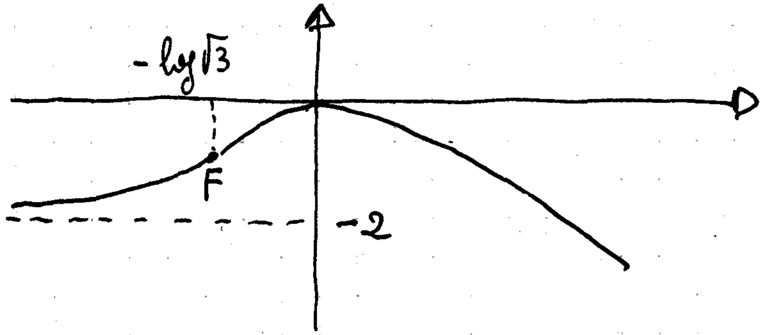
Compito di Matematica Generale del 17/9/2020 $\boxed{CMG1}$

1) $f(x)=3e^x-e^{3x}-2$. C.E.: $\mathbb{R}$. $\lim_{x\to-\infty} f(x)=-2$; $\lim_{x\to+\infty} f(x)=-\infty$; $f(0)=0$.

$f'(x)=3e^x-3e^{3x}=3e^x(1-e^{2x})\geq 0$

per $e^{2x}\leq 1 \Rightarrow x\leq 0$



$f''(x)=3e^x-9e^{3x}=3e^x(1-3e^{2x})\geq 0$

per $e^{2x}\leq \frac{1}{3} \Rightarrow x\leq -\log\sqrt{3}$

In $x=0$ punto di massimo con $f(0)=0$ quindi $f(x)\leq 0 \ \forall x\in\mathbb{R}$.

2) $\lim_{x\to 0} \frac{3^x-1}{1-2^x}=\lim_{x\to 0}\frac{3^x-1}{x}\cdot\frac{(-x)}{2^x-1}=\log 3\cdot\frac{-1}{\log 2}=-\frac{\log 3}{\log 2}$.

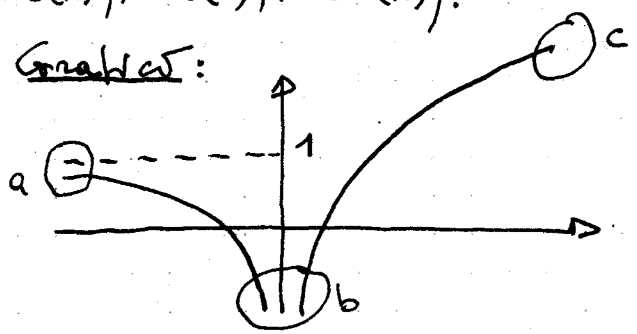
$\lim_{x\to 0}\frac{x^4-3x^2+5x}{2x^3-4x^2-x}=\lim_{x\to 0}\frac{5x}{-x}=-5$ $\left(x^4=o(x);\ x^2=o(x);\ x^3=o(x)\right)$.

3) a) $\forall\varepsilon>0\ \exists\delta(\varepsilon): x<\delta(\varepsilon)\Rightarrow |f(x)-1|<\varepsilon$: $\lim_{x\to-\infty} f(x)=1$;

b) $\forall\varepsilon\ \exists\delta(\varepsilon): |x|<\delta(\varepsilon)\Rightarrow f(x)<\varepsilon$: $\lim_{x\to 0} f(x)=-\infty$;

c) $\forall\varepsilon\ \exists\delta(\varepsilon): x>\delta(\varepsilon)\Rightarrow f(x)>\varepsilon$: $\lim_{x\to+\infty} f(x)=+\infty$

Grafico:



4) $f^{-1}(x)=\frac{x-1}{x+1}=y\Rightarrow x-1=xy+y\Rightarrow x(1-y)=y+1\Rightarrow x=\frac{y+1}{1-y}$: $f(x)=\frac{x+1}{1-x}$;

$g^{-1}(x)=3^{2-x}=y\Rightarrow 2-x=\log_3 y\Rightarrow x=2-\log_3 y$: $g(x)=2-\log_3 x$.

