Compito di Matematica Generale del 8/2/2022 $\boxed{\text{CMG1}}$

1) $f(x)=x^2 e^x - e^x = (x^2-1)\cdot e^x$. C.E.: $\mathbb{R}$. $\lim_{x\to -\infty} f(x)=0^+$; $\lim_{x\to+\infty} f(x)=+\infty$.

$f(x)\geq 0$ per $x^2\geq 1 \Rightarrow x\leq -1 \cup x\geq 1$.

$f'(x)=2x e^x+(x^2-1)e^x = e^x(x^2+2x-1)\geq 0$

$x=-1\pm\sqrt{1+1} \Rightarrow f'(x)\geq 0$ per

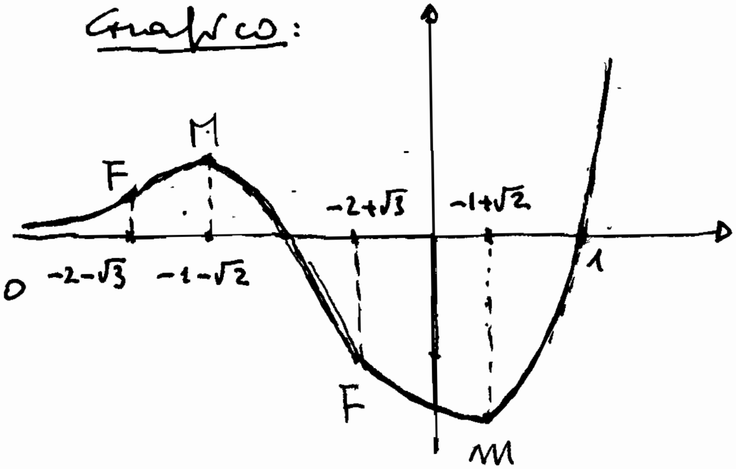
$x\leq -1-\sqrt{2} \cup x\geq -1+\sqrt{2}$ $\quad -1-\sqrt{2} \quad -1+\sqrt{2}$

$f''(x)=(2x+2)e^x+(x^2+2x-1)e^x=e^x(x^2+4x+1)\geq 0$

$x=-2\pm\sqrt{4-1} \Rightarrow f''(x)\geq 0$ per

$x\leq -2-\sqrt{3} \cup x\geq -2+\sqrt{3}$ $\quad -2-\sqrt{3} \quad -2+\sqrt{3}$

Grafico:



2) $\lim_{x\to 0} \frac{e^{\operatorname{sen}x}-e^{-\operatorname{sen}x}}{x} = \lim_{x\to 0}\left(\frac{e^{\operatorname{sen}x}-1}{\operatorname{sen}x}\cdot\frac{\operatorname{sen}x}{x} - \frac{e^{-\operatorname{sen}x}-1}{-\operatorname{sen}x}\cdot\left(-\frac{\operatorname{sen}x}{x}\right)\right)=1\cdot 1-1\cdot(-1)=2.$

$\lim_{x\to+\infty}\left(\frac{4+x^2}{3+x^2}\right)^{3-x^2}=\lim_{x\to+\infty}\left(\frac{3+x^2+1}{3+x^2}\right)^{3-x^2}=\lim_{x\to+\infty}\left[\left(1+\frac{1}{3+x^2}\right)^{3+x^2}\right]^{\frac{3-x^2}{3+x^2}}=e^{-1}=\frac{1}{e}.$

3) $f(x)=o(g(x)) \Rightarrow \lim_{x\to x_0}\frac{2x}{e^x-1}=0$ vera solo se $x\to+\infty$.

4) $f(x)=e^x$; $g(x)=1+x^3$; $h(x)=\log x$. $f(g(\log x))=f(1+\lg^3 x)=e^{1+\lg^3 x}=y \Rightarrow$

$\Rightarrow 1+\lg^3 x=\log y \Rightarrow \lg^3 x=\log y-1 \Rightarrow \log x=\sqrt[3]{\lg y-1} \Rightarrow x=e^{\sqrt[3]{\lg y-1}} \Rightarrow$

$\Rightarrow$ Inversa: $F^{-1}(x)=e^{\sqrt[3]{\lg x-1}}$.

5) $\int_0^1 \frac{x+1}{x^2+1}dx=\int_0^1 \frac{x}{x^2+1}+\frac{1}{x^2+1}dx=\left(\frac{1}{2}\log(x^2+1)+\operatorname{arctg} x\right|_0^1=\frac{1}{2}\log 2+\frac{\pi}{4}-\cancel{(0+0)}.$

6) $f(x)=e^x-x$. Teo. Lagrange: $\exists\, x_0$: $f'(x_0)=\frac{f(b)-f(a)}{b-a} \Rightarrow$

$\Rightarrow e^x-1=\frac{(e-1)-(1-0)}{1-0} \Rightarrow e^x-1=e-2 \Rightarrow e^x=e-1 \Rightarrow x_0=\log(e-1).$

