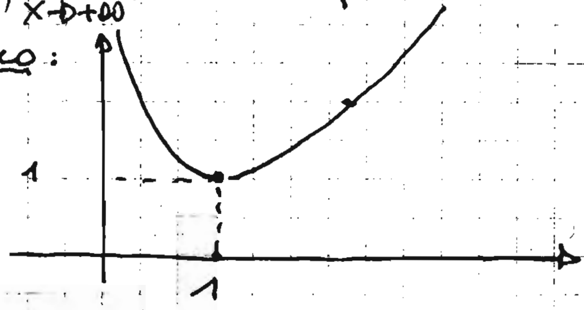
Compito di Matematica Generale del 21/4/2020

$\boxed{\text{CMG1}}$

1) $f(x) = x - \log x$. C.E.: $x > 0$. $\lim_{x \to 0^+} f(x) = +\infty$; $\lim_{x \to +\infty} f(x) = +\infty$. $f(x) > 0 \ \forall x > 0$.

$f'(x) = 1 - \frac{1}{x} = \frac{x-1}{x} \geq 0$ per $x \geq 1$ Grafico:



$0 \searrow 1 \nearrow$ min $\qquad f(1) = 1$.

$f''(x) = -\left(-\frac{1}{x^2}\right) = \frac{1}{x^2} > 0 \ \forall x$

funzione sempre convessa.

2) $\lim_{x \to 0} \frac{\operatorname{sen}^2 x}{1 - \cos x} = \lim_{x \to 0} \frac{\operatorname{sen}^2 x}{x^2} \cdot \frac{x^2}{1 - \cos x} = 1 \cdot 2 = 2$.

$\lim_{x \to +\infty} \frac{\log x - \log^2 x + x}{3x + 3} = \lim_{x \to +\infty} \frac{x}{3x} = \frac{1}{3}$. $\left(\log x = o(x);\ \log^2 x = o(x);\ 3 = o(x)\right)$

3) $\lim_{x \to 0} \frac{2^{kx} - 1}{2x} = \lim_{x \to 0} \frac{2^{kx} - 1}{kx} \cdot \frac{k}{2} = \frac{k}{2} \cdot \log 2 = 1 \Rightarrow k = \frac{2}{\log 2} = \log_2 e^2$.

4) Se $f'(x) = 2e^{2x} \Rightarrow f(x) = \int 2e^{2x}\,dx + k \Rightarrow f(x) = e^{2x} + k$; $f(0) = 0 \Rightarrow 1 + k = 0 \Rightarrow$
$\Rightarrow k = -1 \Rightarrow f(x) = e^{2x} - 1$.

5) $$A \cdot B \cdot X = \left\| \begin{matrix} 1 & 0 & 2 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{matrix} \right\| \cdot \left\| \begin{matrix} -1 & 2 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{matrix} \right\| \cdot \left\| \begin{matrix} 1 \\ 1 \\ 1 \end{matrix} \right\| = \left\| \begin{matrix} 1 & 0 & 2 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{matrix} \right\| \cdot \left\| \begin{matrix} -1+2+0 \\ 0+0+1 \\ 1-1+0 \end{matrix} \right\| = \left\| \begin{matrix} 1 & 0 & 2 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{matrix} \right\| \cdot \left\| \begin{matrix} 1 \\ 1 \\ 0 \end{matrix} \right\| =$$

$$= \left\| \begin{matrix} 1+0+0 \\ 1+0+0 \\ 0+1+0 \end{matrix} \right\| = \left\| \begin{matrix} 1 \\ 1 \\ 1 \end{matrix} \right\| \Rightarrow \|A \cdot B \cdot X\| = \sqrt{1+1+1} = \sqrt{3}.$$

6) $f(x) = x^2 - 3x + 1$ è un polinomio quindi continua e derivabile $\forall x \in \mathbb{R}$.
Quindi $\exists x_0 : f'(x_0) = \frac{f(1) - f(0)}{1 - 0} \Rightarrow 2x_0 - 3 = \frac{-1-1}{1} \Rightarrow 2x_0 - 3 = -2 \Rightarrow 2x_0 = 1 \Rightarrow x_0 = \frac{1}{2}$.

