Compito di Matematica Generale del 2/9/2022 $\boxed{MG1}$

1) $f(x) = x - \log x$. C.E.: $x > 0$. $\lim_{x\to 0^+} f(x) = (0-(-\infty)) = +\infty$; $\lim_{x\to +\infty} f(x) = +\infty$.

$f(x) \geq 0$ per $x \geq \log x$ vera $\forall x>0$

$f'(x) = 1 - \frac{1}{x} = \frac{x-1}{x} \geq 0$ per $x \geq 1$

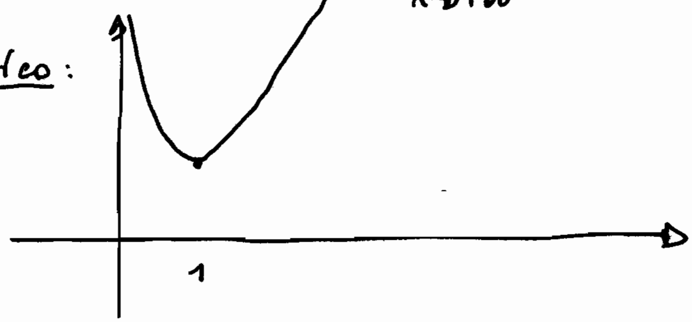
0 ↘ 1 ↗
m

$f(1) = 1$

$f''(x) = -\left(-\frac{1}{x^2}\right) = \frac{1}{x^2} > 0 \ \forall x$

0

Grafico:



2) $\lim_{x\to 0} \frac{\pi^x - e^x}{x} = \lim_{x\to 0} \frac{\pi^x - 1}{x} - \frac{e^x-1}{x} = \log \pi - 1 = \log\pi - \log e = \log\frac{\pi}{e}$.

$\lim_{x\to+\infty} \left(1+\frac{3}{2x}\right)^{x+1} = \lim_{x\to+\infty} \left[\left(1+\frac{3}{2}\cdot\frac{1}{x}\right)^x\right]^{\frac{x+1}{x}} = \left(e^{\frac{3}{2}}\right)^1 = e^{\frac{3}{2}} = e\sqrt{e}$.

3) $f(x) = \frac{e^x}{1-e^x}$. C.E.: $1-e^x \neq 0 \Rightarrow e^x \neq 1 \Rightarrow x \neq 0$. $-\infty$ ——— 0 ——— $+\infty$

$\lim_{x\to-\infty} f(x) = \frac{0}{1-0} = 0$: As. orizzontale; $\lim_{x\to+\infty} f(x) = -1$: As. orizzontale.

$\lim_{x\to 0^-} f(x) = \left(\frac{1}{1-1^-}\right) = +\infty$; $\lim_{x\to 0^+} f(x) = \left(\frac{1}{1-1^+}\right) = -\infty$: As. Verticale in $x = 0$.

4) $f(x) = \log(1+x)$; $g(x) = \frac{x+1}{x} \Rightarrow f(g(x)) = \log\left(1+\frac{x+1}{x}\right) = \log\left(\frac{2x+1}{x}\right)$.

C.E.: $\frac{2x+1}{x} > 0 \Rightarrow \begin{cases} 2x+1>0 \Rightarrow x > -\frac{1}{2} \\ x > 0 \end{cases}$ $-\frac{1}{2}$ 0 $+$ $-$ $+$ C.E.: $x < -\frac{1}{2} \cup x > 0$.

$\log\left(\frac{2x+1}{x}\right) = \log\left(2+\frac{1}{x}\right) = y \Rightarrow 2 + \frac{1}{x} = e^y \Rightarrow \frac{1}{x} = e^y - 2 \Rightarrow x = \frac{1}{e^y-2}$.

Inversa: $y = \frac{1}{e^x-2}$.

5) $\int_0^1 \sqrt{x+1} - e^{1-x}\,dx = \int_0^1 (x+1)^{\frac{1}{2}} - e^{1-x}\,dx = \left(\frac{1}{\frac{1}{2}+1}(x+1)^{\frac{1}{2}+1} - (-e^{1-x})\right|_0^1 =$

$= \left(\frac{2}{3}\sqrt{(x+1)^3} + e^{1-x}\right|_0^1 = \left(\frac{2}{3}\sqrt{8} + e^0\right) - \left(\frac{2}{3} + e\right) = \frac{4}{3}\sqrt{2} - e + \frac{1}{3}$.

6) $f(x) = 3x^2 - 2x + 1$; $y = x + \frac{1}{4}$ con $m = 1 \Rightarrow$

$\Rightarrow f'(x) = 6x - 2 = 1 \Rightarrow 6x = 3 \Rightarrow x = \frac{1}{2}$.

