Compito di Matematica Generale del 6/10/2020 $\boxed{\text{CMG1}}$

1) $f(x)=\dfrac{x^2+x+1}{x-1}$. C.E.: $x\neq 1$. $\lim\limits_{x\to-\infty} f(x)=-\infty$; $\lim\limits_{x\to 1^-} f(x)=-\infty$; $\lim\limits_{x\to 1^+} f(x)=+\infty$; $\lim\limits_{x\to+\infty} f(x)=+\infty$.

$f(x)\geq 0$ per $x\geq 1$. $f(0)=-1$.

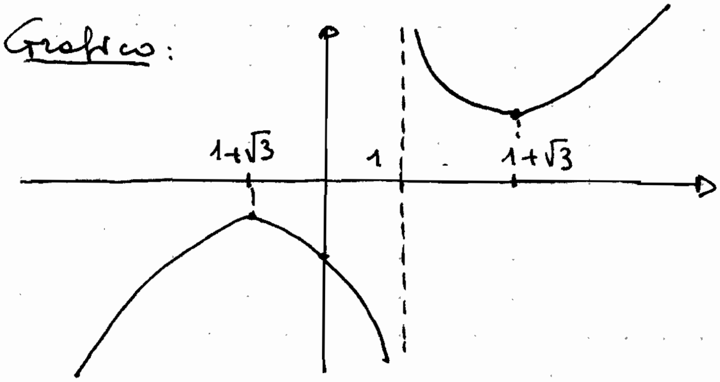
$$f'(x)=\frac{(2x+1)(x-1)-(x^2+x+1)}{(x-1)^2}=\frac{x^2-2x-2}{(x-1)^2}\geq 0$$

$x=1\pm\sqrt{1+2}=1\pm\sqrt{3}$ $\quad 1-\sqrt{3} \quad 1 \quad 1+\sqrt{3}$

$$f''(x)=\frac{(2x-2)(x-1)^2-(x^2-2x-2)\cdot 2(x-1)}{(x-1)^4}=$$

$$f''(x)=\frac{6}{(x-1)^3}\geq 0 \text{ per } x\geq 1 \quad 1$$

Grafico:



2) $$\lim_{x\to 0}\frac{3^{\operatorname{sen}3x}-1}{6x}=\lim_{x\to 0}\frac{3^{\operatorname{sen}3x}-1}{\operatorname{sen}3x}\cdot\frac{\operatorname{sen}3x}{3x}\cdot\frac{1}{2}=\log 3\cdot 1\cdot\frac{1}{2}=\frac{1}{2}\cdot\log 3=\log\sqrt{3}.$$

$$\lim_{x\to+\infty}\left(\frac{1+x^2}{1+x}\right)^{1-x}=(\to+\infty)^{(\to-\infty)}=0^+.$$

3) $f(x)=3x-1$; $g(x)=2^{1-x}$. $g(f(f(x)))=g(f(3x-1))=g(3(3x-1)-1)=g(9x-4)=$

$=2^{1-9x+4}=2^{5-9x}=y \Rightarrow 5-9x=\log_2 y \Rightarrow 9x=5-\log_2 y \Rightarrow x=\frac{1}{9}(5-\log_2 y)$. Inversa $y=\frac{1}{9}(5-\log_2 x)$.

4) $f(x)=o(g(x))$: $\lim\limits_{x\to x_0}\dfrac{\log x}{x+1}=0$ se $x\to 1$ e se $x\to+\infty$;

$g(x)=o(f(x))$: $\lim\limits_{x\to x_0}\dfrac{x+1}{\log x}=0$ se $x\to 0^+$ $\left(\dfrac{\to 1}{\to-\infty}\right)$

5) $$\int_1^2 x-\frac{1}{x}+\frac{1}{x^2}\,dx=\left(\frac{x^2}{2}-\log x-\frac{1}{x}\right|_1^2=\left(2-\log 2-\frac{1}{2}\right)-\left(\frac{1}{2}-0-1\right)=2-\log 2-1+1=2-\log 2.$$

6) $f(x)=\log x-2x$. C.E.: $x>0$. $f'(x)=\frac{1}{x}-2$. Se tangente parallela a $y=1-x$ deve essere: $\frac{1}{x}-2=-1 \Rightarrow \frac{1}{x}=1 \Rightarrow x=1$. $f(1)=-2$: Eq. retta tg.: $y+2=-1(x-1) \Rightarrow y=-x-1$.

