TASKS Quantitative Methods/Mathematics for Economic Applications AA. 2019/20

Intermediate Test December 2019

I M 1) Given the polynomial equation $x^2 - \sqrt{2}\,x + k = 0$, determine the value of k for which this equation admits the solution $x = e^{\frac{\pi}{4}i}$. Then calculate the square roots of the other solution.

I M 2) Given the matrix $\mathbb{A} = \begin{Vmatrix} 1 & k & 0 \\ k & 1 & -1 \\ 0 & -1 & 1 \end{Vmatrix}$ determine the value of the parameter k such that the matrix admits the eigenvalue $\lambda = 0$ and, for this value of k, find one modal matrix which diagonalizes $\mathbb{A}$.

I M 3) If in the basis $\mathbb{V} = \{(1,1,0);(1,-1,0);(2,0,1)\}$ the vector $\mathbb{X}$ has coordinates $(1,2,-1)$, determine its coordinates in the basis $\mathbb{W} = \{(1,1,0);(1,-1,0);(0,2,1)\}$.

I M 4) Given a linear map $f : \mathbb{R}^3 \to \mathbb{R}^3$, $\mathbb{Y} = \mathbb{A} \cdot \mathbb{X}$, determine the matrix $\mathbb{A}$ knowing that such linear map satisfies these three conditions:

a) $f(1,2,1) = (1,3,3)$;

b) $(1,1,0) \in \text{Ker}(f)$;

c) $(1,1,1)$ is an eigenvector of $\mathbb{A}$ corresponding to the eigenvalue $\lambda = 1$.

I M 5) Given the linear system $\begin{cases} x_1 - 2x_2 + 2x_3 + x_4 = 1 \\ 3x_1 - x_2 + 2x_4 = 1 \\ x_1 + 3x_2 + mx_3 + k\,x_4 = 0 \end{cases}$, check, depending on the parameters m and k, the existence and the number of its solutions.

I Winter Exam Session 2020

I M 1) If $z = \sqrt{2}\left(2+\sqrt{3}\right)e^{\frac{\pi}{4}i} - 2\left(1+\sqrt{3}\right)e^{\frac{\pi}{3}i}$, calcolate $\sqrt{z}$.

I M 2) Given the matrix $\mathbb{A} = \begin{Vmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & k & 1 \end{Vmatrix}$ determine the two values of the parameter k for which the matrix admits multiple eigenvalues. For these values of k, check if the corresponding matrix is diagonalizable or not.

I M 3) Consider the linear map $f : \mathbb{R}^4 \to \mathbb{R}^4$, $\mathbb{Y} = \mathbb{A} \cdot \mathbb{X}$ for which:
$f(x_1,x_2,x_3,x_4) = (x_1 + 3x_2; 2x_1 + 2x_2; x_3 + 2x_4; 2x_3 + 4x_4)$.
Determine the dimensions of the Kernel and of the Image of this linear map, and then find a basis for the Kernel.

I M 4) Given the matrix $\mathbb{A} = \begin{Vmatrix} 1 & 2 \\ 2 & 1 \end{Vmatrix}$ determine at least a matrix $\mathbb{B}$, different from $\mathbb{A}$, that is similar to $\mathbb{A}$.

II M 1) Given $f(x,y) = x^2y - xy + xy^2$ and $v = (\cos\alpha, \sin\alpha)$, determine at least two values of α for which it results $D_v f(1,1) = D^2_{v,v} f(1,1)$.

II M 2) Solve the problem $\begin{cases} \text{Max/min } f(x,y) = x^2 + y^2 \\ \text{u.c.:} \begin{cases} y \le 1 - x^2 \\ 1 - x \le y \end{cases} \end{cases}$.

