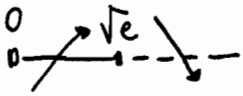
Compito di Matematica Generale del 9/2/2021 Compito A $\boxed{\text{MGA1}}$

1) $f(x)=\dfrac{\log x}{x^2}$. C.E.: $x>0$. $\lim\limits_{x\to 0^+} f(x)=-\infty$; $\lim\limits_{x\to+\infty} f(x)=0^+$. $f(x)\geq 0$ per $x\geq 1$.

$$f'(x)=\frac{\frac{1}{x}\cdot x^2-2x\log x}{x^4}=\frac{1-2\log x}{x^3}\geq 0$$

per $\log x\leq\frac{1}{2}\Rightarrow x\leq\sqrt{e}$



$$f''(x)=\frac{-\frac{2}{x}\cdot x^3-3x^2(1-2\log x)}{x^6}=\frac{6\log x-5}{x^4}\geq 0$$

per $\log x\geq\frac{5}{6}\Rightarrow x\geq\sqrt[6]{e^5}$

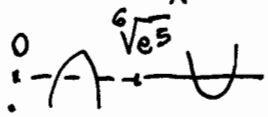
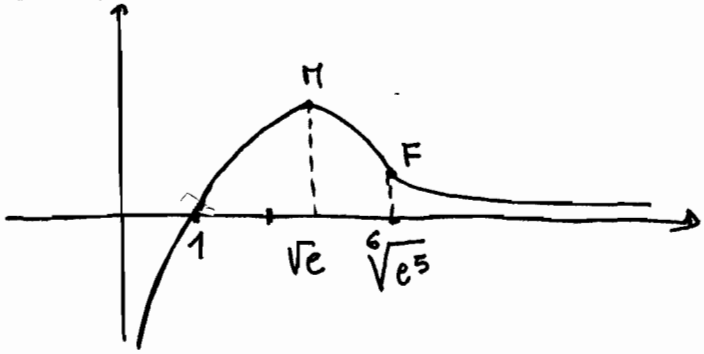


Grafico:



2) $$\lim_{x\to 0}\frac{\log(1+\operatorname{sen}2x)}{e^{\operatorname{sen}3x}-1}=\lim_{x\to 0}\frac{\log(1+\operatorname{sen}2x)}{\operatorname{sen}2x}\cdot\frac{\operatorname{sen}2x}{2x}\cdot\frac{2x}{3x}\cdot\frac{3x}{\operatorname{sen}3x}\cdot\frac{\operatorname{sen}3x}{e^{\operatorname{sen}3x}-1}=1\cdot 1\cdot\frac{2}{3}\cdot 1\cdot 1=\frac{2}{3}.$$

$$\lim_{x\to-\infty}\frac{1-x+x^3-2^x}{2^x-x}=\lim_{x\to-\infty}\frac{x^3}{-x}=-\infty\quad\left(2^x\to 0;\ x=o(x^3)\right).$$

3) $$\lim_{x\to 0}\frac{\operatorname{sen}kx}{(k+1)x}=\lim_{x\to 0}\frac{\operatorname{sen}kx}{kx}\cdot\frac{k}{k+1}=\frac{k}{k+1}=2\Rightarrow k=2k+2\Rightarrow k=-2.$$

4) $f(x)=2x-1$; $g(x)=x^3$. $F(x)=f(g(f(x)))=f(g(2x-1))=f((2x-1)^3)=2(2x-1)^3-1$.

$$F'(x)=2\cdot 3\cdot(2x-1)^2\cdot 2=12\cdot(2x-1)^2.$$

5) $$\int_1^{+\infty} e^{1-2x}-\frac{1}{x^7}\,dx=\lim_{m\to+\infty}\int_1^m e^{1-2x}-x^{-7}\,dx=\lim_{m\to+\infty}\left(-\frac{1}{2}e^{1-2x}-\frac{1}{-6}\cdot x^{-6}\right)\Bigg|_1^m=$$

$$=\lim_{m\to+\infty}\left(-\frac{1}{2}e^{1-2m}+\frac{1}{6m^6}\right)-\left(-\frac{1}{2}e^{-1}+\frac{1}{6}\right)=(0+0)-\left(-\frac{1}{2e}+\frac{1}{6}\right)=\frac{1}{2e}-\frac{1}{6}.$$

6) Da $F(x)=\dfrac{1}{f(x)}\Rightarrow F'(x)=-\dfrac{f'(x)}{f^2(x)}:\left(-\dfrac{(-)}{(+)}\right)\Rightarrow F'(x)\geq 0\Rightarrow F(x)$ crescente.

