Prova Intermedia Matematica Generale 4/11/2019 $\boxed{\text{Compito A1}}$

1) $f(x)=\dfrac{\arcsin\frac{x}{3}}{x}-\dfrac{x^2-x-2}{x^2+3x+2}=\dfrac{\arcsin\frac{x}{3}}{x}-\dfrac{(x-2)(x+1)}{(x+2)(x+1)}$. C.E.: $[-3;3]\setminus\{-2;-1;0\}$

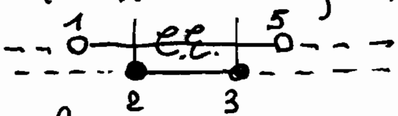
$\lim_{x\to 0} f(x)=\frac{1}{3}+1=\frac{4}{3}$; in $x=0$ Discontinuità III specie; $\lim_{x\to -1} f(x)=-\arcsin\left(-\frac{1}{3}\right)+3$; in $x=-1$ Discontinuità di III specie; $\lim_{x\to -2} f(x)=-\dfrac{\arcsin\left(-\frac{2}{3}\right)}{2}-(\to\infty)=\infty$: in $x=-2$ punto di discontinuità di II specie.

2) $\lim_{x\to 0}\dfrac{(1+x^2)^3-\cos x}{\log(1+x^2)}=\lim_{x\to 0}\left[\dfrac{(1+x^2)^3-1}{x^2}+\dfrac{1-\cos x}{x^2}\right]\cdot\dfrac{x^2}{\log(1+x^2)}=\left(3+\frac{1}{2}\right)\cdot 1=\frac{7}{2}$.

$\lim_{x\to+\infty}\left(\dfrac{x+\sin x+2^{-x}}{3x-5+\arctan x}\right)^{1-x}=\lim_{x\to+\infty}\left(\dfrac{x}{3x}\right)^{1-x}=\left(\to\frac{1}{3}\right)^{\to-\infty}=+\infty$ $\left(\begin{matrix}2^{-x}\to 0;\ \sin x=o(x)\\ \arctan x-5=o(3x)\end{matrix}\right)$.

3) $f(x)=\log\left(\dfrac{x-1}{5-x}\right)-\sqrt{(x-2)(3-x)}$. C.E.: $\left\{x:\dfrac{x-1}{5-x}>0\right\}\cap\{x:(x-2)(3-x)\geq 0\}\Rightarrow$

$\Rightarrow \dfrac{x-1}{5-x}<0$ per $1<x<5$; $(x-2)(3-x)\geq 0$ per $2\leq x\leq 3$.



Quindi C.E. $=[2;3]$ intervallo limitato e chiuso.

4) $f(x)=1-\dfrac{2}{x}=y\Rightarrow\dfrac{2}{x}=1-y\Rightarrow x=\dfrac{2}{1-y}\Rightarrow f^{-1}(x)=\dfrac{2}{1-x}$;

$g(x)=2^{1-3x}=y\Rightarrow 1-3x=\log_2 y\Rightarrow x=\frac{1}{3}(1-\log_2 y)\Rightarrow g^{-1}(x)=\frac{1}{3}(1-\log_2 x)$.

$f^{-1}(g(x))=f^{-1}(2^{1-3x})=\dfrac{2}{1-2^{1-3x}}$; $f(g^{-1}(x))=f\left(\frac{1}{3}(1-\log_2 x)\right)=1-\dfrac{2}{\frac{1}{3}(1-\log_2 x)}=\dfrac{\log_2 x+5}{\log_2 x-1}$.

5)

A B C	(A e C)	(A ⇒ B)	[(A e C) e (A ⇒ B)]	[(A e C) e (A ⇒ B)] ⇒ B
1 1 1	1	1	1	1
1 1 0	0	1	0	1
1 0 1	1	0	0	1
1 0 0	0	0	0	1
0 1 1	0	1	0	1
0 1 0	0	1	0	1
0 0 1	0	1	0	1
0 0 0	0	1	0	1

La proposizione $[(A \text{ e } C) \text{ e } (A\Rightarrow B)]\Rightarrow B$ risulta una tautologia.