II M 3) Given the equation $f(x,y) = x^3y + xy^3 - 2xy - 2x + 2y = 0$, satisfied at $(1,1)$, verify that an implicit function $x \to y(x)$ can be defined with it and that such function has a stationary point. Then determine the nature of this stationary point.
II M 4) Given the vectors $\mathbb{X} = (xy, 2-3y)$ and $\mathbb{Y} = (x+4, xy)$, determine if pairs (x,y) exist for which the scalar product of the two vectors $\mathbb{X} \cdot \mathbb{Y} = f(x,y)$ is maximum or minimum.

II Winter Exam Session 2020

I M 1) Transform $z = \dfrac{1+\sqrt{3}}{1+i} + \dfrac{1-\sqrt{3}}{1-i}$ in trigonometric form and then calculate z^3.

I M 2) Given the matrix $\mathbb{A} = \begin{Vmatrix} 1 & 2 & 1 \\ -1 & m & 2 \\ 1 & 1 & k \end{Vmatrix}$ and the vector $\mathbb{X} = \begin{Vmatrix} 1 \\ 1 \\ 1 \end{Vmatrix}$, determine the values of m and k for which the vector $\mathbb{Y} = \mathbb{A} \cdot \mathbb{X}$ is perpendicular to the vector $(0,1,-1)$ and with modulus equal to $\sqrt{24}$.

I M 3) Given the matrix $\mathbb{A} = \begin{Vmatrix} 1 & 0 & 1 \\ 1 & k & 1 \\ 1 & 0 & 1 \end{Vmatrix}$ determine the two values of the parameter k for which the matrix admits multiple eigenvalues. For these values of k, check if the corresponding matrix is diagonalizable or not.

I M 4) Given the basis for $\mathbb{R}^{3:}$ $\mathbb{W} = \{(1,1,0);(1,0,1);(0,1,1)\}$, find the coordinates of the vector $\mathbb{Y} = (0,1,-1)$ in such basis.

II M 1) Given the system $\begin{cases} f(x,y,z) = x\,e^y + y\,e^x - 2\,e^z = 0 \\ g(x,y,z) = x^2y + xz^2 - 2xyz = 0 \end{cases}$ satisfied at $\mathrm{P}_0 = (1,1,1)$, verify that an implicit function $x \to (y,z)$ can be defined with it, and then calculate the first order derivatives of this function.

II M 2) Solve the problem $\begin{cases} \text{Max/min } f(x,y) = x^2 + y^2 - x\,y - 3x \\ \text{u.c.: } \begin{cases} x \geq 0 \\ y \geq 0 \\ y \leq 4 - x \end{cases} \end{cases}$.

II M 3) Given $f(x,y) = x\,y$ and the vectors $\mathbb{V} = (1,1)$ and $\mathbb{W} = (-1,1)$, let v and w be their unit vectors. If $\mathcal{D}_v f(x_0,y_0) = \sqrt{2}$ and $\mathcal{D}_w f(x_0,y_0) = 0$, determine the coordinates of the point (x_0,y_0).

II M 4) Given the function $f(x,y) = x^2 - xy + y^3$, determine the nature of its stationary points.

I Additional Exam Session QMfEA 2020

I M 1) After having determined the complex number z which is the solution of the equation $\dfrac{z}{1+i} + \dfrac{z}{1-i} = 2i$, calculate its cubic roots $\sqrt[3]{z}$.

I M 2) Given the matrix $\mathbb{A} = \begin{Vmatrix} 1 & 1 & 1 \\ 0 & k & k-1 \\ 0 & 0 & k^2 \end{Vmatrix}$, since $\lambda = -1$ is an eigenvalue of $\mathbb{A}$, find the value of the real parameter k, verify that $\mathbb{A}$ is diagonalizable and then find a matrix that diagonalizes $\mathbb{A}$.

I M 3) Since the linear system $\begin{cases} x_1 + x_2 + x_3 = 0 \\ mx_1 + x_2 + x_3 = 0 \\ x_1 + x_2 + kx_3 = 0 \end{cases}$ has the solution $(1, 0, -1)$, calculate the values of the real parameters m and k and then find a basis for the space of the solutions of the system.

