Prova Intermedia MG del 26/11/2021 $\boxed{\text{COMPITO A}}$

1) Se $B \Leftrightarrow C$ è sempre falsa allora B e C hanno sempre casi diversi.

A	B	C	(A o B)	non C	[(A o B) e non C]	(A e C)	[(A e C) $\Leftrightarrow$ B]	$P_1 \Rightarrow P_2$
1	1	0	1	1	1	0	0	0
1	0	1	1	0	0	1	0	1
0	1	0	1	1	1	0	0	0
0	0	1	0	0	0	0	1	1
					P_1		P_2	

2) $$\lim_{x\to 0}\frac{(1+5x)^3-1}{(1-3x)^5-1}=\lim_{x\to 0}\frac{(1+5x)^3-1}{5x}\cdot\frac{5x}{-3x}\cdot\frac{1}{\frac{(1-3x)^5-1}{-3x}}=3\cdot\left(-\frac{5}{3}\right)\cdot\frac{1}{5}=-1.$$

$$\lim_{x\to+\infty}\left(\frac{5+\sqrt{x}}{3+\sqrt{x}}\right)^{1+\sqrt{x}}=\lim_{x\to+\infty}\left(\frac{(3+\sqrt{x})+2}{3+\sqrt{x}}\right)^{1+\sqrt{x}}=\lim_{x\to+\infty}\left[\left(1+\frac{2}{3+\sqrt{x}}\right)^{3+\sqrt{x}}\right]^{\frac{1+\sqrt{x}}{3+\sqrt{x}}}=(e^2)^1=e^2.$$

3) $$\lim_{x\to 0}\frac{1-\cos Kx}{x^3+x^2}=\lim_{x\to 0}\frac{1-\cos(Kx)}{K^2x^2}\cdot\frac{K^2x^2}{x^2(x+1)}=\frac{1}{2}K^2=5\Rightarrow K^2=10\Rightarrow$$
$$\Rightarrow K=\pm\sqrt{10}.$$

4) $$f(x)=\frac{x+1}{x+2}\Rightarrow f(g(x))=\frac{g(x)+1}{g(x)+2}=\log(x+1)\Rightarrow g(x)+1=\log(x+1)\cdot(g(x)+2)\Rightarrow$$
$$\Rightarrow g(x)\cdot(\log(x+1)-1)=1-2\log(x+1)\Rightarrow g(x)=\frac{1-2\log(x+1)}{\log(x+1)-1}=y\Rightarrow$$
$$\Rightarrow y(\log(x+1)-1)=1-2\log(x+1)\Rightarrow\log(x+1)\cdot(y+2)=y+1\Rightarrow\log(x+1)=\frac{y+1}{y+2}\Rightarrow$$
$$\Rightarrow x+1=e^{\frac{y+1}{y+2}}\Rightarrow x=e^{\frac{y+1}{y+2}}-1.$$ Inversa di $g(x)$: $y=e^{\frac{x+1}{x+2}}-1$.

5) $f(x)=\log\left(\frac{1-\log x}{e^x+1}\right)$. C.E.: $\begin{cases} x>0 \\ \frac{1-\log x}{e^x+1}>0\end{cases}$ Ma $e^x+1>0\ \forall x\in\mathbb{R}$

e quindi $\begin{cases}x>0\\ 1-\log x>0\end{cases}\Rightarrow\begin{cases}x>0\\ \log x<1\end{cases}\Rightarrow\begin{cases}x>0\\ x<e\end{cases}\Rightarrow$ C.E.: $]0;e[$.

Prova Intermedia MG del 26/11/2021 | COMPITO B

1) Se $B \Leftrightarrow A$ è sempre vera allora A e B hanno sempre casi uguali:

A	B	C	(A o B)	[(A o B) => C]	(non C)	(A e non C)	[(A e non C) o B]	P₁ <=> P₂
1	1	1	1	1	0	0	1	1
1	1	0	1	0	1	1	1	0
0	0	1	0	1	0	0	0	0
0	0	0	0	1	1	0	0	0
				P_1			P_2	

2) $$\lim_{x\to 0}\frac{2-\cos 2x-\cos 3x}{3^{x^2}-1}=\lim_{x\to 0}\left(\frac{1-\cos 2x}{4x^2}\cdot 4+\frac{1-\cos 3x}{9x^2}\cdot 9\right)\frac{x^2}{3^{x^2}-1}=\left(\frac{1}{2}\cdot 4+\frac{1}{2}\cdot 9\right)\frac{1}{\log 3}=$$
$$=\left(2+\frac{9}{2}\right)\cdot \log_3 e=\frac{13}{2}\cdot \log_3 e.$$

$$\lim_{x\to+\infty}\left(\frac{1+x+x^2}{3+x+x^2}\right)^{x-1}=\lim_{x\to+\infty}\left(\frac{3+x+x^2-2}{3+x+x^2}\right)^{x-1}=\lim_{x\to+\infty}\left[\left(1-\frac{2}{3+x+x^2}\right)^{3+x+x^2}\right]^{\frac{x-1}{3+x+x^2}}=\left(e^{-2}\right)^0=1.$$

3) $$\lim_{x\to 0}\frac{e^{x^2}-\cos kx}{x^2}=\lim_{x\to 0}\frac{e^{x^2}-1}{x^2}+\frac{1-\cos kx}{k^2x^2}\cdot k^2=1+\frac{1}{2}k^2=5\ \Rightarrow$$
$$\Rightarrow \frac{1}{2}k^2=4\Rightarrow k^2=8\Rightarrow k=\pm\sqrt{8}=\pm 2\sqrt{2}.$$

4) $$f(x)=\frac{2x}{x+1}\Rightarrow f(g(x))=\frac{2g(x)}{g(x)+1}=e^{x-1}\Rightarrow 2g(x)=(g(x)+1)\cdot e^{x-1}\Rightarrow$$
$$\Rightarrow g(x)\cdot(2-e^{x-1})=e^{x-1}\Rightarrow g(x)=\frac{e^{x-1}}{2-e^{x-1}}=y\Rightarrow e^{x-1}=y(2-e^{x-1})\Rightarrow$$
$$\Rightarrow e^{x-1}(1+y)=2y\Rightarrow e^{x-1}=\frac{2y}{y+1}\Rightarrow x-1=\log\frac{2y}{y+1}\Rightarrow x=1+\log\frac{2y}{y+1}\Rightarrow$$
$\Rightarrow$ Inversa di $g(x)$: $y=1+\log\dfrac{2x}{x+1}$.

5) $f(x)=\sqrt{\dfrac{x\log x}{x^4+1}}$. C.E. $\begin{cases} x>0 \\ \dfrac{x\log x}{x^4+1}\geq 0\end{cases}$. Ma $x>0$ e $x^4+1>0\ \forall x\in\mathbb{R}$

e quindi $\begin{cases} x>0 \\ \log x\geq 0\end{cases}\Rightarrow\begin{cases} x>0 \\ x\geq 1\end{cases}\Rightarrow$ C.E.: $[1; +\infty[$.