Compito di Matematica Generale del 30/5/2022 $\boxed{\text{CMG1}}$

1) $f(x)=2e^{x}-e^{-x}=2e^{x}-\frac{1}{e^{x}}=\frac{2e^{2x}-1}{e^{x}}$. C.E. $=\mathbb{R}$. $\lim_{x\to-\infty} f(x)=-\infty$; $\lim_{x\to+\infty} f(x)=+\infty$.

$f(x)\geq 0$ per $2e^{2x}\geq 1 \Rightarrow e^{2x}\geq \frac{1}{2} \Rightarrow 2x\geq \log\frac{1}{2}=-\log 2 \Rightarrow x\geq -\frac{1}{2}\log 2=-\log\sqrt{2}$.

$f'(x)=2e^{x}+e^{-x}\geq 0\ \forall x\in\mathbb{R}$

$f''(x)=2e^{x}-e^{-x}=f(x)\geq 0$ per $x\geq -\log\sqrt{2}$

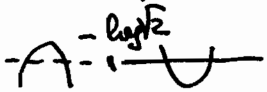
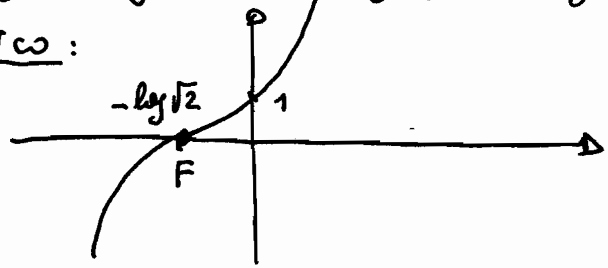


Grafico: $f(0)=1$



2) $\lim_{x\to 0}\frac{3^{x}-2^{x}}{3^{x}-e^{x}}=\lim_{x\to 0}\left(\frac{3^{x}-1}{x}-\frac{2^{x}-1}{x}\right)\cdot\left(\frac{1}{\frac{3^{x}-1}{x}-\frac{e^{x}-1}{x}}\right)=(\log 3-\log 2)\cdot\frac{1}{\log 3-1}=\frac{\log\frac{3}{2}}{\log\frac{3}{e}}$.

$\lim_{x\to+\infty}\left(\frac{1+3^{x}}{1+2^{x}}\right)^{\frac{1-x}{x}}=(\to+\infty)^{(\to-1)}=0^{+}$.

3) $\lim_{x\to 0}\frac{2-\cos x-\cos kx}{x^{2}}=\lim_{x\to 0}\frac{1-\cos x}{x^{2}}+\frac{1-\cos kx}{k^{2}x^{2}}\cdot k^{2}=\frac{1}{2}+\frac{k^{2}}{2}=1$ se

$\frac{k^{2}}{2}=1-\frac{1}{2}=\frac{1}{2}\Rightarrow k^{2}=1\Rightarrow k=\pm 1$.

4) $f(x)=\frac{x-1}{x+1}\Rightarrow f(g(x))=\frac{g(x)-1}{g(x)+1}=\log x\Rightarrow g(x)-1=g(x)\cdot\log x+\log x\Rightarrow$

$\Rightarrow g(x)(1-\log x)=\log x+1\Rightarrow g(x)=\frac{\log x+1}{1-\log x}=y\Rightarrow \log x+1=y-y\cdot\log x\Rightarrow$

$\Rightarrow \log x(1+y)=y-1\Rightarrow \log x=\frac{y-1}{y+1}\Rightarrow x=e^{\frac{y-1}{y+1}}\Rightarrow g^{-1}(x)=e^{\frac{x-1}{x+1}}$.

5) $\int_0^1\frac{x-1}{x^{2}+1}\,dx=\int_0^1\frac{1}{2}\frac{2x}{x^{2}+1}\,dx-\int_0^1\frac{1}{1+x^{2}}\,dx=\left(\frac{1}{2}\log(x^{2}+1)\right|_0^1-\left(\operatorname{arctg}x\right|_0^1=\frac{1}{2}\log 2-\frac{\pi}{4}$.