I M 4) Determine the vectors parallel to the vector $\mathbb{X} = (1, -1, 2)$ and with modulus equal to $\sqrt{6}$.

II M 1) Given the equation $f(x, y) = e^{x^2+y^2} - e^{x-y} = 0$ satisfied at the point $(0, 0)$, verify that an implicit function $x \to y(x)$ can be defined with it, and then calculate the first order derivative of this function.

II M 2) Solve the problem $\begin{cases} \text{Max/min } f(x, y) = x - 2y + 3z \\ \text{s.v. } x^2 + y^2 + z^2 = 14 \end{cases}$.

II M 3) Given $f(x, y) = x\,y$ and $g(x, y) = x^2 + y^2$ determine for which values of parameter α it is $\mathcal{D}_v f(1, 1) = \mathcal{D}_v g(1, 1)$, with $v = (\cos\alpha, \sin\alpha)$.

II M 4) Given the function $f(x, y, z) = x^2 y z^3 - \log(y - x) + e^{y-z}$, determine the gradient vector of the function at $P_0 = (1, 2, 2)$.

I Additional Exam Session MfEA 2020

I M 1) After having determined the complex number z which is the solution of the equation $\dfrac{z}{1-i} - \dfrac{z}{1+i} = 1$, calculate its square roots $\sqrt{z}$.

I M 2) Given the matrix $\mathbb{A} = \begin{Vmatrix} 1 & 0 & 0 \\ 1 & k-1 & 0 \\ 1 & k & k \end{Vmatrix}$, since $\lambda = 1$ is a multiple eigenvalue for $\mathbb{A}$, find the values of the real parameter k and check, for such values, if $\mathbb{A}$ is diagonalizable or not.

I M 3) Since the linear system $\begin{cases} kx_1 + 2x_2 - x_3 = 2 \\ 2x_1 - kx_2 + x_3 = 2 \\ 3x_1 + x_2 - kx_3 = 3 \end{cases}$ has the solution $(1, 1, 1)$, calculate the value of the real parameter k and then find the number of the solutions of the system.

I M 4) Determine all the vectors orthogonal to the vector $\mathbb{X}_1 = (2, -1, 1)$ and to the vector $\mathbb{X}_2 = (1, 1, 1)$.

II M 1) Given the equation $f(x, y) = e^{x+y} - e^{x-y} = 0$ satisfied at the point $(0, 0)$, verify that an implicit function $x \to y(x)$ can be defined with it, and then calculate the first order derivative of this function.

II M 2) Solve the problem $\begin{cases} \text{Max/min } f(x, y) = x - y + z \\ \text{s.v. } x^2 + y^2 + z^2 = 3 \end{cases}$.

II M 3) Given $f(x, y) = x\,y$ and $g(x, y) = x - y$ determine for which values of parameter α it is $\mathcal{D}_v f(1, 1) = \mathcal{D}_v g(1, 1)$, with $v = (\cos\alpha, \sin\alpha)$.

II M 4) Given the function $f(x, y, z) = x^2 y^3 z + \log(z - x) - e^{y-z}$, determine the gradient vector of the function at $P_0 = (1, 2, 2)$.

I Summer Exam Session 2020

I M 1) Calculate the cubic roots of the number $z = \dfrac{\sqrt{2}\,i}{1+i}$.

