TASKS Quantitative Methods/Mathematics for Economic Applications AA. 2020/21

Intermediate Test December 2020

NOT DONE

I Winter Exam Session 2021

I M 1) If $e^z = \dfrac{\sqrt[3]{e}}{2}\left(\sqrt{3} - i\right)$, find z.

I M 2) Given the vectors $\mathbb{X}_1 = (1, 0, -2, 2)$, $\mathbb{X}_2 = (2, 1, -1, 2)$ and $\mathbb{X}_3 = (1, 2, 4, k)$, check, depending on the parameter k, when they are linearly independent.

I M 3) Determine the only value of the parameter k for which the matrix $\left\|\begin{matrix} 1 & 2 & 3 \\ 0 & k & 2 \\ 0 & 0 & 1 \end{matrix}\right\|$ is diagonalizable.

I M 4) Consider the linear map $f : \mathbb{R}^4 \to \mathbb{R}^3$, $\mathbb{Y} = \mathbb{A} \cdot \mathbb{X}$ whith $\mathbb{A} = \left\|\begin{matrix} 1 & -1 & m & 1 \\ 0 & 2 & 1 & k \\ 1 & 1 & 2k & m \end{matrix}\right\|$.
Since $f(1,1,1,1) = (4,5,9)$, determine the values of the parameters m and k and then find the dimensions of the Kernel and of the Image of this linear map.

II M 1) Given $f(x,y) = x^2 - 3xy + 2y^2$, v and w the unit vectors of $(1,1)$ and $(1,-1)$, since $\mathcal{D}_v f(\mathrm{P}_0) = \sqrt{2}$ and $\mathcal{D}_w f(\mathrm{P}_0) = 2\sqrt{2}$, determine P_0 and then calcolate $D^2_{v,w} f(\mathrm{P}_0)$.

II M 2) Solve the problem $\begin{cases} \text{Max/min } f(x,y) = x(y+1) \\ \text{u.c.: } \begin{cases} x \le 1 - y^2 \\ 1 \le x + y \end{cases} \end{cases}$.

II M 3) Given the equation $f(x,y,z) = 2xy - e^{z-x} - e^{z-y} = 0$, satisfied at $(1,1,1)$, verify that an implicit function $(x,y) \to z(x,y)$ can be defined with it and then calcolate the first order derivatives of such function.

II M 4) Given the function $f(x,y,z) = x^2 + y^2 + z^2 - x^2 y - yz$, analyze the nature of its stationary points.

II Winter Exam Session 2021

I M 1) Two complex numbers are given z_1 and z_2 such that, written in trigonometric form, have moduli equal respectively to 4 and $\dfrac{1}{2}$, and arguments respectively equal to $\dfrac{17}{5}\pi$ and $\dfrac{23}{20}\pi$. Calculate their quotient $\dfrac{z_1}{z_2}$.

I M 2) Determine an orthogonal matrix that diagonalizes $\mathbb{A} = \left\|\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{matrix}\right\|$.

I M 3) Given the matrix $\mathbb{A} = \left\|\begin{matrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{matrix}\right\|$, after having determined the dimensions of the Kernel and the Image of the linear map $f : \mathbb{R}^3 \to \mathbb{R}^3$, $f(\mathbb{X}) = \mathbb{A} \cdot \mathbb{X}$, find all the vectors $\mathbb{X}$ that in this linear map coincide with their image: $f(\mathbb{X}) = \mathbb{X}$.

I M 4) Given the matrix $\mathbb{A} = \begin{Vmatrix} 1 & -2 & 0 & 1 \\ 1 & 1 & 1 & -1 \\ 5 & -4 & 2 & 1 \end{Vmatrix}$ and the linear map $f : \mathbb{R}^4 \to \mathbb{R}^3$, $f(\mathbb{X}) = \mathbb{A} \cdot \mathbb{X}$, determine if the vector $\mathbb{Y} = (1, 1, 4)$ belongs to the Image of that linear map.

