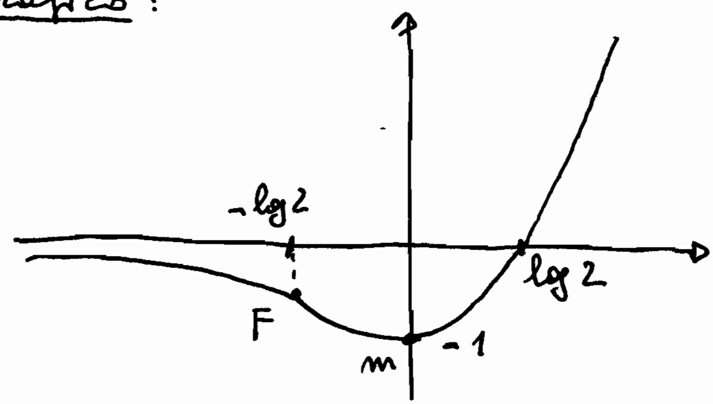
Compito di Matematica Generale del 27/6/2022 $\boxed{\text{CMG1}}$

$f(x)=e^{2x}-2e^{x}$. C.E.: $\mathbb{R}$. $\lim\limits_{x\to-\infty} f(x)=0^-$; $\lim\limits_{x\to+\infty} f(x)=+\infty$. $f(0)=-1$.

$f(x)=e^{2x}-2e^{x}=e^{x}(e^{x}-2)\geq 0$

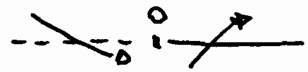
per $e^{x}\geq 2 \Rightarrow x\geq \log 2$.

Grafico:



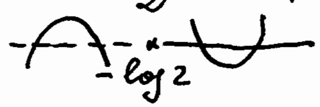
$f'(x)=2e^{2x}-2e^{x}=2e^{x}(e^{x}-1)\geq 0$

per $e^{x}\geq 1 \Rightarrow x\geq 0$



$f''(x)=4e^{2x}-2e^{x}=2e^{x}(2e^{x}-1)\geq 0$

per $e^{x}\geq \frac{1}{2} \Rightarrow x\geq \log\frac{1}{2}=-\log 2$



2) $\lim\limits_{x\to 0}\dfrac{\cos x-\cos 2x}{x^2}=\lim\limits_{x\to 0}\dfrac{1-\cos 2x}{4x^2}\cdot 4-\left(\dfrac{1-\cos x}{x^2}\right)=4\cdot\frac{1}{2}-\frac{1}{2}=\frac{3}{2}$.

$\lim\limits_{x\to+\infty}\left(\dfrac{1+3x}{1+2x}\right)^{1-x}=\left(\to\frac{3}{2}\right)^{(\to-\infty)}=0^+$.

3) $\lim\limits_{x\to 0}\dfrac{3^{kx}-2^{x}}{x}=\lim\limits_{x\to 0}\dfrac{3^{kx}-1}{kx}\cdot k-\dfrac{2^{x}-1}{x}=k\cdot\log 3-\log 2=1 \Rightarrow$

$\Rightarrow k\cdot\log 3=\log 2+1=\log 2e \Rightarrow k=\dfrac{\log 2e}{\log 3}$.

4) $f(x)=\dfrac{2x-1}{3x+1}$; $g(x)=3^{x}\Rightarrow f(g(x))=\dfrac{2\cdot 3^{x}-1}{3\cdot 3^{x}+1}=y\Rightarrow 2\cdot 3^{x}-1=3y3^{x}+y\Rightarrow$

$\Rightarrow 3^{x}(2-3y)=y+1\Rightarrow 3^{x}=\dfrac{y+1}{2-3y}\Rightarrow x=\log_3\dfrac{y+1}{2-3y}$. Inversa $y=\log_3\dfrac{x+1}{2-3x}$.

5) $\displaystyle\int_0^1\frac{e^{x}}{e^{x}+1}\,dx=\left(\log(e^{x}+1)\right|_0^1=\log(e+1)-\log 2=\log\frac{e+1}{2}$.