7) $f(x;y)=x^2-3y-2x+y^3$.

$$\nabla f(x;y)=(0;0)\Rightarrow\begin{cases}f'_x=2x-2=2(x-1)=0\\ f'_y=3y^2-3=3(y^2-1)=0\end{cases}\Rightarrow\begin{cases}x=1\\ y=\pm 1\end{cases}$$

Ci sono 2 punti stazionari: $(1;1)$ e $(1;-1)$

$H(x;y)=\begin{Vmatrix} 2 & 0 \\ 0 & 6y \end{Vmatrix}$. $H(1;1)=\begin{Vmatrix} 2 & 0 \\ 0 & 6 \end{Vmatrix}$: P. Minimo; $H(1;-1)=\begin{Vmatrix} 2 & 0 \\ 0 & -6 \end{Vmatrix}$: Punto di Sella.

8) $A\cdot X=\begin{Vmatrix} 1 & -1 & k \\ 0 & k & 1 \end{Vmatrix}\cdot\begin{Vmatrix} k \\ 1 \\ 1 \end{Vmatrix}=\begin{Vmatrix} k-1+k \\ 0+k+1 \end{Vmatrix}=\begin{Vmatrix} 2k-1 \\ k+1 \end{Vmatrix}$.

$\|A\cdot X\|=\sqrt{(2k-1)^2+(k+1)^2}=\sqrt{2} \Rightarrow 5k^2-2k+2=2 \Rightarrow k(5k-2)=0 \Rightarrow k=0,\ k=\frac{2}{5}$.

$A\cdot X // (2;3) \Rightarrow \frac{2k-1}{2}=\frac{k+1}{3} \Rightarrow 6k-3=2k+2 \Rightarrow 4k=5 \Rightarrow k=\frac{5}{4}$.

$A\cdot X \perp (1;2) \Rightarrow (2k-1)\cdot 1+(k+1)\cdot 2=2k-1+2k+2=4k+1=0 \Rightarrow k=-\frac{1}{4}$.

9)

A	B	C	non A	(A δ B)	[non A e (A δ B)]	e	δ	⇒	⇔	C
1	1	1	0	1	0	0	1	1	0	1
1	1	0	0	1	0	0	0	1	1	0
1	0	1	0	1	0	0	1	1	0	1
1	0	0	0	1	0	0	0	1	1	0
0	1	1	1	1	1	1	1	1	1	1
0	1	0	1	1	1	0	1	0	0	0
0	0	1	1	0	0	0	1	1	0	1
0	0	0	1	0	0	0	0	1	1	0

Come si vede, nessun connettivo rende [non A e (A δ B)] * C una tautologia.

10) $f(x)=x^3\cdot e^x$. $f\in\mathcal{C}(\mathbb{R})$

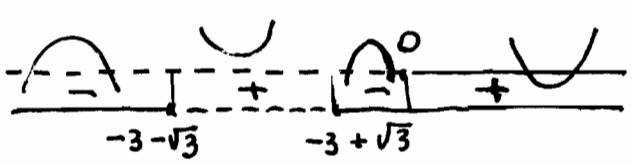
$f'(x)=3x^2\cdot e^x+x^3\cdot e^x=x^2e^x(3+x)$.

$f''(x)=6x\cdot e^x+3x^2\cdot e^x+3x^2e^x+x^3e^x=e^x(x^3+6x^2+6x) \Rightarrow$

$f''(x)=x\cdot e^x\cdot(x^2+6x+6)$. $x^2+6x+6=0 \Rightarrow x=-3\pm\sqrt{9-6}=-3\pm\sqrt{3}$

$x\geq 0$ per $x\geq 0$

$x^2+6x+6\geq 0$ per $x\leq -3-\sqrt{3} \cup x\geq -3+\sqrt{3}$



Funzione Concava: $]-\infty;-3-\sqrt{3}[$ e $]-3+\sqrt{3};0[$

Funzione Convessa: $]-3-\sqrt{3};-3+\sqrt{3}[$ e $]0;+\infty[$

Punti di flesso in $x=0$; $x=-3-\sqrt{3}$ e $x=-3+\sqrt{3}$.