7) $$f(x)=\begin{cases}2^{1-x}: & x\leq 0\\ mx+q: & 0<x<1.\\ x^2-2x+5: & 1<x\end{cases}$$

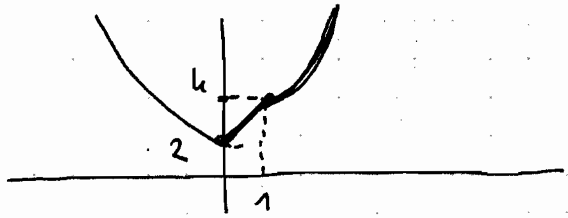
$\lim\limits_{x\to 0^-} f(x)=2=\lim\limits_{x\to 0^+} f(x)=q \Rightarrow q=2$;

$\lim\limits_{x\to 1^-} f(x)=m+q=\lim\limits_{x\to 1^+} f(x)=4 \Rightarrow q+m=4 \Rightarrow m=+2$.

$y=mx+q \Rightarrow y=+2x+2$.

CMG2

Grafico di $f(x)$:



8) $A = \begin{Vmatrix} 1 & 2 \\ 1 & 3 \end{Vmatrix}$; $X = \begin{Vmatrix} x \\ y \end{Vmatrix}$. $A \cdot A \cdot X = \begin{Vmatrix} 1 & 2 \\ 1 & 3 \end{Vmatrix} \cdot \begin{Vmatrix} 1 & 2 \\ 1 & 3 \end{Vmatrix} \cdot \begin{Vmatrix} x \\ y \end{Vmatrix} = \begin{Vmatrix} 1+2 & 2+6 \\ 1+3 & 2+9 \end{Vmatrix} \cdot \begin{Vmatrix} x \\ y \end{Vmatrix} =$

$= \begin{Vmatrix} 3 & 8 \\ 4 & 11 \end{Vmatrix} \cdot \begin{Vmatrix} x \\ y \end{Vmatrix} = \begin{Vmatrix} 3x+8y \\ 4x+11y \end{Vmatrix} = Y$. $Y \perp X_0 : (3x+8y; 4x+11y) \cdot (1; -1) = 0 \Rightarrow$

$\Rightarrow 3x+8y-4x-11y = -x-3y = 0 \Rightarrow x = -3y$. $X = (-3k; k)$, $k \in \mathbb{R}$.

9) $f(x;y) = x^3 - 3xy + y^2 + y$. $\nabla f(x;y) = (0;0) \Rightarrow$

$\begin{cases} f'_x = 3x^2 - 3y = 3(x^2 - y) = 0 \\ f'_y = -3x + 2y + 1 = 0 \end{cases} \Rightarrow \begin{cases} y = x^2 \\ 2x^2 - 3x + 1 = 0 \end{cases} \Rightarrow x = \dfrac{3 \pm \sqrt{9-8}}{4} = \dfrac{3 \pm 1}{4}$

$\Rightarrow \begin{cases} x = 1 \\ y = 1 \end{cases} \cup \begin{cases} x = \frac{1}{2} \\ y = \frac{1}{4} \end{cases}$. $H(x;y) = \begin{Vmatrix} 6x & -3 \\ -3 & 2 \end{Vmatrix}$.

$H(1;1) = \begin{Vmatrix} 6 & -3 \\ -3 & 2 \end{Vmatrix} \Rightarrow \begin{cases} |H_1| = 6 > 0;\ 2 > 0 \\ |H_2| = 12 - 9 > 0 \end{cases} \Rightarrow (1;1)$ è Punto di minimo;

$H(\frac{1}{2}; \frac{1}{4}) = \begin{Vmatrix} 3 & -3 \\ -3 & 2 \end{Vmatrix} \Rightarrow |H_2| = 6 - 9 < 0 : (\frac{1}{2}; \frac{1}{4})$ è Punto di Selle.

10)

A	B	$(A \Rightarrow B)$	$[A\ \sigma\ (A \Rightarrow B)]$	(non B)	(non A)	(non B $\Rightarrow$ non A)	$P_1 \Leftrightarrow P_2$
1	1	1	1	0	0	1	1
1	0	0	1	1	0	0	0
0	1	1	1	0	1	1	1
0	0	1	1	1	1	1	1

Le due proposizioni non risultano logicamente equivalenti.