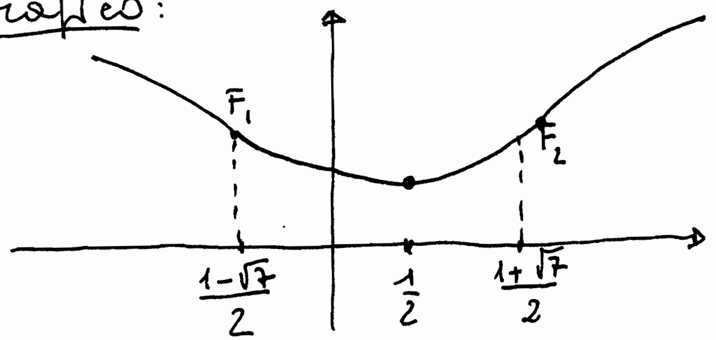
Compito di Matematica Generale del 13/7/2020 $\boxed{EMG1}\boxed{A}$

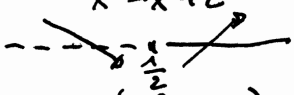
1) $f(x) = \log(x^2 - x + 2)$. C.E. $= \mathbb{R}$ in quanto $x^2 - x + 2 > 0 \ \forall x \in \mathbb{R}$ $(\Delta = 1 - 8 < 0)$.

$\lim_{x \to -\infty} f(x) = \lim_{x \to +\infty} f(x) = +\infty$. $f(x) \geq 0$ per $x^2 - x + 2 > 1 \Rightarrow x^2 - x + 1 > 0$ vera $\forall x \in \mathbb{R}$.

Grafico:



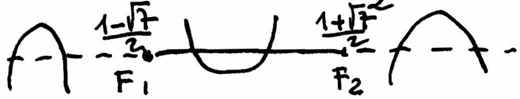
$f'(x) = \dfrac{2x - 1}{x^2 - x + 2} \geq 0$ per $2x - 1 \geq 0 \Rightarrow x \geq \frac{1}{2}$



$f\left(\frac{1}{2}\right) = \log\left(\frac{7}{4}\right) > 0$

$f''(x) = \dfrac{2(x^2 - x + 2) - (2x - 1)^2}{(x^2 - x + 2)^2} = \dfrac{-2x^2 + 2x + 3}{(x^2 - x + 2)^2} \geq 0$

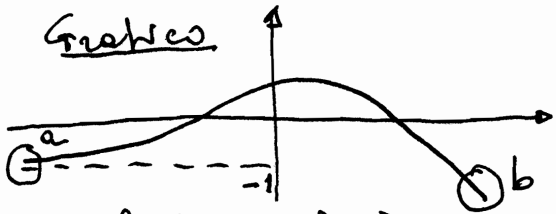
per $2x^2 - 2x - 3 \leq 0 \Rightarrow \frac{1-\sqrt{7}}{2} \leq x \leq \frac{1+\sqrt{7}}{2}$



2) $\lim_{x \to 0} \dfrac{\sin(x + x^2)}{x - x^2} = \lim_{x \to 0} \dfrac{\sin(x + x^2)}{x + x^2} \cdot \dfrac{\cancel{x}(1 + x)}{\cancel{x}(1 - x)} = 1 \cdot 1 = 1$.

$\lim_{x \to +\infty} \left(\dfrac{x + 3}{x + 1}\right)^{2x} = \lim_{x \to +\infty} \left(\dfrac{x + 1 + 2}{x + 1}\right)^{2x} = \lim_{x \to +\infty} \left(1 + \dfrac{2}{x + 1}\right)^{2x} = (e^2)^2 = e^4$.

Grafico



3) $\forall \varepsilon > 0 \ \exists \delta(\varepsilon) : x < \delta(\varepsilon) \Rightarrow |f(x) + 1| < \varepsilon : \lim_{x \to -\infty} f(x) = -1$;

$\forall \varepsilon \ \exists \delta(\varepsilon) : \delta(\varepsilon) < x \Rightarrow f(x) < \varepsilon : \lim_{x \to +\infty} f(x) = -\infty$.

4) $f(x) = \dfrac{x - 1}{3x} \Rightarrow f(g(x)) = \dfrac{g(x) - 1}{3g(x)} = \log 2x \Rightarrow g(x) - 1 = 3 \cdot \log 2x \cdot g(x) \Rightarrow$

$\Rightarrow g(x)(1 - 3\log 2x) = 1 \Rightarrow g(x) = \dfrac{1}{1 - 3\log 2x} = y \Rightarrow 1 - 3\log 2x = \dfrac{1}{y} \Rightarrow 3\log 2x = 1 - \dfrac{1}{y} \Rightarrow$

$\Rightarrow \log 2x = \frac{1}{3}\left(1 - \frac{1}{y}\right) \Rightarrow 2x = e^{\frac{1}{3}\left(1 - \frac{1}{y}\right)} \Rightarrow x = \frac{1}{2} e^{\frac{1}{3}\left(1 - \frac{1}{y}\right)}$; Inversa: $y = \frac{1}{2} \cdot e^{\frac{1}{3}\left(1 - \frac{1}{x}\right)}$.