Prova Intermedia di Matematica Generale 4/11/2019 $\boxed{\text{Compito B1}}$

1) $f(x)=\dfrac{\operatorname{arctg}2x}{x}-\dfrac{x^2+3x+2}{x^2+x-2}=\dfrac{\operatorname{arctg}2x}{2}-\dfrac{(x+1)(x+2)}{(x-1)(x+2)}$. C.E.: $\mathbb{R}\setminus\{-2;0;1\}$.

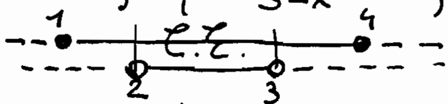
$\lim_{x\to 0} f(x)=2+1=3$; in $x=0$ Discontinuità $\underline{\text{III}}$ specie; $\lim_{x\to -2} f(x)=\dfrac{\operatorname{arctg}(-4)}{-2}-\dfrac{1}{3}$; in $x=-2$ Discontinuità di $\underline{\text{III}}$ specie; $\lim_{x\to 1} f(x)=\operatorname{arctg}2-(\pm\infty)=\infty$; in $x=1$ Discontinuità di $\underline{\text{II}}$ specie.

2) $\lim_{x\to 0}\dfrac{(1-2x)^3-2^x}{\operatorname{sen}x}=\lim_{x\to 0}\dfrac{(1-2x)^3-1}{(-2x)}\cdot\dfrac{-2x}{\operatorname{sen}x}-\dfrac{2^x-1}{x}\cdot\dfrac{x}{\operatorname{sen}x}=3\cdot(-2)-\log 2=-6-\log 2.$

$\lim_{x\to+\infty}\left(\dfrac{3x+\operatorname{arctg}x+3^{-x}}{2x-2+\cos x}\right)^{1-x}=\lim_{x\to+\infty}\left(\dfrac{3x}{2x}\right)^{1-x}=\left(\to\dfrac{3}{2}\right)^{(\to-\infty)}=0^+\ \left(\begin{array}{l}3^{-x}\to 0;\ \operatorname{arctg}x=o(3x)\\ \cos x-2=o(2x)\end{array}\right).$

3) $f(x)=\sqrt{(x-4)(1-x)}-\log\left(\dfrac{x-2}{3-x}\right)$. C.E.: $\{x:(x-4)(1-x)\geq 0\}\cap\left\{x:\dfrac{x-2}{3-x}>0\right\}\Rightarrow$

$\Rightarrow (x-4)(1-x)\geq 0$ per $1\leq x\leq 4$; $\dfrac{x-2}{3-x}>0$ per $2<x<3$.



Quindi C.E. $=\,]2;3[$ intervallo limitato e aperto.

4) $f(x)=\dfrac{2}{x}+1=y\Rightarrow\dfrac{2}{x}=y-1\Rightarrow x=\dfrac{2}{y-1}\Rightarrow f^{-1}(x)=\dfrac{2}{x-1}$;

$g(x)=\log_2(3x+1)=y\Rightarrow 3x+1=2^y\Rightarrow x=\dfrac{1}{3}(2^y-1)\Rightarrow g^{-1}(x)=\dfrac{1}{3}(2^x-1)$.

$f^{-1}(g(x))=f^{-1}(\log_2(3x+1))=\dfrac{2}{\log_2(3x+1)-1}$; $f(g^{-1}(x))=f\left(\dfrac{1}{3}(2^x-1)\right)=\dfrac{2}{\frac{1}{3}(2^x-1)}+1=\dfrac{2^x+5}{2^x-1}$.

5)

A	B	C	$(A\delta B)$	$(B\Rightarrow C)$	$[(A\delta B)\text{ e }(B\Rightarrow C)]$	$[(A\delta B)\text{ e }(B\Rightarrow C)]\Rightarrow C$	
1	1	1	1	1	1	1	
1	1	0	1	0	0	1	
1	0	1	1	1	1	1	
1	0	0	1	1	1	0	*
0	1	1	1	1	1	1	
0	1	0	1	0	0	1	
0	0	1	0	1	0	1	
0	0	0	0	1	0	1	

La proposizione $[(A\delta B)\text{ e }(B\Rightarrow C)]\Rightarrow C$ NON è una tautologia.

