow \log_3(2x+1)=1-y\Rightarrow 2x+1=3^{1-y}\Rightarrow$

$\Rightarrow 2x=3^{1-y}-1 \Rightarrow x=\frac{1}{2}\left(3^{1-y}-1\right)\Rightarrow g(x)=\frac{1}{2}\left(3^{1-x}-1\right)$.

5) $\int_0^{\pi}\operatorname{sen}x-\cos 2x\,dx=\left(-\cos x-\frac{1}{2}\operatorname{sen}2x\right|_0^{\pi}=\left(-\cos\pi-\frac{1}{2}\operatorname{sen}2\pi\right)-\left(-\cos 0-\frac{1}{2}\operatorname{sen}0\right)=$

$=(1-0)-(-1-0)=2$.

6) $f(x;y)=x^3+y^3-9x-3y$. $\nabla f(x;y)=(0;0)\Rightarrow\begin{cases}3x^2-9=3(x^2-3)=0\\3y^2-3=3(y^2-1)=0\end{cases}\Rightarrow$

$\Rightarrow\begin{cases}x^2=3\\y^2=1\end{cases}\Rightarrow$ 4 punti stazionari: $(\sqrt{3};1)$; $(\sqrt{3};-1)$; $(-\sqrt{3};1)$; $(-\sqrt{3};-1)$.

MGA2

$H(x;y)=\begin{Vmatrix}6x & 0\\ 0 & 6y\end{Vmatrix}$. $H(\sqrt{3};1)=\begin{Vmatrix}6\sqrt{3} & 0\\ 0 & 6\end{Vmatrix} \Rightarrow \begin{cases}|H_1|>0\\ |H_2|>0\end{cases}$: Punto di minimo;

$H(-\sqrt{3};-1)=\begin{Vmatrix}-6\sqrt{3} & 0\\ 0 & -6\end{Vmatrix} \Rightarrow \begin{cases}|H_1|<0\\ |H_2|>0\end{cases}$: Punto di Massimo;

$H(\sqrt{3};-1)=\begin{Vmatrix}6\sqrt{3} & 0\\ 0 & -6\end{Vmatrix}$ e $H(-\sqrt{3};1)=\begin{Vmatrix}-6\sqrt{3} & 0\\ 0 & 6\end{Vmatrix}$ hanno $|H_2|<0 \Rightarrow$ Punti di Sella.

7) $f(x)=x^2-2x+3$; $f'(x)=2x-2 \Rightarrow f(-1)=6$; $f'(-1)=-4$; $f(2)=3$; $f'(2)=2$.

Equazione tangente in $x=-1$: $y-6=-4(x+1) \Rightarrow y=-4x+2$

Equazione tangente in $x=2$: $y-3=2(x-2) \Rightarrow y=2x-1$

$\Rightarrow \begin{cases}2x-1=-4x+2\\ y=2x-1\end{cases} \Rightarrow \begin{cases}6x=3\\ y=2x-1\end{cases} \Rightarrow \begin{cases}x_0=\frac{1}{2}\\ y_0=0\end{cases}$.

8) $A\cdot B\cdot X=\begin{Vmatrix}1 & K\\ 2 & 1\end{Vmatrix}\cdot\begin{Vmatrix}K & 1\\ -1 & 2\end{Vmatrix}\cdot\begin{Vmatrix}1\\ -1\end{Vmatrix}=\begin{Vmatrix}1 & K\\ 2 & 1\end{Vmatrix}\cdot\begin{Vmatrix}K-1\\ -3\end{Vmatrix}=\begin{Vmatrix}K-1-3K\\ 2K-2-3\end{Vmatrix}=\begin{Vmatrix}-1-2K\\ 2K-5\end{Vmatrix}=Y$.

$\|Y\|=\sqrt{(-1-2K)^2+(2K-5)^2}=\sqrt{1+4K^2+4K+4K^2+25-20K}=\sqrt{8K^2-16K+26}=\sqrt{18} \Rightarrow$

$\Rightarrow 8K^2-16K+26=18 \Rightarrow 8K^2-16K+8=8(K^2-2K+1)=8(K-1)^2=0 \Rightarrow K=1$.

9) $f(x)=\dfrac{3^{2x}+\log(1+2x)}{3x^4-1}$. $\mathcal{D}\left(\dfrac{f(x)}{g(x)}\right)=\dfrac{f'(x)\cdot g(x)-f(x)\cdot g'(x)}{g^2(x)}$.

$$f'(x)=\frac{\left(3^{2x}\cdot 2\cdot\log 3+\frac{1}{1+2x}\cdot 2\right)\cdot(3x^4-1)-\left(3^{2x}+\log(1+2x)\right)\cdot(12x^3)}{(3x^4-1)^2}.$$

10)

A B C	$(A\Rightarrow B)$	(non B)	(A o non B)	[C $\Leftrightarrow$ (A o non B)]	P: $(A\Rightarrow B)$ e [C $\Leftrightarrow$ (A o non B)]
1 1 0	1	0	1	0	0
1 0 0	0	1	1	0	0

La proposizione P è sempre falsa.