6) Da $f(x)\simeq f(x_0)+f'(x_0)(x-x_0)+\cancel{o(x-x_0)}$ otteniamo:

da $f(x)=\sqrt[3]{x}\Rightarrow f'(x)=\frac{1}{3}\cdot\frac{1}{\sqrt[3]{x^{2}}}$ e quindi:

$\sqrt[3]{8{,}12}\simeq\sqrt[3]{8}+\frac{1}{3}\frac{1}{\sqrt[3]{8^{2}}}\cdot(8{,}12-8)\Rightarrow\sqrt[3]{8{,}12}=2+\frac{1}{3}\cdot\frac{1}{4}\cdot\frac{12}{100}=2+\frac{1}{100}=2{,}01$.

$\boxed{CMG2}$

7) $f(x;y) = x^2+y^2-x^2y^2 \Rightarrow \nabla f(x;y) = (0;0) \Rightarrow$

$$\begin{cases} f'_x = 2x-2xy^2 = 2x(1-y^2) = 0 \\ f'_y = 2y-2x^2y = 2y(1-x^2) = 0 \end{cases} \Rightarrow \begin{cases} x=0 \\ y=0 \end{cases} \cup \cancel{\begin{cases} x=0 \\ x^2=1 \end{cases}} \cup \cancel{\begin{cases} y^2=1 \\ y=0 \end{cases}} \cup \begin{cases} y^2=1 \\ x^2=1 \end{cases} \Rightarrow$$

$(0;0)$; $(1;1)$; $(1;-1)$; $(-1;1)$; $(-1;-1)$ $\quad H(x;y) = \begin{Vmatrix} 2-2y^2 & -4xy \\ -4xy & 2-2x^2 \end{Vmatrix}$

$H(0;0) = \begin{Vmatrix} 2 & 0 \\ 0 & 2 \end{Vmatrix} \Rightarrow \begin{cases} |H_1| = 2 > 0 \\ |H_2| = 4 > 0 \end{cases}$ Punto di Minimo.

$H(1;1) = H(-1;-1) = \begin{Vmatrix} 0 & -4 \\ -4 & 0 \end{Vmatrix}$:

$|H_2| < 0$: Punti di Sella

$H(1;-1) = H(-1;1) = \begin{Vmatrix} 0 & 4 \\ 4 & 0 \end{Vmatrix}$.

$|H_2| < 0$ Punti di Sella.

8) $A \circ X = \begin{Vmatrix} 1 & 1 & K \\ 1 & K & -1 \\ K-1 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} 1 \\ 1 \\ 1 \end{Vmatrix} = \begin{Vmatrix} 1+1+K \\ 1+K-1 \\ K-1+1 \end{Vmatrix} = \begin{Vmatrix} 2+K \\ K \\ K \end{Vmatrix}$

$A \circ X \perp (1;-1;2) \Rightarrow (2+K;K;K)\cdot(1;-1;2) = 2+\cancel{K}-\cancel{K}+2K = 0$ per $K=-1$.

9)

A B C	$(B \Leftrightarrow C)$	non $(B \Leftrightarrow C)$	[non $(B \Leftrightarrow C) \Rightarrow A$]	$A \Leftrightarrow C$	
1 1 1	1	0	1	1	
1 1 0	0	1	1	0	
1 0 1	0	1	1	1	
1 0 0	1	0	1	0	
0 1 1	1	0	1	0	
0 1 0	0	1	0	1	*
0 0 1	0	1	0	0	
0 0 0	1	0	1	1	

non $(B \Leftrightarrow C) \Rightarrow A$ è falsa mentre $A \Leftrightarrow C$ è vera in $\underline{\text{VI}}$ riga con A falsa B vera e C falsa.

10) $f(x) = \log\left(\dfrac{e^x-1}{\log(x^2+1)}\right)$. Dato che $\log(x^2+1) \geq 0$ $\forall x$ e $=0$ per $x=0$

avremo C.E. : $\begin{cases} e^x-1 > 0 \\ \log(x^2+1) \neq 0 \end{cases} \Rightarrow \begin{cases} e^x > 1 \\ x \neq 0 \end{cases} \Rightarrow \begin{cases} x > 0 \\ x \neq 0 \end{cases} \Rightarrow$ C.E.: $x > 0$.