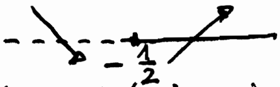
Compito di Matematica Generale del 14/7/2020 $\boxed{\text{CMG1}}\boxed{\text{B}}$

1) $f(x)=\log(x^2+x+3)$. C.E. $=\mathbb{R}$ in quanto $x^2+x+3>0\ \forall x\in\mathbb{R}$. $(\Delta=1-12)<0$.

$\lim_{x\to-\infty} f(x)=\lim_{x\to+\infty} f(x)=+\infty$. $f(x)\geq 0$ per $x^2+x+3>1 \Rightarrow x^2+x+2>0$ vera $\forall x\in\mathbb{R}$.

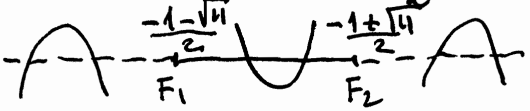
$f'(x)=\frac{2x+1}{x^2+x+3}\geq 0$ per $2x+1\geq 0 \Rightarrow x\geq -\frac{1}{2}$



$f(-\frac{1}{2})=\log\left(\frac{11}{4}\right)>0$.

$$f''(x)=\frac{2\cdot(x^2+x+3)-(2x+1)^2}{(x^2+x+3)^2}=\frac{-2x^2-2x+5}{(x^2+x+3)^2}\geq 0$$

per $2x^2+2x-5\leq 0 \Rightarrow \frac{-1-\sqrt{11}}{2}\leq x\leq \frac{-1+\sqrt{11}}{2}$



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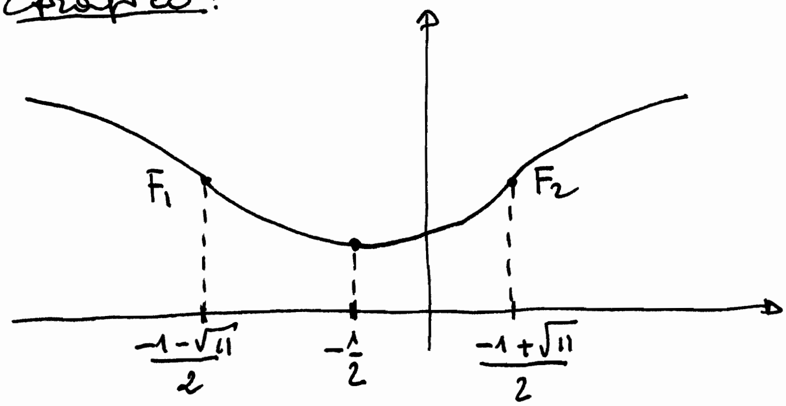


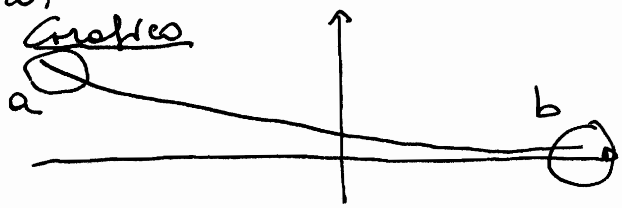
2) $\lim_{x\to 0}\frac{\log(1+(x+x^2))}{x-x^2}=\lim_{x\to 0}\frac{\log(1+(x+x^2))}{x+x^2}\cdot\frac{x(1+x)}{x(1-x)}=1\cdot 1=1.$

$\lim_{x\to+\infty}\left(\frac{x+3}{x+2}\right)^{3x}=\lim_{x\to+\infty}\left(\frac{x+2+1}{x+2}\right)^{3x}=\lim_{x\to+\infty}\left(1+\frac{1}{x+2}\right)^{3x}=e^3.$

3) $\forall\varepsilon\ \exists\delta(\varepsilon): x<\delta(\varepsilon)\Rightarrow f(x)>\varepsilon: \lim_{x\to-\infty}f(x)=+\infty;$

$\forall\varepsilon>0\ \exists\delta(\varepsilon): x>\delta(\varepsilon)\Rightarrow |f(x)|<\varepsilon: \lim_{x\to+\infty}f(x)=0.$

Grafico



4) $f(x)=\frac{2x-1}{x}\Rightarrow f(g(x))=\frac{2g(x)-1}{g(x)}=\log 3x\Rightarrow 2g(x)-1=\log 3x\cdot g(x)\Rightarrow g(x)(2-\log 3x)=1\Rightarrow$

$\Rightarrow g(x)=\frac{1}{2-\log 3x}=y\Rightarrow 1=2y-y\cdot\log 3x\Rightarrow y\cdot\log 3x=2y-1\Rightarrow \log 3x=\frac{2y-1}{y}=2-\frac{1}{y}\Rightarrow$

$\Rightarrow 3x=e^{2-\frac{1}{y}}\Rightarrow x=\frac{1}{3}e^{2-\frac{1}{y}}$; Inversa: $y=\frac{1}{3}e^{2-\frac{1}{x}}$.