5) $A \cdot B = \begin{Vmatrix} 1 & m & 1 \\ K & 0 & -1 \end{Vmatrix} \cdot \begin{Vmatrix} 1 & 2 \\ 1 & -1 \\ K & 0 \end{Vmatrix} = \begin{Vmatrix} 1 + m + K & 2 - m + 0 \\ K + 0 - K & 2K + 0 + 0 \end{Vmatrix} = \begin{Vmatrix} 1 + m + K & 2 - m \\ 0 & 2K \end{Vmatrix} = \begin{Vmatrix} 3 & 1 \\ 0 & 2 \end{Vmatrix}$

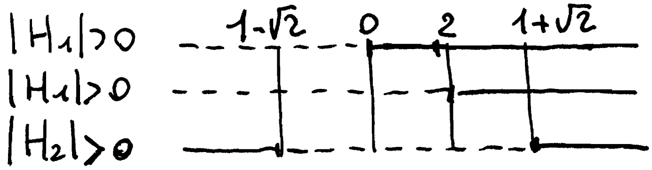
$\Rightarrow \begin{cases} 2 - m = 1 \\ 2K = 2 \end{cases} \Rightarrow \begin{cases} m = 1 \\ K = 1 \end{cases} \Rightarrow 1 + m + K = +1 + 1 + 1 = 3$.

6) $\displaystyle\int_1^2 e^{3x} - \frac{1}{x^3}\,dx = \left(\frac{1}{3}e^{3x} - \frac{1}{-3+1} \cdot x^{-3+1}\right)\Bigg|_1^2 = \left(\frac{1}{3}e^{3x} + \frac{1}{2}\frac{1}{x^2}\right)\Bigg|_1^2 = \left(\frac{1}{3}e^6 + \frac{1}{8}\right) - \left(\frac{1}{3}e^3 + \frac{1}{2}\right) = \frac{1}{3}(e^6 - e^3) - \frac{3}{8}$.

7) $H(x;y)=\begin{Vmatrix} k & 1 \\ 1 & k-2 \end{Vmatrix}$ 1) $|H_1| = k > 0$ per $k>0$

1) $|H_1| = k-2 > 0$ per $k>2$

2) $|H_2| = k(k-2)-1 = k^2-2k-1 \geq 0$; $k = 1 \pm \sqrt{1+1} = 1 \pm \sqrt{2}$: $|H_2|>0$ per $k<1-\sqrt{2} \cup k>1+\sqrt{2}$.



Se $k<1-\sqrt{2}$: $|H_1|<0$; $|H_2|>0$: Punto di Max

Se $1-\sqrt{2}<k<1+\sqrt{2}$: $|H_2|<0$: Punto di Sella

Se $k>1+\sqrt{2}$: $|H_1|>0$; $|H_2|>0$: Punto di MIN.

Nulla si può dire se $k=1-\sqrt{2}$; $k=1+\sqrt{2}$; se $k=0$ o $k=2$: $|H_2|<0$: Punto di Sella.

8) $f(x;y;z) = x \cdot e^{y-2z} + x \cdot \log(2y-x)$

$\nabla f(x;y;z) = \left(e^{y-2x} + \log(2y-x) - \frac{x}{2y-x} ; x e^{y-2z} + \frac{2x}{2y-x} ; -2x e^{y-2z} \right)$

$\nabla f(1;1;1) = \left(e^{-1} + \log 1 - \frac{1}{1} ; e^{-1}+2 ; -2e^{-1} \right) = \left(\frac{1}{e} - 1 ; \frac{1}{e} + 2 ; -\frac{2}{e} \right)$.

9)

A	B	C	(non B)	(non B ⇒ C)	(A ⇒ (non B ⇒ C))	(A ⇒ B)	(non C)	((A ⇒ B) ⇒ non C)	$P_1 \Leftrightarrow P_2$
1	1	1	0	1	1	1	0	0	0
1	1	0	0	1	1	1	1	1	1
1	0	1	1	1	1	0	0	1	1
1	0	0	1	0	0	0	1	1	0
0	1	1	0	1	1	1	0	0	0
0	1	0	0	1	1	1	1	1	1
0	0	1	1	1	1	1	0	0	0
0	0	0	1	0	1	1	1	1	1

Le due proposizioni NON sono logicamente equivalenti.

10) $f(x) = e^{x^2-2x}$. C.E.: $\mathbb{R}$. $f'(x) = (2x-2)\, e^{x^2-2x} \Rightarrow$

$f''(x) = 2 \cdot e^{x^2-2x} + (2x-2) \cdot (2x-2) \cdot e^{x^2-2x} = (4x^2-8x+6)\, e^{x^2-2x} \geq 0$

per $2x^2-4x+3 \geq 0 \Rightarrow x = \frac{2 \pm \sqrt{4-6}}{2}$.: Dato che $\Delta = -2 < 0$

risulta $f''(x) > 0 \ \forall x \in \mathbb{R}$.

Quindi la funzione $f(x) = e^{x^2-2x}$ è convessa su tutto $\mathbb{R}$.