$f\left(\frac{1}{2}\right) = \frac{3}{4} - 1 + 1 = \frac{3}{4}$; $y\left(\frac{1}{2}\right) = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$. Quindi la retta $y = x + \frac{1}{4}$ è la tangente al grafico di $f(x)$ in $x_0 = \frac{1}{2}$.

7) $f(x;y) = 2x - x^2 + 3y - y^3$. $\nabla f(x;y) = \underline{0} \Rightarrow \begin{cases} f'_x = 2 - 2x = 2(1-x) = 0 \\ f'_y = 3 - 3y^2 = 3(1-y^2) = 0 \end{cases} \Rightarrow$

$\Rightarrow P_1 : (1;1)$ e $P_2 : (1;-1)$. $H(x;y) = \begin{Vmatrix} -2 & 0 \\ 0 & -6y \end{Vmatrix}$.

$H(1;1) = \begin{Vmatrix} -2 & 0 \\ 0 & -6 \end{Vmatrix} \Rightarrow \begin{cases} |H_1| < 0 \\ |H_2| = 12 > 0 \end{cases}$: P. MAX ; $H(1;-1) = \begin{Vmatrix} -2 & 0 \\ 0 & 6 \end{Vmatrix}$: $|H_2| < 0$: P. Sella.

8) $A = \begin{Vmatrix} 1 & 2 \\ 3 & 1 \end{Vmatrix}$; $B = \begin{Vmatrix} 5 & 0 \\ 0 & 10 \end{Vmatrix}$; $A \circ X = \begin{Vmatrix} 1 & 2 \\ 3 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} x_1 & x_2 \\ x_3 & x_4 \end{Vmatrix} = \begin{Vmatrix} x_1 + 2x_3 & x_2 + 2x_4 \\ 3x_1 + x_3 & 3x_2 + x_4 \end{Vmatrix} = \begin{Vmatrix} 5 & 0 \\ 0 & 10 \end{Vmatrix} \Rightarrow$

$$\begin{cases} x_1 + 2x_3 = 5 \\ 3x_1 + x_3 = 0 \\ x_2 + 2x_4 = 0 \\ 3x_2 + x_4 = 10 \end{cases} \Rightarrow \begin{cases} x_1 - 6x_1 = 5 \\ x_3 = -3x_1 \\ x_2 = -2x_4 \\ -6x_4 + x_4 = 10 \end{cases} \Rightarrow \begin{cases} x_1 = -1 \\ x_3 = 3 \\ x_2 = 4 \\ x_4 = -2 \end{cases} \Rightarrow X = \begin{Vmatrix} -1 & 4 \\ 3 & -2 \end{Vmatrix}.$$

$X \circ A = \begin{Vmatrix} -1 & 4 \\ 3 & -2 \end{Vmatrix} \cdot \begin{Vmatrix} 1 & 2 \\ 3 & 1 \end{Vmatrix} = \begin{Vmatrix} -1+12 & -2+4 \\ 3-6 & 6-2 \end{Vmatrix} = \begin{Vmatrix} 11 & 2 \\ -3 & 4 \end{Vmatrix} \neq \begin{Vmatrix} 5 & 0 \\ 0 & 10 \end{Vmatrix} \Rightarrow A \circ X \neq X \circ A.$

9)

A B C	(B e C)	non (B e C)	A e non (B e C)	non B	A e non B	⇔	
1 1 1	1	0	0	0	0	1	
1 1 0	0	1	1	0	0	0	*
1 0 1	0	1	1	1	1	1	
1 0 0	0	1	1	1	1	1	
0 1 1	1	0	0	0	0	1	
0 1 0	0	1	0	0	0	1	
0 0 1	0	1	0	1	0	1	
0 0 0	0	1	0	1	0	1	

La proposizione P: (A e non (B e C) ⇔ (A e non B) NON è una tautologia.

10) $f(x) = \log(\log x)$; $f(e) = \log(\log e) = \log 1 = 0$;

$f'(x) = \frac{1}{\log x} \cdot \frac{1}{x} = \frac{1}{x \cdot \log x}$; $f'(e) = \frac{1}{e \cdot \log e} = \frac{1}{e}$;

$f''(x) = -\frac{1 \cdot \log x + x \cdot \frac{1}{x}}{(x \cdot \log x)^2} = -\frac{\log x + 1}{(x \cdot \log x)^2}$; $f''(e) = -\frac{1+1}{(e \cdot 1)^2} = -\frac{2}{e^2}$.

$$P_2(x;e) = 0 + \frac{1}{e} \cdot (x-e) + \frac{1}{2} \cdot \left(-\frac{2}{e^2}\right) \cdot (x-e)^2 = \frac{1}{e}(x-e) - \frac{1}{e^2}(x-e)^2.$$