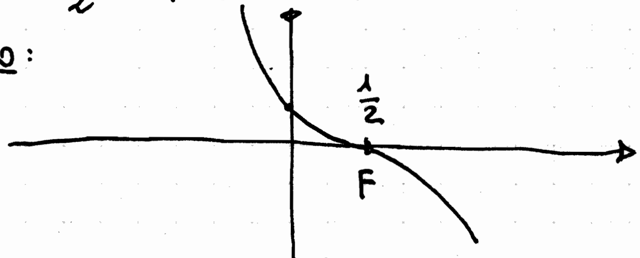
Compito Matematica Generale del 15/6/2020 $\boxed{\text{CMG1}}$

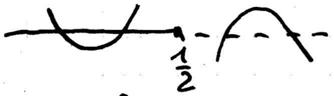
1) $f(x)=e^{1-x}-e^{x}=\frac{e}{e^{x}}-e^{x}=\frac{e-e^{2x}}{e^{x}}$. C.E.: $\mathbb{R}$. $\lim_{x\to-\infty} f(x)=+\infty$; $\lim_{x\to+\infty} f(x)=-\infty$.

$f(x)\geq 0$ per $e-e^{2x}\geq 0 \Rightarrow e^{2x}\leq e \Rightarrow 2x\leq 1 \Rightarrow x\leq \frac{1}{2}$. $f(\frac{1}{2})=0$; $f(0)=e-1$

$f'(x)=-e^{1-x}-e^{x}<0\ \forall x$ Grafico:



$f''(x)=e^{1-x}-e^{x}\geq 0$ per $x\leq\frac{1}{2}$



2) $\lim_{x\to0}\frac{\log(1+2x)}{\log(1-3x)}=\lim_{x\to0}\frac{\log(1+2x)}{2x}\cdot\frac{2x}{(-3x)}\cdot\frac{-3x}{\log(1-3x)}=1\cdot\left(-\frac{2}{3}\right)\cdot1=-\frac{2}{3}$.

$\lim_{x\to+\infty}\frac{3^{x}-2^{x}+x}{3^{x}+3^{-x}}=\lim_{x\to+\infty}\frac{3^{x}}{3^{x}}=1.\ \left(2^{x}=o(3^{x});\ x=o(3^{x});\ 3^{-x}\to0^{+}\right)$

3) $\lim_{x\to0}\frac{K\cdot \text{sen}^{2}x}{1-\cos 2x}=\lim_{x\to0}K\cdot\frac{\text{sen}^{2}x}{x^{2}}\cdot\frac{4x^{2}}{1-\cos 2x}\cdot\frac{1}{4}=K\cdot1\cdot2\cdot\frac{1}{4}=\frac{K}{2}=3\Rightarrow K=6.$

4) $f(x)=1-5x$; $g(x)=3^{x}$; $h(x)=\log_{2}x$.

$f(g(h(x)))=f(g(\log_{2}x))=f\left(3^{\log_{2}x}\right)=1-5\cdot3^{\log_{2}x}=y\Rightarrow 5\cdot3^{\log_{2}x}=1-y\Rightarrow$

$\Rightarrow 3^{\log_{2}x}=\frac{1-y}{5}\Rightarrow \log_{2}x=\log_{3}\left(\frac{1-y}{5}\right)\Rightarrow x=2^{\log_{3}\left(\frac{1-y}{5}\right)}\Rightarrow y=2^{\log_{3}\left(\frac{1-x}{5}\right)}$.

$h(g(f(x)))=h(g(1-5x))=h\left(3^{1-5x}\right)=\log_{2}3^{1-5x}=y\Rightarrow 3^{1-5x}=2^{y}\Rightarrow$

$\Rightarrow 1-5x=\log_{3}2^{y}\Rightarrow 5x=1-\log_{3}2^{y}\Rightarrow x=\frac{1}{5}\left(1-\log_{3}2^{y}\right)\Rightarrow y=\frac{1}{5}\left(1-\log_{3}2^{x}\right)$.