Compito di Matematica Generale del 3/2/2020 $\boxed{MGB1}$

1) $f(x)=(2-x)\cdot e^{x-1}$. C.E.: $\mathbb{R}$. $\lim_{x\to-\infty} f(x)=0^+$; $\lim_{x\to+\infty} f(x)=-\infty$. $f(x)\geq 0$ per $x\leq 2$. $f(0)=\frac{2}{e}$.

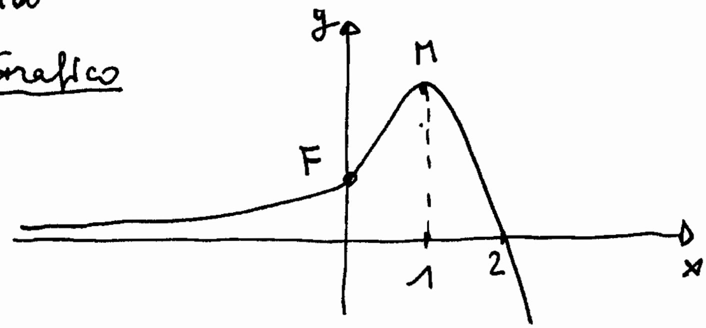
$f'(x)=-1\cdot e^{x-1}+(2-x)e^{x-1}=(1-x)\cdot e^{x-1}\geq 0$ per

$1-x\geq 0$: $x\leq 1$

$f''(x)=-e^{x-1}+(1-x)e^{x-1}=-xe^{x-1}\geq 0$ per

$x\leq 0$

Grafico



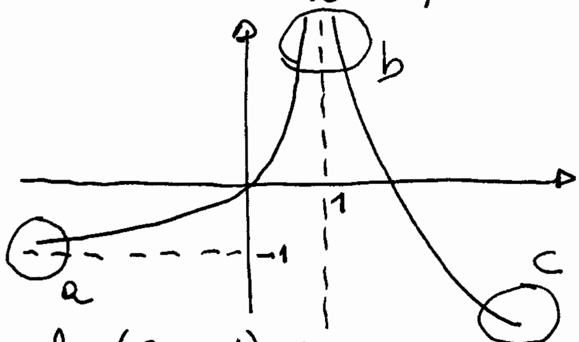
2) $\lim_{x\to 0}\frac{\operatorname{sen}2x\cdot\log(1+x)}{(1-\cos x)}=\lim_{x\to 0}\frac{\operatorname{sen}2x}{2x}\cdot 2\cdot\frac{\log(1+x)}{x}\cdot\frac{x^2}{1-\cos x}=1\cdot 2\cdot 1\cdot 2=4.$

$\lim_{x\to+\infty}\frac{3^x-e^x}{2^x-x}=\lim_{x\to+\infty}\frac{3^x}{2^x}=\lim_{x\to+\infty}\left(\frac{3}{2}\right)^x=+\infty\quad\left(e^x=o(3^x);\ x=o(2^x);\ \frac{3}{2}>1\right).$

3) a) $\forall\varepsilon>0\ \exists\delta(\varepsilon): x<\delta(\varepsilon)\Rightarrow|f(x)+1|<\varepsilon$: $\lim_{x\to-\infty}f(x)=-1$;

b) $\forall\varepsilon\ \exists\delta(\varepsilon): 0<|x-1|<\delta(\varepsilon)\Rightarrow f(x)>\varepsilon$: $\lim_{x\to 1}f(x)=+\infty$;

c) $\forall\varepsilon\ \exists\delta(\varepsilon): x>\delta(\varepsilon)\Rightarrow f(x)<\varepsilon$: $\lim_{x\to+\infty}f(x)=-\infty$.



4) $f(x)=2^{1+x}$; $f(g^{-1}(x))=2^{1+g^{-1}(x)}=3x-1\Rightarrow 1+g^{-1}(x)=\log_2(3x-1)\Rightarrow$

$\Rightarrow g^{-1}(x)=\log_2(3x-1)-1=y\Rightarrow\log_2(3x-1)=y+1\Rightarrow 3x-1=2^{y+1}\Rightarrow 3x=2^{y+1}+1\Rightarrow$

$\Rightarrow x=\frac{1}{3}\left(2^{y+1}+1\right)\Rightarrow g(x)=\frac{1}{3}\left(2^{x+1}+1\right).$