$\boxed{CMG\ 2}$

7) $f(x;y) = 3x + 2y - \log(xy) \Rightarrow \nabla f(x;y) = (0;0) \Rightarrow$

$$\begin{cases} f'_x = 3 - \frac{1}{xy}\cdot y = 3 - \frac{1}{x} = 0 \\ f'_y = 2 - \frac{1}{xy}\cdot x = 2 - \frac{1}{y} = 0 \end{cases} \Rightarrow \begin{cases} x = \frac{1}{3} \\ y = \frac{1}{2} \end{cases} . \quad H(x;y) = \begin{Vmatrix} \frac{1}{x^2} & 0 \\ 0 & \frac{1}{y^2} \end{Vmatrix}$$

$$H\left(\tfrac{1}{3};\tfrac{1}{2}\right) = \begin{Vmatrix} 9 & 0 \\ 0 & 4 \end{Vmatrix} \Rightarrow \begin{cases} |H_1| = 9 > 0;\ |H_1| = 4 > 0 \\ |H_2| = 36 > 0 \end{cases} \Rightarrow \left(\tfrac{1}{3};\tfrac{1}{2}\right) \text{ Punto di Minimo.}$$

8) $$A\cdot B\cdot X = \begin{Vmatrix} 1 & 1 \\ 0 & 1 \end{Vmatrix}\cdot\begin{Vmatrix} 2 & 1 \\ 1 & 1 \end{Vmatrix}\cdot\begin{Vmatrix} 1 \\ k \end{Vmatrix} = \begin{Vmatrix} 1 & 1 \\ 0 & 1 \end{Vmatrix}\cdot\begin{Vmatrix} 2+K \\ 1+K \end{Vmatrix} = \begin{Vmatrix} 2+K+1+K \\ 0+1+K \end{Vmatrix} = \begin{Vmatrix} 3+2K \\ 1+K \end{Vmatrix};$$

$$B\cdot A\cdot X = \begin{Vmatrix} 2 & 1 \\ 1 & 1 \end{Vmatrix}\cdot\begin{Vmatrix} 1 & 1 \\ 0 & 1 \end{Vmatrix}\cdot\begin{Vmatrix} 1 \\ K \end{Vmatrix} = \begin{Vmatrix} 2 & 1 \\ 1 & 1 \end{Vmatrix}\cdot\begin{Vmatrix} 1+K \\ 0+K \end{Vmatrix} = \begin{Vmatrix} 2+2K+K \\ 1+K+K \end{Vmatrix} = \begin{Vmatrix} 2+3K \\ 1+2K \end{Vmatrix}.$$

$$A\cdot B\cdot X \perp B\cdot A\cdot X = (3+2k;1+k)\cdot(2+3K;1+2k) = 0 \Rightarrow$$

$$\Rightarrow (3+2k)(2+3k) + (1+k)(1+2k) = 8K^2 + 16K + 7 = 0 \Rightarrow$$

$$\Rightarrow K = \frac{-8 \pm \sqrt{64-56}}{8} = \frac{-8 \pm \sqrt{8}}{8} = \frac{-8 \pm 2\sqrt{2}}{8} = -1 \pm \frac{\sqrt{2}}{4}.$$

9)

A	B	$(A\Rightarrow B)$	$(B\Rightarrow A)$	$[(A\Rightarrow B)\Rightarrow(B\Rightarrow A)]$	$(A\Leftrightarrow B)$	$\{[(A\Rightarrow B)\Rightarrow(B\Rightarrow A)]\Leftrightarrow(A\Leftrightarrow B)]\}$
1	1	1	1	1	1	1
1	0	0	1	1	0	0
0	1	1	0	0	0	1
0	0	1	1	1	1	1

La proposizione $[(A\Rightarrow B)\Rightarrow(B\Rightarrow A)] \Leftrightarrow (A\Leftrightarrow B)$ non è una tautologia.

10) $f(x) = \log^2 x - \log^3 x$. C.E.: $x > 0$. $\lim_{x\to 0^+} f(x) = +\infty$; $\lim_{x\to +\infty} f(x) = -\infty$.

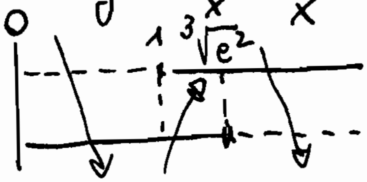
$$f'(x) = 2\log x \cdot \frac{1}{x} - 3\log^2 x \cdot \frac{1}{x} = \frac{1}{x}\cdot\log x\cdot(2 - 3\log x) \geq 0 .$$

$\log x \geq 0: x \geq 1$

$2 - 3\log x \geq 0: x \leq \sqrt[3]{e^2}$

$\log x \leq \frac{2}{3}$

$x \leq e^{\frac{2}{3}}$



In $x = 1$ punto di minimo relativo

in $x = \sqrt[3]{e^2}$ punto di massimo relativo

Funzione crescente in $]1;\sqrt[3]{e^2}[$.