Compito Matematica Generale 16/6/2020 — CMG1

1) $f(x) = e^x + e^{1-x} = e^x + \frac{e}{e^x} = \frac{e^{2x}+e}{e^x}$. C.E.: $\mathbb{R}$. $\lim_{x\to-\infty} f(x) = +\infty$; $\lim_{x\to+\infty} f(x) = +\infty$.

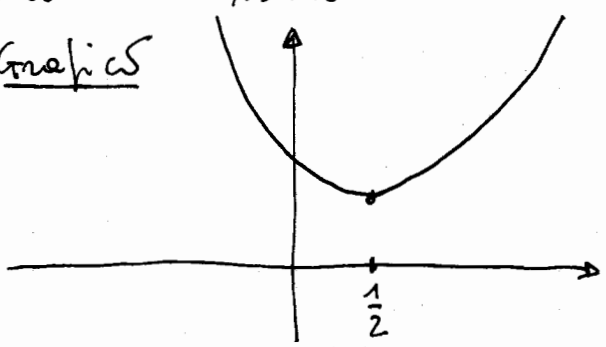
$f(x) \geq 0 \ \forall x \in \mathbb{R}$. $f(0) = 1 + e$.

$f'(x) = e^x - e^{1-x} = \frac{e^{2x}-e}{e^x} \geq 0$ se $e^{2x} \geq e \Rightarrow 2x \geq 1$

$\Rightarrow x \geq \frac{1}{2}$ — $\frac{1}{2}$ — $f(\frac{1}{2}) = 2\sqrt{e}$

$f''(x) = e^x + e^{1-x} \geq 0 \ \forall x \in \mathbb{R}$

Grafico



2) $\lim_{x\to0} \frac{e^{3x}-1}{e^{2x}-1} = \lim_{x\to0} \frac{e^{3x}-1}{3x} \cdot \frac{3x}{2x} \cdot \frac{2x}{e^{2x}-1} = 1 \cdot \frac{3}{2} \cdot 1 = \frac{3}{2}$.

$\lim_{x\to+\infty} \frac{3^x + 2^x - x}{2^x + 3^{-x}} = \lim_{x\to+\infty} \frac{3^x}{2^x} = \lim_{x\to+\infty} \left(\frac{3}{2}\right)^x = +\infty$ $\left(2^x = o(3^x);\ x = o(3^x);\ 3^{-x} \to 0\right)$

3) $\lim_{x\to0} \frac{\log(1+kx)}{\log(1+2x)} = \lim_{x\to0} \frac{\log(1+kx)}{kx} \cdot \frac{kx}{2x} \cdot \frac{2x}{\log(1+2x)} = 1 \cdot \frac{k}{2} \cdot 1 = 2 \Rightarrow k = 4$.

4) $f(x) = \log_2 x$; $g(x) = 2x - 1$; $h(x) = e^x$.

$f(g(h(x))) = f(g(e^x)) = f(2e^x - 1) = \log_2(2e^x - 1) = y \Rightarrow 2e^x - 1 = 2^y \Rightarrow$

$\Rightarrow 2e^x = 2^y + 1 \Rightarrow e^x = \frac{1}{2}(2^y + 1) \Rightarrow x = \log\left(\frac{1}{2}(2^y+1)\right) \Rightarrow y = \log\left(\frac{1}{2}(2^x+1)\right)$.

$h(g(f(x))) = h(g(\log_2 x)) = h(2\log_2 x - 1) = e^{2\log_2 x - 1} = y \Rightarrow 2\log_2 x - 1 = \log y \Rightarrow$

$\Rightarrow 2\log_2 x = \log y + 1 \Rightarrow \log_2 x = \frac{1}{2}(\log y + 1) \Rightarrow x = 2^{\frac{1}{2}(\log y + 1)} \Rightarrow y = 2^{\frac{1}{2}(\log x + 1)}$.

