Prova Intermedia Matematica Generale 7/11/2022 **Compito A1**

1)

A	B	C	non C	(non C $\Leftrightarrow$ A)	(non A $\Rightarrow$ C) e (non B $\Rightarrow$ A)
1	1	1	0	0	1 1 1
1	1	0	1	1	1 1 1
1	0	1	0	0	1 1 1
1	0	0	1	1	1 1 1
0	1	1	0	1	1 1 1
0	1	0	1	0	0 0 1
0	0	1	0	1	1 0 0
0	0	0	1	0	0 0 0

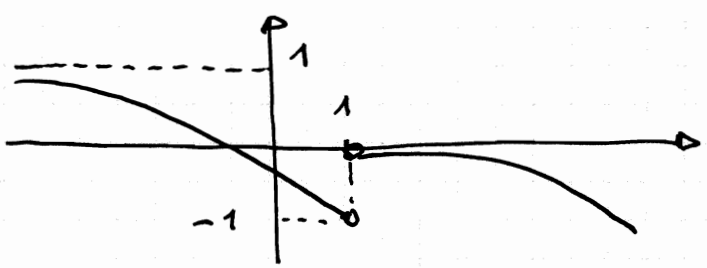
2) $\lim_{x\to 0} \frac{(1+\sin x)^3-1}{x} = \lim_{x\to 0} \frac{(1+\sin x)^3-1}{\sin x} \cdot \frac{\sin x}{x} = 3\cdot 1 = 3.$

$\lim_{x\to+\infty} \left(\frac{5+2x}{3+2x}\right)^{1-x} = \lim_{x\to+\infty} \left(\frac{3+2x+2}{3+2x}\right)^{1-x} = \lim_{x\to+\infty} \left[\left(1+\frac{2}{3+2x}\right)^{3+2x}\right]^{\frac{1-x}{3+2x}} = (e^2)^{-\frac{1}{2}} = \frac{1}{e}.$

3) $\lim_{x\to-\infty} f(x) = 1$; $\lim_{x\to+\infty} f(x) = -\infty$

b) $\lim_{x\to 1^-} f(x) = -1$

c) Disc. I specie in $x = 1$



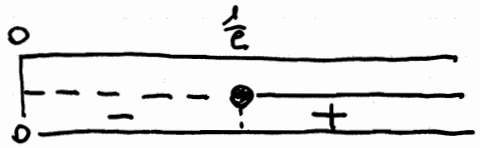
4) $F(x) = \frac{3^x-1}{3^x+2} = y \Rightarrow 3^x - 1 = 3^x\cdot y + 2y \Rightarrow 3^x(1-y) = 2y+1 \Rightarrow 3^x = \frac{2y+1}{1-y} \Rightarrow$

$x = \log_3 \frac{2y+1}{1-y} \Rightarrow f(x) = \log_3 \frac{2x+1}{1-x}$; $G(x) = \frac{3x+1}{x} = y \Rightarrow 3x+1 = xy \Rightarrow x(y-3) = 1 \Rightarrow$

$\Rightarrow x = \frac{1}{y-3} \Rightarrow g(x) = \frac{1}{x-3} \Rightarrow f(g(x)) = \log_3 \frac{2\cdot\frac{1}{x-3}+1}{1-\frac{1}{x-3}} = \log_3 \frac{x-1}{x-4}$;

$g(f(x)) = \frac{1}{\log_3 \frac{2x+1}{1-x} - 3}$.

5) $f(x) = \sqrt{\frac{1+\log x}{e^x-1}}$. C.E.: $\begin{cases} x>0 \\ \frac{1+\log x}{e^x-1} \geq 0 \end{cases} \Rightarrow \begin{cases} x>0 \\ \log x \geq -1 \\ e^x > 1 \end{cases} \Rightarrow \begin{cases} x>0 \\ x \geq \frac{1}{e} \\ x>0 \end{cases}$



C.E.: $x \geq \frac{1}{e}$.

Prova Intermedia di Matematica Generale 7/11/2022 | Compito B1

1)

A	B	C	non B	(A ⇔ non B)	(A ⇒ C)	e	(B ⇒ non A)
1	1	1	0	0	1	0	0
1	1	0	0	0	0	0	0
1	0	1	1	1	1	1	1
1	0	0	1	1	0	0	1
0	1	1	0	1	1	1	1
0	1	0	0	1	1	1	1
0	0	1	1	0	1	1	1
0	0	0	1	0	1	1	1

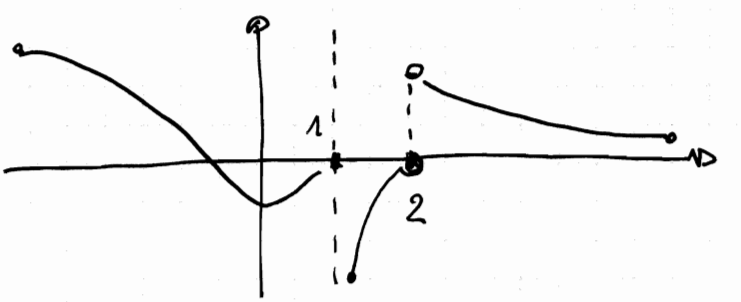
2) $\lim_{x\to 0} \frac{2^{\sin x}-1}{x} = \lim_{x\to 0} \frac{2^{\sin x}-1}{\sin x}\cdot\frac{\sin x}{x} = \log 2 \cdot 1 = \log 2.$