Prova Intermedia Matematica Generale 4/11/2019 $\boxed{\text{Compito C1}}$

1) $f(x)=\frac{\operatorname{sen}5x}{x}-\frac{x^2-x-2}{x^2-3x+2}=\frac{\operatorname{sen}5x}{x}-\frac{(x+1)(x-2)}{(x-1)(x-2)}$. C.E.: $\mathbb{R}\setminus\{0;1;2\}$.

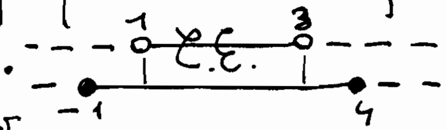
$\lim_{x\to0} f(x)=5+1=6$; In $x=0$ Discontinuità di III specie; $\lim_{x\to2} f(x)=\frac{\operatorname{sen}10}{2}-3$; in $x=2$ Discontinuità di III specie; $\lim_{x\to1} f(x)=\operatorname{sen}5-(\to\infty)=\infty$; In $x=1$ Discontinuità di II specie.

2) $\lim_{x\to0}\frac{2^{x^2}-\cos2x}{\arcsin x^2}=\lim_{x\to0}\frac{2^{x^2}-1}{x^2}\cdot\frac{x^2}{\arcsin x^2}+\frac{1-\cos2x}{4x^2}\cdot\frac{4x^2}{\arcsin x^2}=\log2\cdot1+\frac{1}{2}\cdot4=\log2+2$.

$\lim_{x\to+\infty}\left(\frac{3x^2+3^{-x}-5}{2x+\cos x+x^2}\right)^{x-1}=\lim_{x\to+\infty}\left(\frac{3x^2}{x^2}\right)^{x-1}=(\to3)^{\to+\infty}=+\infty \quad \left(\begin{matrix}3^{-x}\to0;\ 5=o(3x^2)\\ 2x+\cos x=o(x^2)\end{matrix}\right)$.

3) $f(x)=\log\left(\frac{x-1}{3-x}\right)-\sqrt{(x+1)(4-x)}$. C.E.: $\left\{x:\frac{x-1}{3-x}>0\right\}\cap\left\{x:(x+1)(4-x)\geq0\right\}\Rightarrow$

$\Rightarrow \frac{x-1}{3-x}>0$ per $1<x<3$; $(x+1)(4-x)\geq0$ per $-1\leq x\leq4$.



Quindi C.E. $=]+1;3[$ intervallo limitato e aperto.

4) $f(x)=3^{1+2x}=y\Rightarrow 1+2x=\log_3 y\Rightarrow x=\frac{1}{2}(\log_3 y-1)\Rightarrow f^{-1}(x)=\frac{1}{2}(\log_3 x-1)$;

$g(x)=\frac{1}{x}+3=y\Rightarrow\frac{1}{x}=y-3\Rightarrow x=\frac{1}{y-3}\Rightarrow g^{-1}(x)=\frac{1}{x-3}$.

$f^{-1}(g(x))=f^{-1}\left(\frac{1}{x}+3\right)=\frac{1}{2}\left(\log_3\left(\frac{1}{x}+3\right)-1\right)$; $f(g^{-1}(x))=f\left(\frac{1}{x-3}\right)=3^{1+\frac{2}{x-3}}=3^{\frac{x-1}{x-3}}$.

5)

A B C	(A e B)	(A ⇒ C)	[(A e B) e (A ⇒ C)]	[(A e B) e (A ⇒ C)] ⇒ C
1 1 1	1	1	1	1
1 1 0	1	0	0	1
1 0 1	0	1	0	1
1 0 0	0	0	0	1
0 1 1	0	1	0	1
0 1 0	0	1	0	1
0 0 1	0	1	0	1
0 0 0	0	1	0	1

La proposizione [(A e B) e (A ⇒ C)] ⇒ C risulta una tautologia.