I M 2) Given the matrix $\mathbb{A}=\begin{Vmatrix}1 & -2 & 0\\ 2 & -3 & 0\\ 3 & 1 & k\end{Vmatrix}$, determine, on varying the parameter k, its eigenvalues and their multiplicity, then establishing if there are values of k for which the given matrix is diagonalizable.
I M 3) Determine, on varying the parameters m and k, the dimensions of the Image and the Kernel of the linear map $\mathbb{R}^5\to\mathbb{R}^3$, $f(\mathbb{X})=\mathbb{A}\cdot\mathbb{X}$, with $\mathbb{A}=\begin{Vmatrix}1 & 0 & 1 & 0 & 1\\ 1 & 1 & 1 & m & k\\ 0 & 1 & 0 & 1 & m\end{Vmatrix}$.
I M 4) Given the vectors $\mathbb{X}_1=(1,2,1)$, $\mathbb{X}_2=(1,-1,2)$ and $\mathbb{X}_3=(2,2,k)$, determine the value of the parameter k for which the three vectors are linearly dependent.
II M 1) Solve the problem : $\begin{cases}\text{Max/min } f(x,y)=x^2+y\\ \text{u.c. : } (x-1)^2-1\le y\le 1\end{cases}$.
II M 2) Given the function $f(x,y)=e^{x-y}$, find the directions $v=(\cos\alpha,\sin\alpha)$ for which it results $\mathcal{D}_v f(0,0)=0$.
II M 3) Given the function $f(x,y,z)=x^2+2y^2+z^2-2xy$, check the nature of its stationary points.
II M 4) Given the equation $f(x,y)=x\,e^{x-y}-y\,e^{y-x}=0$ satisfied at the point $(0,0)$, verify that an implicit function $x\to y(x)$ can be defined with it, and then calculate the first order derivative of this function.

II Summer Exam Session 2020

I M 1) Calculate the square roots of the number $z=\dfrac{1-i}{1+i}$.
I M 2) Determine an orthogonal matrix that diagonalizes the matrix $\mathbb{A}=\begin{Vmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{Vmatrix}$.
I M 3) Determine, on varying the parameter m, the dimensions of the Image and the Kernel of the linear map $\mathbb{R}^3\to\mathbb{R}^3$, $f(\mathbb{X})=\mathbb{A}\cdot\mathbb{X}$, with $\mathbb{A}=\begin{Vmatrix}1 & 1 & 2\\ 2 & -1 & 2\\ 1 & 2 & m\end{Vmatrix}$.
I M 4) Determine the value of the parameter k for which the vector $\mathbb{X}=(1,k,1)$ can be expressed as a linear combination of the vectors $\mathbb{X}_1=(1,2,-1)$ and $\mathbb{X}_2=(1,1,2)$ and then find the coefficients of such combination.
II M 1) Solve the problem : $\begin{cases}\text{Max/min } f(x,y)=x^2+x\,y^2\\ \text{u.c.: } 4x^2+y^2\le 4\end{cases}$.
II M 2) Given the function $f(x,y)=x\,y$, let v and w be the unit vectors of $\mathbb{V}=(1,1)$ and $\mathbb{W}=(1,-1)$; determine the point (x_0,y_0) if $\mathcal{D}_v f(x_0,y_0)=\sqrt{2}$ and $\mathcal{D}_w f(x_0,y_0)=0$.
II M 3) In a stationary point of a function of two variables $f(x,y)$ the Hessian matrix is equal to $\mathbb{H}=\begin{Vmatrix}k & 1\\ 1 & k-2\end{Vmatrix}$. Determine, on varying the parameter k, the nature of such stationary point.
II M 4) Given the system $\begin{cases}f(x,y,z)=\text{sen}\,(xy)+\cos\,(xz)=1\\ g(x,y,z)=x^3y^2-xz^3+zy^3=1\end{cases}$ satisfied at the point $P_0=(0,1,1)$, verify that with it we can define an implicit function $z\to(x,y)$ and then calculate its first derivatives at $z=1$.

II Autumn Exam Session 2020

I M 1) Calculate the cubic roots of the number $z = \dfrac{1}{1-i} - \dfrac{1}{1+i}$.

I M 2) Given the matrix $\mathbb{A} = \begin{Vmatrix} k & 0 & k \\ 0 & 2 & 0 \\ k & 0 & k \end{Vmatrix}$, determine the values of the parameter k for which the matrix admits a multiple eigenvalue.