II M 1) Given the equation $f(x, y) = e^{x^2 - y} - e^{y^2 - x} = 0$, verify that at the point $\mathrm{P}_0 = (1, 1)$ the hypotheses of Dini's Theorem are satisfied and then find the equation of the tangent line to the implicit function $y = y(x)$ so determined at $x_0 = 1$.

II M 2) Solve the problem $\begin{cases} \text{Max/min } f(x, y) = 2x^2 + y^2 \\ \text{u.c.} : x^2 + 2y^2 \leq 1 \end{cases}$.

II M 3) Given the function $f(x, y) = x^2 + y^2 + x\,y$ find all directions $v = (\cos\alpha, \sin\alpha)$ for which it results $\mathcal{D}_v f(1, 1) = 0$.

II M 4) Given the function $f(x, y, z) = x^2 + y^2 + z^2 - x - xy^2$, determine the nature of its stationary points.

I Additional Exam Session QMfEA 2021

I M 1) If $z = e\,(\cos\alpha + i\sin\alpha)$, since $e^{1-3i} \cdot z = e^{2+i}$, determine α.

I M 2) In the basis for $\mathbb{R}^3$ formed by the vectors $\mathbb{V}_1 = (1, 2, -1)$, $\mathbb{V}_2 = (2, 1, 1)$ and $\mathbb{V}_3 = (x_1, x_2, x_3)$, the coordinates of the vector $\mathbb{Y} = (1, 3, 0)$ are $(2, -2, 1)$. Determine $\mathbb{X}_3$.

I M 3) Given the matrix $\mathbb{A} = \begin{Vmatrix} 2 & 0 & 1 \\ 2 & 1 & 2 \\ 1 & m & 4 \end{Vmatrix}$, determine, on varying the parameter m, the existence of multiple eigenvalues and determine if, for these values, the matrix is diagonalizable.

I M 4) Given the linear system $\begin{cases} x_1 - x_2 + 2x_3 + x_4 = 2 \\ 2x_1 + x_2 - x_3 + 2x_4 = 1 \\ x_1 - 4x_2 + 7x_3 + x_4 = k \end{cases}$, find the values of the parameter k for which the system has solutions.

II M 1) Given the equation $f(x, y) = x^3 - 3xy + y^3 - 3 = 0$ and the point $P = (-1, 1)$ that satisfies such equation, find the equation of the tangent line to the graphic of the implicit function $y = y(x)$ defined at the point $x = -1$.

II M 2) Solve the problem $\begin{cases} \text{Max/min } f(x, y) = x - y \\ \text{u.c.:}\ y^2 - 1 \leq x \leq 1 \end{cases}$.

II M 3) Given $f(x, y) = xy - 2x + y$, let v and w be the unit vectors of $(1, 1)$ and $(1, -1)$; determine if there are points P where $\mathcal{D}_v f(P) = \sqrt{2}$ and $\mathcal{D}_w f(P) = -\sqrt{2}$.

II M 4) Find the pair (x, y) for which the determinant of the matrix $\mathbb{A} = \begin{Vmatrix} xy & y \\ x & x + y \end{Vmatrix}$ has a minimum value.

I Summer Exam Session 2021

I M 1) Find z if $z^3 = 1 - i$.

I M 2) Given the matrix $\mathbb{A} = \begin{Vmatrix} 1 & 3 & -1 \\ 2 & 0 & 1 \\ 1 & -3 & 2 \end{Vmatrix}$ and linear map $\mathbb{R}^3 \to \mathbb{R}^3 : \mathbb{A} \cdot \mathbb{X} = \mathbb{Y}$, calculate the dimensions of its Kernel and its Image. Then given the two linearly independent vectors $\mathbb{X}_1=(2,1,0)$ and $\mathbb{X}_2=(1,2,2)$ determine if $\mathbb{Y}_1=\mathbb{A}\cdot\mathbb{X}_1$ and $\mathbb{Y}_2=\mathbb{A}\cdot\mathbb{X}_2$ are also linearly independent.

I M 3) Given the linear system $\begin{cases} 2x+3y-z=1 \\ 2x+y+2z=2 \\ 2x+5y+kz=m \end{cases}$, determine for which values of the parameters k and m it admits none, only one or infinite solutions.