6) $f(x)=e^{x}-2x$. Se perpendicolare a $y=1-x \Rightarrow m=1=f'(x_0)\Rightarrow$

$\Rightarrow f'(x)=e^{x}-2=1=e^{x}=3\Rightarrow x_0=\log 3$.

7) $f(x;y)=x^2+y^2-2x^2y\Rightarrow\nabla f(x;y)=(0;0)\Rightarrow$

$\boxed{CMG2}$

$$\Rightarrow \begin{cases} f'_x = 2x-4xy = 2x(1-2y) = 0 \\ f'_y = 2y-2x^2 = 2(y-x^2) = 0 \end{cases} \Rightarrow \begin{cases} x=0 \\ y=0 \end{cases} \cup \begin{cases} y=\frac{1}{2} \\ x^2=\frac{1}{2} \end{cases} \Rightarrow \begin{cases} x=\frac{1}{\sqrt{2}} \\ y=\frac{1}{2} \end{cases} \cup \begin{cases} x=-\frac{1}{\sqrt{2}} \\ y=\frac{1}{2} \end{cases}.$$

$H(x;y) = \begin{Vmatrix} 2-4y & -4x \\ -4x & 2 \end{Vmatrix} \cdot H(0;0) = \begin{Vmatrix} 2 & 0 \\ 0 & 2 \end{Vmatrix} \Rightarrow \begin{cases} |H_1| = 2 > 0 \\ |H_2| = 4 > 0 \end{cases}$: Punto di minimo;

$H\left(\frac{1}{\sqrt{2}};\frac{1}{2}\right) = \begin{Vmatrix} 0 & -2\sqrt{2} \\ -2\sqrt{2} & 2 \end{Vmatrix} : |H_2| < 0$: P. Sella; $H\left(-\frac{1}{\sqrt{2}};\frac{1}{2}\right) = \begin{Vmatrix} 0 & 2\sqrt{2} \\ 2\sqrt{2} & 2 \end{Vmatrix} : |H_2| < 0$: P. Sella.

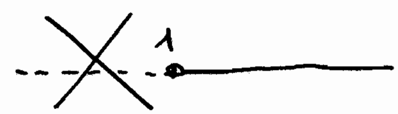
8) $A \cdot X = \begin{Vmatrix} 1 & 1 & K \\ 1 & K & -1 \\ K-1 & 1 & \end{Vmatrix} \cdot \begin{Vmatrix} 1 \\ 1 \\ 1 \end{Vmatrix} = \begin{Vmatrix} 1+1+K \\ 1+K-1 \\ K-1+1 \end{Vmatrix} = \begin{Vmatrix} 2+K \\ K \\ K \end{Vmatrix}$ che risulta parallelo a $(4;2;2)$ se

$\frac{2+K}{4} = \frac{K}{2} = \frac{K}{2} \Rightarrow$ vera per $K = 2$.

9)

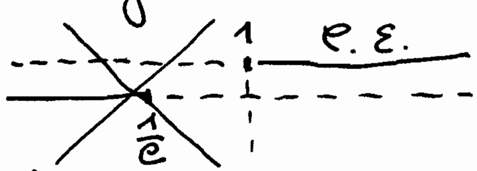
A B C D	(A o B)	(C e D)	(A o B) ⇒ (C e D)	non B	[(A o B) ⇒ (C e D)] ⇔ non B
0 1 1 1	1	1	1	0	0
0 1 0 1	1	0	0	0	1
0 0 1 1	0	1	1	1	1
0 0 0 1	0	0	1	1	1

10) $f(x) = \log(\log x)$. C.E.: $\log x > 0 \Rightarrow x > 1$



$f'(x) = \frac{1}{\log x} \cdot \frac{1}{x} = \frac{1}{x \cdot \log x}$; $f''(x) = -\frac{\log x + x \cdot \frac{1}{x}}{(x \cdot \log x)^2} = -\frac{\log x + 1}{(x \cdot \log x)^2} \geq 0$

per $\log x + 1 \leq 0 \Rightarrow \log x \leq -1 \Rightarrow \log x \leq e^{-1} = \frac{1}{e}$.



quindi $f''(x) < 0 \ \forall x > 1$.

La funzione è sempre concava in C.E.: $]1;+\infty[$.