7) $X \cdot Y = (y; x; -2) \cdot (x - y; 1 - x; y) = xy - y^2 + x - x^2 - 2y = xy - x^2 - y^2 + x - 2y$.

$$\begin{cases} f'_x = y - 2x + 1 = 0 \\ f'_y = x - 2y - 2 = 0 \end{cases} \Rightarrow \begin{cases} y = 2x - 1 \\ x - 4x + 2 - 2 = -3x = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = -1 \end{cases}.$$

$\boxed{CMG\ 2}$

$$H(x;y)=H(0;-1)=\begin{Vmatrix} -2 & 1 \\ 1 & -2 \end{Vmatrix} \Rightarrow \begin{cases} |\bar{H}_1| = -2 < 0 \\ |\bar{H}_2| = 4-1=3>0 \end{cases} \Rightarrow (0;-1) \text{ punto di MAX.}$$

8)

A	B	non B	(A ⇒ non B)	(B ⇒ A)	[(A ⇒ non B) ⇔ (B ⇒ A)]
1	1	0	0	1	0
1	0	1	1	1	1
0	1	0	1	0	0
0	0	1	1	1	1

La proposizione P non è una tautologia.

9) a) $\lim\limits_{x\to-\infty} f(x) = -\infty$;

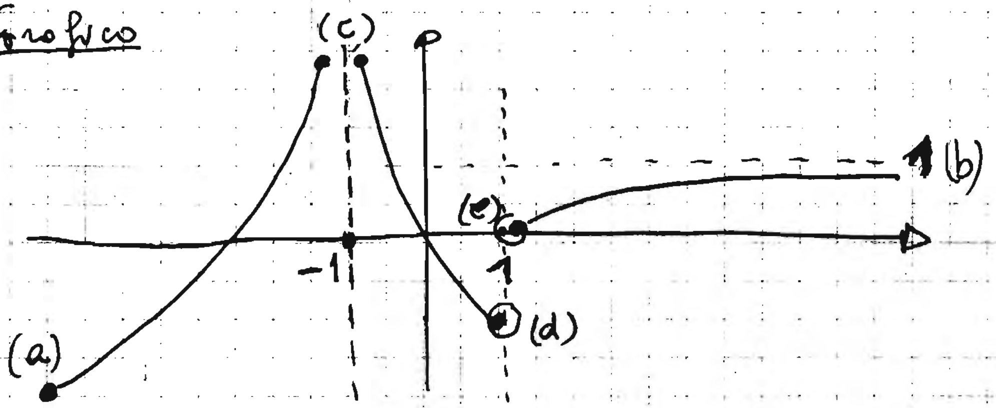
b) $\lim\limits_{x\to+\infty} f(x) = 1$;

c) $\lim\limits_{x\to-1} f(x) = +\infty$;

d) $\lim\limits_{x\to1^-} f(x) = -1$;

e) $\lim\limits_{x\to1^+} f(x) = 0$.

Grafico



10) $f(x) = 1-\cos 2x$. $f(0) = 1-1=0$;

$f'(x) = -(-\operatorname{sen} 2x)\cdot 2 = +2\operatorname{sen} 2x$; $f'(0)=0$;

$f''(x) = 4\cos 2x$; $f''(0) = 4$;

$f'''(x) = -8\operatorname{sen} 2x$; $f'''(0) = 0$;

$f^{(4)}(x) = -16\cos 2x$; $f^{(4)}(0) = -16$.

Quindi $P_4(x;0) = \dfrac{4}{2!}\cdot x^2 - \dfrac{16}{4!}x^4 = 2x^2 - \dfrac{2}{3}x^4$.