5) $X\cdot A\cdot Y=\begin{Vmatrix}1 & -e^{x}\end{Vmatrix}\cdot\begin{Vmatrix}e^{2x} & e^{x}\\ 1 & 1\end{Vmatrix}\cdot\begin{Vmatrix}e^{x}\\ e^{2x}\end{Vmatrix}=\begin{Vmatrix}1 & -e^{x}\end{Vmatrix}\cdot\begin{Vmatrix}e^{3x}+e^{3x}\\ e^{x}+e^{2x}\end{Vmatrix}=$

$=2e^{3x}-e^{2x}-e^{3x}=e^{3x}-e^{2x}=e^{2x}(e^{x}-1)=0\Rightarrow e^{x}=1\Rightarrow x=0.$

6) $\int_{1}^{2}\frac{3+x}{1+x}dx=\int_{1}^{2}\frac{1+x+2}{1+x}dx=\int_{1}^{2}1+\frac{2}{1+x}dx=\left(x+2\log(1+x)\right|_{1}^{2}=$

$=\left(2+2\log3\right)-\left(1+2\log2\right)=1+2\left(\log3-\log2\right)=1+2\log\frac{3}{2}=1+\log\frac{9}{4}=\log\frac{9}{4}e.$

CMG2

7) $f(x;y) = x^2 - x + y^2 - xy^2$. $\nabla f(x;y) = (0;0) \Rightarrow$

$$\Rightarrow \begin{cases} f'_x = 2x - 1 - y^2 = 0 \\ f'_y = 2y - 2xy = 2y(1-x) = 0 \end{cases} \Rightarrow \begin{cases} x = \frac{1}{2} \\ y = 0 \end{cases} \cup \begin{cases} x = 1 \\ y^2 = 1 \end{cases} \Rightarrow \begin{cases} x = 1 \\ y = 1 \end{cases} \cup \begin{cases} x = 1 \\ y = -1. \end{cases}$$

$P_1: \left(\frac{1}{2}; 0\right); P_2: (1;1); P_3: (1;-1)$. $H(x;y) = \begin{Vmatrix} 2 & -2y \\ -2y & 2-2x \end{Vmatrix}$. $H\left(\frac{1}{2};0\right) = \begin{Vmatrix} 2 & 0 \\ 0 & 1 \end{Vmatrix} : \begin{cases} 2>0; 1>0 \\ 2>0 \end{cases}$: MIN.

$H(1;1) = \begin{Vmatrix} 2 & -2 \\ -2 & 0 \end{Vmatrix} : |H_2| = -4 < 0$: Sella; $H(1;-1) = \begin{Vmatrix} 2 & 2 \\ 2 & 0 \end{Vmatrix} : |H_2| = -4 < 0$: Sella.

8) $X \cdot X_1 = 0 \Rightarrow (x;1;y) \cdot (1;1;1) = x + 1 + y = 0 \Rightarrow y = -x - 1$.

$X = (x; 1; -x-1) \Rightarrow \|X\| = \sqrt{x^2 + 1 + (-x-1)^2} = \sqrt{x^2 + 1 + x^2 + 1 + 2x} = \sqrt{2x^2 + 2x + 2} \Rightarrow$

$\Rightarrow \sqrt{2x^2 + 2x + 2} = \sqrt{2} \Rightarrow 2x^2 + 2x + 2 = 2 \Rightarrow 2x^2 + 2x = 0 \Rightarrow 2x(x+1) = 0 \Rightarrow$

$\Rightarrow x = 0 \cup x = -1 \Rightarrow X = (0;1;-1)$ oppure $X = (-1;1;0)$.

9)

A	B	$(A \Rightarrow B)$	$[A \Rightarrow (A \Rightarrow B)]$	$(B \Rightarrow A)$	$[A \Rightarrow (A \Rightarrow B)] \Leftrightarrow (B \Rightarrow A)$
1	1	1	1	1	1
1	0	0	0	1	0
0	1	1	1	0	0
0	0	1	1	1	1

La proposizione data NON è una tautologia.

10) $df(x_0) = f'(x_0) \cdot dx$

$f(x) = e^{3x} - \operatorname{sen} 2x \Rightarrow f'(x) = 3e^{3x} - 2\cos 2x \Rightarrow f'(0) = 3 - 2 = 1$.

$df(0) = 1 \cdot 0{,}1 = 1 \cdot \frac{1}{10} = \frac{1}{10}$.