I M 3) Given the linear system $\begin{cases} x_1 - 2x_3 = 1 \\ 2x_1 - x_2 + k\,x_3 = 2 \\ x_1 - 2x_2 + 6x_3 = k \end{cases}$, check existence and number of its solutions on varying the parameter k.

I M 4) Determine the matrix $\mathbb{A} = \begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix}$ knowing that $(1,\,-1)$ is an eigenvector relative to the eigenvalue $\lambda = 0$ and that $(1,1)$ is an eigenvector relative to $\lambda = 2$.

II M 1) Solve the problem : $\begin{cases} \text{Max/min } f(x,y) = x^2 + y^2 - y \\ \text{u.c.: } \begin{cases} y \le 1 - 2x \\ x \ge 0 \\ y \ge 0 \end{cases} \end{cases}$.

II M 2) Given the function $f(x,y) = x^2 - xy^2$, and the unit vector $v = (\cos\alpha, \sin\alpha)$, calculate $\mathcal{D}_v f(1,1)$ and $\mathcal{D}^2_{v,v} f(1,1)$. And then calculate $\mathcal{D}^2_{v,v} f(1,1)$ when $\alpha = \dfrac{\pi}{4}$.

II M 3) Given the function $f(x,y) = x^2 + x^2\,y + y^2$ determine existence and nature of its stationary points.

II M 4) Given the equation $f(x,y) = x\,e^{x+y} - y\,e^{x-y} = 0$, verify if at $\mathrm{P}_0 = (0,0)$ it is possible to define an implicit function having y as dependent variable of x and then calculate the partial derivative of the first order of $y(x)$ at $x = 0$.

II Additional Exam Session 2020

I M 1) Calcolate $\sqrt{\left(1 + \sqrt{3}\,i\right)^3}$.

I M 2) Given the linear map $f : \mathbb{R}^4 \to \mathbb{R}^3$ having matrix $\mathbb{A} = \begin{Vmatrix} 1 & -1 & 2 & 1 \\ 0 & 2 & 1 & 2 \\ 2 & 0 & k & m \end{Vmatrix}$, determine the value of the parameters k and m if the Kernel and the Image have the same dimension. For such values find a basis for the Kernel of the linear map generated by $\mathbb{A}$.

I M 3) Given the matrix $\mathbb{A} = \begin{Vmatrix} 2 & 2 & -1 \\ 3 & 0 & -2 \\ k & 2 & 1 \end{Vmatrix}$, find the value of the parameter k knowing that the matrix admits the eigenvalue $\lambda = -1$ and then determine, for this value of the parameter, if the matrix is diagonalizable or not.

I M 4) Find the coordinates of the vector $\mathbb{X}$ in the basis $\mathbb{V} = \{(1,0,0),(1,1,0),(1,0,1)\}$ if it has coordinates $(2,0,-1)$ in the basis $\mathbb{W} = \{(1,1,0),(1,-1,1),(1,0,1)\}$.

II M 1) Solve the problem : $\begin{cases} \text{Max/min } f(x,y) = x^2 - y^2 \\ \text{u.c. : } x^2 \le y \le 1 \end{cases}$.

II M 2) Given the function $f(x,y,z)=x^2-xy+y^2+z^2$ determine the nature of its stationary point.
II M 3) Given the function $f(x,y)=e^{x^2+y^2}$ and the unit vector $v=(\cos\alpha,\sin\alpha)$ calculate its directional derivatives $\mathcal{D}^2_{vv}f(0,0)$.
II M 4) Given the equation $f(x,y)=x\sin y-y\cos x=0$, verify if at $\mathrm{P}_0=(0,0)$ it is possible to define an implicit function having y as dependent variable of x and then calculate the partial derivative of the first order of $y(x)$ at $x=0$.