5) $X \cdot A \cdot Y = \|1 - e^{-x}\| \cdot \begin{Vmatrix} 1 & 1 \\ e^x & e^{2x} \end{Vmatrix} \cdot \begin{Vmatrix} e^{2x} \\ e^x \end{Vmatrix} = \|1 - e^{-x}\| \cdot \begin{Vmatrix} e^{2x} + e^x \\ e^{3x} + e^{3x} \end{Vmatrix} =$

$= e^{2x} + e^x - e^{2x} - e^{2x} = e^x - e^{2x} = e^x(1 - e^x) = 0$ se $e^x = 1 \Rightarrow x = 0$.

6) $\int_1^2 \frac{x-1}{x+2}\,dx = \int_1^2 \frac{x+2-3}{x+2}\,dx = \int_1^2 1 - \frac{3}{x+2}\,dx = \left(x - 3\log(x+2)\right)\Big|_1^2 =$

$= \left(2 - 3\log 4\right) - \left(1 - 3\log 3\right) = 1 - 3\left(\log 4 - \log 3\right) = 1 - 3\log\frac{4}{3} = 1 - \log\frac{64}{27} = \log\frac{27e}{64}$.

CMG2

7) $f(x;y)=x^2+y^2-y-x^2y$. $\nabla f(x;y)=(0;0) \Rightarrow$

$$\Rightarrow \begin{cases} f'_x = 2x-2xy = 2x(1-y)=0 \\ f'_y = 2y-1-x^2=0 \end{cases} \Rightarrow \begin{cases} x=0 \\ y=\frac{1}{2} \end{cases} \cup \begin{cases} x^2=1 \\ y=1 \end{cases} \Rightarrow \begin{cases} x=1 \\ y=1 \end{cases} \cup \begin{cases} x=-1 \\ y=1 \end{cases}.$$

$P_1:\left(0;\frac{1}{2}\right)$; $P_2:(1;1)$; $P_3:(-1;1)$. $H(x;y)=\begin{Vmatrix} 2-2y & -2x \\ -2x & 2 \end{Vmatrix}$. $H\left(0;\frac{1}{2}\right)=\begin{Vmatrix} 1 & 0 \\ 0 & 2 \end{Vmatrix}: \begin{cases} 1>0; 2>0 \\ 2>0 \end{cases}$: MIN.

$H(1;1)=\begin{Vmatrix} 0 & -2 \\ -2 & 2 \end{Vmatrix}: |H_2|=-4<0$: Sella; $H(-1;1)=\begin{Vmatrix} 0 & 2 \\ 2 & 2 \end{Vmatrix}: |H_2|=-4<0$: Sella.

8) $X \cdot X_1 = 0 \Rightarrow (x;y;1)\cdot(1;-1;1) = x-y+1=0 \Rightarrow y=x+1$.

$X=(x;x+1;1) = \|X\| = \sqrt{x^2+(x+1)^2+1} = \sqrt{x^2+x^2+2x+1+1} = \sqrt{2x^2+2x+2} = \sqrt{2} \Rightarrow$

$\Rightarrow 2x^2+2x+2=2 \Rightarrow 2x^2+2x=0 \Rightarrow 2x(x+1)=0 \Rightarrow x=0 \cup x=-1 \Rightarrow$

$X=(0;1;1)$ oppure $X=(-1;0;1)$.

9)

A	B	$(B \Rightarrow A)$	$[A \Rightarrow (B \Rightarrow A)]$	$(A \Rightarrow B)$	$[A \Rightarrow (B \Rightarrow A)] \Leftrightarrow (A \Rightarrow B)$
1	1	1	1	1	1
1	0	1	1	0	0
0	1	0	1	1	1
0	0	1	1	1	1

La proposizione data NON è una tautologia.

10) $df(x_0) = f'(x_0)\cdot dx$

$f(x) = e^{2x} + \sin 3x$; $f'(x) = 2e^{2x} + 3\cos 3x$; $f'(0) = 2+3 = 5$.

$df(0) = 5 \cdot dx = 0,1 = \frac{1}{10} \Rightarrow dx = \frac{1}{50} = 0,02$.