Compito di Matematica Generale del 9/2/2021 Compito B $\boxed{MGB1}$

1) $f(x) = e^{3x-x^2}$. C.E.: $\mathbb{R}$. $\lim_{x\to-\infty} f(x) = \lim_{x\to+\infty} f(x) = 0^+$. $f(x) > 0 \ \forall x \in \mathbb{R}$, $f(0) = 1$.

$f'(x) = (3-2x)\,e^{3x-x^2} \geq 0$ per $3-2x \geq 0$

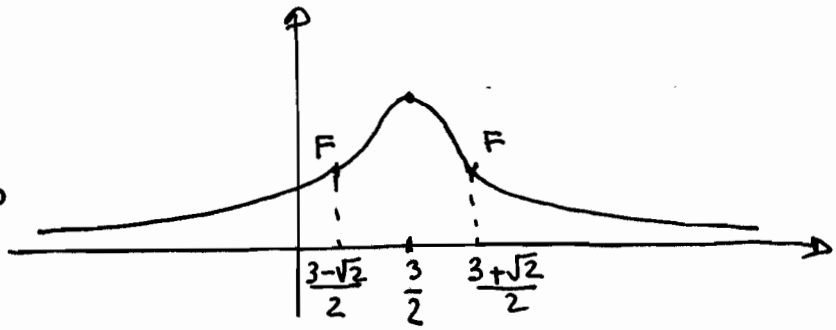
$\Rightarrow x \leq \frac{3}{2}$

$f''(x) = -2e^{3x-x^2} + (3-2x)^2 e^{3x-x^2} =$

$= e^{3x-x^2}\cdot(4x^2-12x+7) \geq 0$; $x = \frac{6 \pm \sqrt{36-28}}{4} \Rightarrow$

$f''(x) \geq 0$ per $x \leq \frac{3-\sqrt{2}}{2} \cup x \geq \frac{3+\sqrt{2}}{2}$

Grafico



2) $\lim_{x\to0} \frac{\operatorname{sen}^2 x}{1-\cos 2x} = \lim_{x\to0} \frac{\operatorname{sen} x}{x}\cdot\frac{\operatorname{sen} x}{x}\cdot\frac{4x^2}{1-\cos 2x}\cdot\frac{1}{4} = 1\cdot1\cdot2\cdot\frac{1}{4} = \frac{1}{2}$.

$\lim_{x\to-\infty} \frac{3^x-2^x+x}{3x-e^x+x^2} = \lim_{x\to-\infty} \frac{x}{x^2} = 0^-$ $\left(3^x\to0;\ e^x\to0;\ 2^x\to0;\ 3x = o(x^2)\right)$.

3) $\lim_{x\to0} \frac{e^{kx}-1}{(k-1)x} = \lim_{x\to0} \frac{e^{kx}-1}{kx}\cdot\frac{k}{k-1} = \frac{k}{k-1} = 5 \Rightarrow k = 5k-5 \Rightarrow k = \frac{5}{4}$.

4) $f(x) = \operatorname{sen} x$; $g(x) = e^{2x}$; $h(x) = x^3$. $F(x) = f(g(h(x))) = f(g(x^3)) = f(e^{2x^3}) =$

$= \operatorname{sen}(e^{2x^3}) \Rightarrow F'(x) = \cos(e^{2x^3})\cdot e^{2x^3}\cdot 6x^2$.

5) $\int_1^{+\infty} e^{2-3x} - \frac{1}{x^5}\,dx = \lim_{m\to+\infty}\int_1^m e^{2-3x} - x^{-5}\,dx = \lim_{m\to+\infty}\left(-\frac{1}{3}e^{2-3x} - \frac{1}{-4}x^{-4}\right|_1^m =$

$= \lim_{m\to+\infty}\left(-\frac{1}{3}e^{2-3m} + \frac{1}{4m^4}\right) - \left(-\frac{1}{3}e^{-1} + \frac{1}{4}\right) = (0+0) - \left(-\frac{1}{3e} + \frac{1}{4}\right) = \frac{1}{3e} - \frac{1}{4}$.