I M 4) Given the matrix $\mathbb{A}=\begin{Vmatrix} 3 & 2 & 1 \\ 1 & 4 & 4 \\ 1 & 2 & 5 \end{Vmatrix}$, knowing that it admits the eigenvalue $\lambda=2$, determine if it is diagonalizable and if it admits orthogonal eigenvectors.

II M 1) Given the equation $f(x,y)=e^{(x-1)\,y}-y^2\,e^{x-1}=0$, find all the points $(1,y)$ that satisfy this equation, verify in these points the applicability of the Dini's Theorem to obtain an implicit function $y=y(x)$, and calculate the first derivative of this function at the points found.

II M 2) Solve the problem $\begin{cases} \text{Max/min } f(x,y)=x\,(y-1) \\ \text{u.c.: } \begin{cases} y^2+x-1\le 0 \\ x\ge 0 \end{cases} \end{cases}$. It may be useful to study the sign of the function.

II M 3) Given $f(x,y)=x\,y^2-x^2$, calculate $\mathcal{D}_v f(P_0)$, where v represents the direction from $P_0=(1,2)$ to the origin $(0,0)$.

II M 4) Determine the value of x that minimizes the value of the determinant of the matrix $\mathbb{A}=\begin{Vmatrix} 2x-1 & 2 & x-5 \\ 2x & 1 & x+1 \\ x+1 & 3 & x+2 \end{Vmatrix}$.

II Summer Exam Session 2021

I M 1) The polynomial equation $x^3-i\,x^2-x+i=0$ has two real solutions and one imaginary solution. Calculate the cubic roots of the product of the three solutions.

I M 2) Given the matrix $\mathbb{A}=\begin{Vmatrix} 2 & 3 & -1 \\ 2 & 1 & 2 \\ 2 & 5 & k \end{Vmatrix}$, the linear map $\mathbb{R}^3\to\mathbb{R}^3:\mathbb{A}\cdot\mathbb{X}=\mathbb{Y}$, and the vector $\mathbb{Y}=\begin{Vmatrix} 1 \\ 2 \\ m \end{Vmatrix}$, determine for which values of the parameters k and m the vector $\mathbb{Y}$ belongs to the Image of the map.

I M 3) Given the matrix $\mathbb{A}=\begin{Vmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{Vmatrix}$, determine an orthogonal matrix that diagonalizes it.

I M 4) Find the coordinates of the vector $\mathbb{X}$ in the basis $\mathbb{A}=\{(1,1,1),(1,1,0),(1,0,1)\}$ if it has coordinates $(2,1,2)$ in the basis $\mathbb{B}=\{(1,0,0),(1,1,0),(1,1,1)\}$

II M 1) Given the system $\begin{cases} f(x,y,z)=e^{x-y}+e^{x-z}-2\,e^{y-z}=0 \\ g(x,y,z)=x^3+y^3+z^2-3xyz=0 \end{cases}$ and $\mathrm{P}=(1,1,1)$, point satisfying it, an implicit function is determined $x\to(y,z)$; of this function calculate the derivatives in the considered point .

II M 2) Solve the problem $\begin{cases} \text{Max/min } f(x,y) = y\,(x-1) \\ \text{u.c.: } \begin{cases} x \le 1 \\ x \ge y^2 \end{cases} \end{cases}$.

II M 3) Given $f(x,y) = 2x\,y - x^2$, calculate $\mathcal{D}_v f(1,1)$, where v represents the direction from $(1,1)$ to $P_0 = (1,2)$.

II M 4) Determine, on varying the parameter k, the nature of the stationary points of the function $f(x,y) = x^3 - 3\,kxy + y^2$.

I Autumn Exam Session 2021

I M 1) Calculate the square roots of the number $z = \dfrac{1}{1+i}$.

I M 2) Given the two orthogonal vectors $\mathbb{X}_1 = (1,1,-2)$ and $\mathbb{X}_2 = (1,1,1)$, determine a third vector $\mathbb{X}_3$ orthogonal to both them and, with their unit vectors, form an orthogonal matrix. Finally, determine the inverse of this orthogonal matrix.