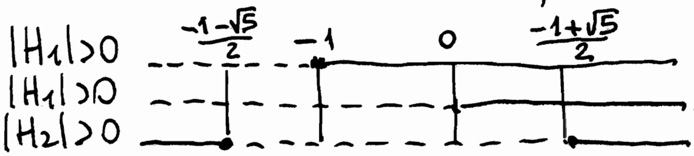
5) $A\cdot B=\begin{Vmatrix}k & 1 & 1\\ 0 & m & -1\end{Vmatrix}\cdot\begin{Vmatrix}1 & 1\\ 0 & 1\\ 1 & k\end{Vmatrix}=\begin{Vmatrix}k+0+1 & k+1+k\\ 0+0-1 & 0+m-k\end{Vmatrix}=\begin{Vmatrix}k+1 & 2k+1\\ -1 & m-k\end{Vmatrix}=\begin{Vmatrix}1 & 1\\ -1 & 2\end{Vmatrix}\Rightarrow\begin{cases}k=0\\ m=2\end{cases}.$

6) $\int_1^2\frac{1}{x^4}-e^{2x}\,dx=\left(\frac{1}{-4+1}\cdot x^{-4+1}-\frac{1}{2}e^{2x}\right)\Big|_1^2=\left(-\frac{1}{3}\cdot\frac{1}{x^3}-\frac{1}{2}e^{2x}\right)\Big|_1^2=\left(-\frac{1}{24}-\frac{1}{2}e^4\right)-\left(-\frac{1}{3}-\frac{1}{2}e^2\right)=\frac{1}{2}\left(e^2-e^4\right)+\frac{7}{24}.$

$\boxed{CMG2}\boxed{B}$

7) $H(x;y)=\begin{Vmatrix} K+1 & 1 \\ 1 & K \end{Vmatrix}$. 1) $|H_1| = K+1 > 0$ per $K > -1$
1) $|H_1| = K > 0$ per $K > 0$

2) $|H_2| = (K+1)\cdot K - 1 = K^2 + K - 1 \geq 0$; $K = \dfrac{-1 \pm \sqrt{1+4}}{2} = \dfrac{-1 \pm \sqrt{5}}{2}$; $|H_2| > 0$ per $K < \dfrac{-1-\sqrt{5}}{2} \cup K > \dfrac{-1+\sqrt{5}}{2}$.



se $K < \dfrac{-1-\sqrt{5}}{2}$: $|H_1|<0$; $|H_2|>0$: Punto di Max;
Se $\dfrac{-1-\sqrt{5}}{2} < K < \dfrac{-1+\sqrt{5}}{2}$: $|H_2|<0$: Punto di Sella;
Se $K > \dfrac{-1+\sqrt{5}}{2}$: $|H_1|>0$; $|H_2|>0$; Punto di Min.

Nulla si può dire se $K = \dfrac{-1-\sqrt{5}}{2}$; $K = \dfrac{-1+\sqrt{5}}{2}$. Se $K=-1$ e $K=0$: $|H_2|<0$ Punto di Sella.

8) $f(x;y;z) = x \cdot e^{2y-x} + \log(3z-x-y)$.

$\nabla f(x;y;z) = \left(e^{2y-x} - x e^{2y-x} - \dfrac{1}{3z-x-y}; 2x e^{2y-x} - \dfrac{1}{3z-x-y}; \dfrac{3}{3z-x-y}\right)$.

$\nabla f(1;1;1) = \left(e - e - \dfrac{1}{1}; 2e - \dfrac{1}{1}; \dfrac{3}{1}\right) = (-1; 2e-1; 3)$.

9)

A B C	(non C)	(B ⇒ non C)	(A ⇒ (B ⇒ non C))	(non B)	(A ⇒ non B)	((A ⇒ non B) ⇒ C)	$P_1 \Leftrightarrow P_2$
1 1 1	0	0	0	0	0	1	0
1 1 0	1	1	1	0	0	1	1
1 0 1	0	1	1	1	1	1	1
1 0 0	1	1	1	1	1	0	0
0 1 1	0	0	1	0	1	1	1
0 1 0	1	1	1	0	1	0	0
0 0 1	0	1	1	1	1	1	1
0 0 0	1	1	1	1	1	0	0

Le due proposizioni NON sono logicamente equivalenti.

10) $f(x) = e^{x^2+3x}$. C.E.: $\mathbb{R}$. $f'(x) = (2x+3)\cdot e^{x^2+3x} \Rightarrow$

$f''(x) = 2\cdot e^{x^2+3x} + (2x+3)(2x+3)\,e^{x^2+3x} = (4x^2+12x+11)\,e^{x^2+3x} \geq 0$

per $4x^2+12x+11 \geq 0 \Rightarrow x = \dfrac{-6 \pm \sqrt{36-44}}{4}$. Dato che $\Delta = -8 < 0$

risulta $f''(x) > 0 \ \forall x \in \mathbb{R}$.

Quindi la funzione $f(x) = e^{x^2+3x}$ è convessa su tutto $\mathbb{R}$.