6) Da $F(x) = e^{f(x)} \Rightarrow F'(x) = f'(x)\cdot e^{f(x)} \Rightarrow F''(x) = f''(x)\cdot e^{f(x)} + (f'(x))^2\cdot e^{f(x)} \geq 0 \ \forall x$

quindi $F(x)$ è convessa su tutto $\mathbb{R}$.

7) $f(x;y) = x^3 + 4y - 3x + y^2$

$\nabla f(x;y) = (0;0)$ $\Rightarrow \begin{cases} f'_x = 3x^2-3 = 3(x^2-1) = 0 \\ f'_y = 2y+4 = 2(y+2) = 0 \end{cases} \Rightarrow \begin{cases} x = \pm1 \\ y = -2 \end{cases}$.

Ci sono due punti stazionari: $(1;-2)$ e $(-1;-2)$.

$H(x;y)=\begin{Vmatrix} 6x & 0 \\ 0 & 2 \end{Vmatrix}$; $H(1;-2)=\begin{Vmatrix} 6 & 0 \\ 0 & 2 \end{Vmatrix}$: P. Minimo; $H(-1;-2)=\begin{Vmatrix} -6 & 0 \\ 0 & 2 \end{Vmatrix}$: P. Sella.

8) $A \cdot A^T \cdot X = \begin{Vmatrix} 1 & 2 & 1 \\ 1 & 1 & -1 \end{Vmatrix} \cdot \begin{Vmatrix} 1 & 1 \\ 2 & 1 \\ 1 & -1 \end{Vmatrix} \cdot \begin{Vmatrix} 1 \\ -2 \end{Vmatrix} = \begin{Vmatrix} 1+4+1 & 1+2-1 \\ 1+2-1 & 1+1+1 \end{Vmatrix} \cdot \begin{Vmatrix} 1 \\ -2 \end{Vmatrix} =$

$= \begin{Vmatrix} 6 & 2 \\ 2 & 3 \end{Vmatrix} \cdot \begin{Vmatrix} 1 \\ -2 \end{Vmatrix} = \begin{Vmatrix} 6-4 \\ 2-6 \end{Vmatrix} = \begin{Vmatrix} 2 \\ -4 \end{Vmatrix} = K \cdot \begin{Vmatrix} 1 \\ -2 \end{Vmatrix} \Rightarrow K = 2.$

9)

A	B	C	(A ⇒ B)	(non B)	[(A ⇒ B) e (non B)]	e	o	⇒	⇔	C
1	1	1	1	0	0	0	1	1	0	1
1	1	0	1	0	0	0	0	1	1	0
1	0	1	0	1	0	0	1	1	0	1
1	0	0	0	1	0	0	0	1	1	0
0	1	1	1	0	0	0	1	1	0	1
0	1	0	1	0	0	0	0	1	1	0
0	0	1	1	1	1	1	1	1	1	1
0	0	0	1	1	1	0	1	0	0	0

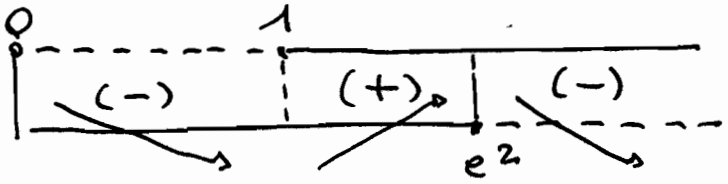
Come si vede, nessun connettivo rende $[(A \Rightarrow B) \text{ e } (\text{non } B)] * C$ una tautologia.

10) $f(x) = \dfrac{\log^2 x}{x}$. Campo d'Esistenza: $x > 0$.

$$f'(x) = \frac{2\log x \cdot \frac{1}{x} \cdot x - 1 \cdot \log^2 x}{x^2} = \frac{2\log x - \log^2 x}{x^2} = \frac{\log x \cdot (2 - \log x)}{x^2} \geq 0$$

$\log x \geq 0 \Rightarrow x \geq 1$

$2 - \log x \geq 0 \Rightarrow \log x \leq 2 \Rightarrow x \leq e^2$



La funzione è crescente in $]1; e^2[$; decrescente in $]0;1[$ e in $]e^2; +\infty[$. In $x=1$ si ha un punto di minimo, in $x = e^2$ si ha un punto di massimo.