Prova Intermedia Matematica Generale 4/11/2019 $\boxed{\text{Compito D1}}$

1) $f(x)=\frac{\log(1+2x)}{x}-\frac{x^2+3x+2}{x^2-x-2}=\frac{\log(1+2x)}{x}-\frac{(x+1)(x+2)}{(x+1)(x-2)}$. C.E.: $\left\{x>-\frac{1}{2} \text{ e } x\neq 0;\ x\neq 2\right\}$. $(-1\notin C.E.)$.

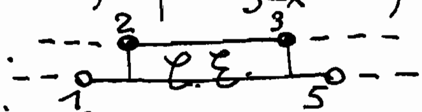
$\lim_{x\to 0} f(x)=2+1=3$; In $x=0$ Discontinuità di III Specie; $\lim_{x\to -\frac{1}{2}^+} f(x)=+\infty$; in $x=-\frac{1}{2}$ Discontinuità di II Specie (solo da destra); $\lim_{x\to 2} f(x)=\frac{\log 5}{2}+(\mp\infty)=\infty$; In $x=2$ punto di Discontinuità di II Specie.

2) $\lim_{x\to 0}\frac{(1+x)^6-(1-x)^4}{\operatorname{tg}x}=\lim_{x\to 0}\frac{(1+x)^6-1}{x}\cdot\frac{x}{\operatorname{tg}x}-\frac{(1-x)^4-1}{(-x)}\cdot\frac{(-x)}{\operatorname{tg}x}=6\cdot 1-4\cdot(-1)=10.$

$$\lim_{x\to+\infty}\left(\frac{2x^2+\cos x-1}{2x+3^{-x}+4x^2}\right)^{x-1}=\lim_{x\to+\infty}\left(\frac{2x^2}{4x^2}\right)^{x-1}=\left(\to\frac{1}{2}\right)^{\to+\infty}=0^+\left(\begin{array}{l}\cos x-1=o(2x^2)\\ 3^{-x}\to 0;\ 2x=o(4x^2)\end{array}\right).$$

3) $f(x)=\sqrt{(x-2)(3-x)}-\log\left(\frac{x-1}{5-x}\right)$. C.E.: $\{x:(x-2)(3-x)\geq 0\}\cap\left\{x:\frac{x-1}{5-x}>0\right\}\Rightarrow$

$\Rightarrow (x-2)(3-x)\geq 0$ per $2\leq x\leq 3$; $\frac{x-1}{5-x}>0$ per $1<x<5$.



Quindi C.E. $=[2;3]$ intervallo limitato e chiuso.

4) $f(x)=\log_3(2x-1)=y\Rightarrow 2x-1=3^y\Rightarrow x=\frac{1}{2}(3^y+1)\Rightarrow f^{-1}(x)=\frac{1}{2}(3^x+1)$;

$g(x)=2-\frac{1}{x}=y\Rightarrow\frac{1}{x}=2-y\Rightarrow x=\frac{1}{2-y}\Rightarrow g^{-1}(x)=\frac{1}{2-x}$.

$f^{-1}(g(x))=f^{-1}\left(2-\frac{1}{x}\right)=\frac{1}{2}\left(3^{2-\frac{1}{x}}+1\right)$; $f(g^{-1}(x))=f\left(\frac{1}{2-x}\right)=\log_3\left(\frac{2}{2-x}-1\right)=\log_3\left(\frac{x}{2-x}\right)$.

5)

A	B	C	(A o C)	(C ⇒ B)	[(A o C) e (C ⇒ B)]	[(A o C) e (C ⇒ B)] ⇒ B	
1	1	1	1	1	1	1	
1	1	0	1	1	1	1	
1	0	1	1	0	0	1	
1	0	0	1	1	1	0	*
0	1	1	1	1	1	1	
0	1	0	0	1	0	1	
0	0	1	1	0	0	1	
0	0	0	0	1	0	1	

La proposizione [(A o C) e (C ⇒ B)] ⇒ B NON è una tautologia.