$f(g(x))=\frac{(2-\log_3 x)+1}{1-(2-\log_3 x)}=\frac{3-\log_3 x}{\log_3 x-1}$; $g(f(x))=2-\log_3\left(\frac{x+1}{1-x}\right)$.

5) $\int_0^1 \frac{e^x}{1+e^x}dx \Rightarrow \int\frac{1}{1+e^x}d(1+e^x)\Rightarrow \left(\log(1+e^x)\right|_0^1=\log(1+e)-\log 2=\log\frac{1+e}{2}$.

6) $f(x)\sim g(x)$ per $x\to+\infty$ se $\lim_{x\to+\infty}\frac{K\cdot 3^x-2^x}{3^x+2^x}=1$ se $K=1$ dato che $2^x=o(3^x)$;

$f(x)=o(g(x))$ per $x\to+\infty$ se $\lim_{x\to+\infty}\frac{K\cdot 3^x-2^x}{3^x+2^x}=0$ se $K=0$ dato che $2^x=o(3^x)$.

CMG 2

7) $A \cdot X = X \Rightarrow \begin{Vmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & -2 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} x \\ y \\ z \end{Vmatrix} = \begin{Vmatrix} x \\ y \\ z \end{Vmatrix} \Rightarrow \begin{cases} x+y+z=x \\ x+z=y \\ x-2y+z=z \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} y+z=0 \\ x+z=y \\ x-2y=0 \end{cases} \Rightarrow \begin{cases} z=-y \\ y=y \\ x=2y \end{cases} \Rightarrow X=(2k;k;-k).\ \|X\|=\sqrt{4k^2+k^2+k^2}=\sqrt{6k^2}=\sqrt{6} \Rightarrow k=\pm 1.$

8) $f(x;y)=x^3-3x-y\,e^{1-y}.\ \nabla f(x;y)=(0;0) \Rightarrow \begin{cases} f'_x=3x^2-3=3(x^2-1)=0 \\ f'_y=-e^{1-y}+ye^{1-y}=e^{1-y}(y-1)=0 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} x=1 \\ y=1 \end{cases} \cup \begin{cases} x=-1 \\ y=1 \end{cases}.\ H(x;y)=\begin{Vmatrix} 6x & 0 \\ 0 & (2-y)e^{1-y} \end{Vmatrix}.$

$H(1;1)=\begin{Vmatrix} 6 & 0 \\ 0 & 1 \end{Vmatrix} \Rightarrow \begin{cases} |H_1|=6>0;1>0 \\ |H_2|=6>0 \end{cases}$: P. di minimo; $H(-1;1)=\begin{Vmatrix} -6 & 0 \\ 0 & 1 \end{Vmatrix}$: $|H_2|<0$: P. Sella.

9)

A B C	non C	(A e non C)	non A	(non A δ B)	[(A e non C) ⇒ (non A δ B)]
~~1 1 1~~					
1 1 0	1	1	0	1	1
~~1 0 1~~					
~~1 0 0~~					
~~0 1 1~~					
~~0 1 0~~					
0 0 1	0	0	1	1	1
~~0 0 0~~					

Le righe 3, 4, 5 e 6 si tolgono perché A e B sono logicamente equivalenti;
Le righe 1 e 8 si tolgono perché B e C non sono logicamente equivalenti.

10) $f(x)=x\cdot e^{1+x}$; $f(0)=0$;

$f'(x)=e^{1+x}+x\,e^{1+x}=(1+x)e^{1+x}$; $f'(0)=e$;

$f''(x)=e^{1+x}+(1+x)e^{1+x}=(2+x)e^{1+x}$; $f''(0)=2e$;

$f'''(x)=e^{1+x}+(2+x)e^{1+x}=(3+x)e^{1+x}$; $f'''(0)=3e$.

$P_3(x;0)=0+ex+\frac{2ex^2}{2}+\frac{3ex^3}{6}=ex+ex^2+\frac{e}{2}x^3.$