5) $\int_0^\pi \operatorname{sen}2x+\cos x\,dx=\left(-\frac{1}{2}\cos 2x+\operatorname{sen}x\right|_0^\pi=\left(-\frac{1}{2}\cos 2\pi+\operatorname{sen}\pi\right)-\left(-\frac{1}{2}\cos 0+\operatorname{sen}0\right)=$

$=\left(-\frac{1}{2}+0\right)-\left(-\frac{1}{2}+0\right)=-\frac{1}{2}+\frac{1}{2}=0.$

6) $f(x;y)=x^3-y^3-12x+6y$. $\nabla f(x;y)=(0;0)\Rightarrow\begin{cases}3x^2-12=3(x^2-4)=0\\ 6-3y^2=3(2-y^2)=0\end{cases}\Rightarrow$

$\Rightarrow\begin{cases}x^2=4\\ y^2=2\end{cases}\Rightarrow$ 4 punti stazionari: $(2;\sqrt{2})$; $(2;-\sqrt{2})$; $(-2;\sqrt{2})$; $(-2;-\sqrt{2})$.

MGB2

$H(x;y)=\begin{Vmatrix} 6x & 0 \\ 0 & -6y \end{Vmatrix}$. $H(2;-\sqrt{2})=\begin{Vmatrix} 12 & 0 \\ 0 & 6\sqrt{2} \end{Vmatrix} \Rightarrow \begin{cases} |H_1|>0 \\ |H_2|>0 \end{cases}$: Punto di Minimo;

$H(-2;\sqrt{2})=\begin{Vmatrix} -12 & 0 \\ 0 & -6\sqrt{2} \end{Vmatrix} \Rightarrow \begin{cases} |H_1|<0 \\ |H_2|>0 \end{cases}$: Punto di Massimo;

$H(2;\sqrt{2})=\begin{Vmatrix} 12 & 0 \\ 0 & -6\sqrt{2} \end{Vmatrix}$ e $H(-2;-\sqrt{2})=\begin{Vmatrix} -12 & 0 \\ 0 & 6\sqrt{2} \end{Vmatrix}$ hanno $|H_2|<0 \Rightarrow$ Punti di Sella.

7) $f(x)=x^2+3x-2$. $f'(x)=2x+3$. $f(-2)=-4$; $f'(-2)=-1$; $f(1)=2$; $f'(1)=5$.

Equazione tangente in $x=-2$: $y+4=-1(x+2) \Rightarrow y=-x-6$

Equazione tangente in $x=1$: $y-2=5(x-1) \Rightarrow y=5x-3$

$\Rightarrow \begin{cases} -x-6=5x-3 \\ y=5x-3 \end{cases} \Rightarrow \begin{cases} 6x=-3 \\ y=5x-3 \end{cases} \Rightarrow \begin{cases} x_0=-\frac{1}{2} \\ y_0=-\frac{11}{2} \end{cases}$.

8) $A \cdot B \cdot X=\begin{Vmatrix} k & -1 \\ 1 & 2 \end{Vmatrix} \cdot \begin{Vmatrix} 1 & 2 \\ k & 1 \end{Vmatrix} \cdot \begin{Vmatrix} 1 \\ 1 \end{Vmatrix} = \begin{Vmatrix} k & -1 \\ 1 & 2 \end{Vmatrix} \cdot \begin{Vmatrix} 3 \\ k+1 \end{Vmatrix} = \begin{Vmatrix} 3k-k-1 \\ 3+2k+2 \end{Vmatrix} = \begin{Vmatrix} 2k-1 \\ 2k+5 \end{Vmatrix} = Y$.

$\|Y\|=\sqrt{(2k-1)^2+(2k+5)^2}=\sqrt{4k^2+1-4k+4k^2+25+20k}=\sqrt{8k^2+16k+26}=\sqrt{18} \Rightarrow$

$\Rightarrow 8k^2+16k+26=18 \Rightarrow 8k^2+16k+8=8(k^2+2k+1)=8(k+1)^2=0 \Rightarrow k=-1$.

9) $f(x)=\dfrac{2^{3x}-\log(1-x)}{4x^3+2}$. $D\left(\dfrac{f(x)}{g(x)}\right)=\dfrac{f'(x)\cdot g(x)-f(x)\cdot g'(x)}{g^2(x)}$.

$$f'(x)=\frac{\left(2^{3x}\cdot 3\cdot\log 2-(-1)\cdot\frac{1}{1-x}\right)(4x^3+2)-\left(2^{3x}-\log(1-x)\right)(12x^2)}{(4x^3+2)^2}.$$

10)

A B C	(non B)	(C e non B)	[A <=> (C e non B)]	(A => C)	P: [A <=> (C e non B)] o (A => C)
1 0 1	1	1	1	1	1
0 0 1	1	1	0	1	1

La proposizione P è sempre vera.