I M 3) Given the matrix $\mathbb{A} = \begin{Vmatrix} k & 0 & k \\ 0 & -1 & 0 \\ k & 0 & k \end{Vmatrix}$, determine the values of k for which the matrix admits a multiple eigenvalue.

I M 4) Given the matrix $\mathbb{A} = \begin{Vmatrix} 1 & m & -1 & 1 \\ 2 & 1 & 1 & 2 \\ 1 & -1 & -4 & 1 \end{Vmatrix}$ and the vector $\mathbb{Y} = (2,6,m)$, determine for which value of m the vector $\mathbb{Y}$ belongs to the Image of the linear map generated by $\mathbb{A}$.

II M 1) Given $f(x,y) = x\,e^y$ and $\mathrm{P}_0 = (1,0)$, find all the directions $v = (\cos\alpha, \sin\alpha)$ for which it is : $\mathcal{D}_v f(\mathrm{P}_0) = 0$.

II M 2) Solve the problem $\begin{cases} \text{Max/min } f(x,y) = 2x - 3y \\ \text{u.c. : } x^2 + y^2 \le 4 \end{cases}$.

II M 3) Given the function $f(x,y,z) = x\,e^{y-z} + x\log(2y - x)$, calculate its gradient at the point $P_0 = (1,1,1)$.

II M 4) Given the equation $f(x,y) = x^3 - 2y^3 - 3x + 4y = 0$, satisfied at the point $(1,1)$, verify that the hypotheses of Dini's theorem for defining an implicit function $x \to y(x)$ are satisfied and then calculate $y'(1)$.

II Autumn Exam Session 2021

I M 1) The two complex numbers z_1 and z_2 have both modulus equal to 1, while the first has for argument $\dfrac{\pi}{4}$ and the second has as its argument $\dfrac{\pi}{3}$. From this informations, deduce the values of $\cos\dfrac{7\,\pi}{12}$ and of $\sin\dfrac{7\,\pi}{12}$.

I M 2) Given the linear map $\mathbb{R}^4 \to \mathbb{R}^3, \mathbb{Y} = \mathbb{A}\cdot\mathbb{X}$, with $\mathbb{A} = \begin{Vmatrix} 1 & -1 & 2 & -2 \\ 2 & 2 & 1 & m \\ 4 & 0 & 5 & k \end{Vmatrix}$, determine the values of m and k for which are equal the dimensions of the Kernel and the Image of this map, and for which the image of the vector $(1,1,1,1)$ is the vector $(0,4,4)$.

I M 3) Given the matrix $\mathbb{A} = \begin{Vmatrix} 0 & 1 & m \\ 1 & 0 & 2 \\ 1 & k & 0 \end{Vmatrix}$, determine some value of m and k for which the eigenvectors of the matrix are all orthogonal each other.

I M 4) Determine the eigenvalues of the matrix $\mathbb{A} = \begin{Vmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & k \end{Vmatrix}$, knowing that the matrix has no inverse matrix.

II M 1) Given the equation $f(x, y) = e^{x-y} + e^{y-x} - 2\, e^{x+y-2} = 0$ satisfied at the point $\mathrm{P} = (1, 1)$, an implicit function $x \to y(x)$ is determined; of this function calculate the first derivative at the point $x = 1$.

II M 2) Solve the problem $\begin{cases} \text{Max/min } f(x, y) = x\, y \\ \text{u.c. : } \begin{cases} x^2 - 2x - y \leq 0 \\ y - x \leq 0 \end{cases} \end{cases}$.

II M 3) Given the function $f(x, y) = x\, e^{y-x} - y\, e^{x-y}$, calculate $\mathcal{D}_v f(1, 1)$, with v the unit vector of the vector from the point $(1, 1)$ to the point $(2, 2)$.

II M 4) Given the function $f(x, y, z) = x^3\, e^{xy-z} + \log\left(2yz - x^2\right)$, calculate its gradient at the point $P_0 = (1, 1, 1)$.