$\lim_{x\to+\infty} \left(\frac{5+2x}{3+3x}\right)^{1-x} = \left(\to \frac{2}{3}\right)^{(\to -\infty)} = +\infty.$

3) $\lim_{x\to-\infty} f(x) = +\infty$; $\lim_{x\to+\infty} f(x) = 0^+$

b) $\lim_{x\to 1^+} f(x) = -\infty$;

c) Disc. I Specie in $x = 2$



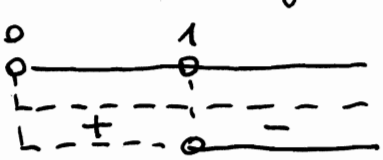
4) $F(x) = \frac{\log x - 1}{\log x + 2} = y \Rightarrow \log x - 1 = y\cdot\log x + 2y \Rightarrow \log x\cdot(1-y) = 2y+1 \Rightarrow$

$\log x = \frac{2y+1}{1-y} \Rightarrow f(x) = e^{\frac{2x+1}{1-x}}$; $G(x) = \frac{x}{2x-1} = y \Rightarrow x = 2xy - y \Rightarrow x(2y-1) = y \Rightarrow$

$\Rightarrow x = \frac{y}{2y-1} \Rightarrow g(x) = \frac{x}{2x-1}$ · $f(g(x)) = e^{\frac{2\cdot\frac{x}{2x-1}+1}{1-\frac{x}{2x-1}}} = e^{\frac{4x-1}{x-1}}$.

$g(f(x)) = \frac{e^{\frac{2x+1}{1-x}}}{2e^{\frac{2x+1}{1-x}} - 1}$.

5) $f(x) = \sqrt{\frac{1-e^x}{\log x}}$ C.E.: $\begin{cases} 1-e^x \geq 0 \\ \log x > 0 \\ x \neq 1 \\ x > 0 \end{cases} \Rightarrow \begin{cases} e^x \leq 1 \\ x \geq 1 \\ x \neq 1 \\ x > 0 \end{cases} \to \begin{cases} x \leq 0 \\ x > 1 \\ x \neq 1 \\ x > 0 \end{cases}$



C.E.: $0 < x < 1$.

Prova Intermedia di Matematica Generale 7/11/2022 **Compito C1**

1)

A	B	C	(non C $\iff$ non A)	(non A o C) $\Rightarrow$ (non B e A)		
1	1	1	1	1	0	0
1	1	0	0	0	1	0
1	0	1	1	1	1	1
1	0	0	0	0	1	1
0	1	1	0	1	0	0
0	1	0	1	1	0	0
0	0	1	0	1	0	0
0	0	0	1	1	0	0

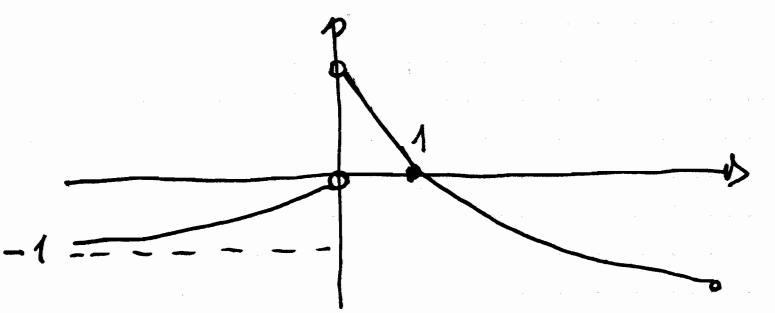
2) $\lim_{x\to 0} \frac{\sqrt{1+\operatorname{tg} 2x}-1}{x} = \lim_{x\to 0} \frac{(1+\operatorname{tg} 2x)^{\frac{1}{2}}-1}{\operatorname{tg} 2x} \cdot \frac{\operatorname{tg} 2x}{2x} \cdot 2 = \frac{1}{2}\cdot 1 \cdot 2 = 1.$

$\lim_{x\to+\infty} \left(\frac{5+2x}{5+3x}\right)^{x-5} = \left(\to \frac{2}{3}\right)^{(\to+\infty)} = 0^+.$

3) $\lim_{x\to-\infty} f(x) = -1$; $\lim_{x\to+\infty} f(x) = -\infty$

b) $\lim_{x\to 1^+} f(x) = 0$

c) Disc. I Sp. in $x=0$



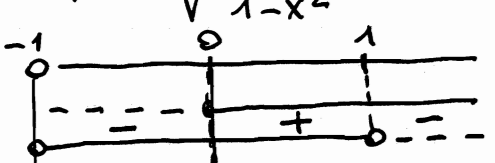
4) $F(x) = \frac{x+2}{x+1} = y \Rightarrow x+2 = xy+y \Rightarrow x(1-y) = y-2 \Rightarrow x = \frac{y-2}{1-y} \Rightarrow$

$f(x) = \frac{x-2}{1-x}$; $G(x) = \frac{e^x+1}{e^x-2} = y \Rightarrow e^x+1 = ye^x - 2y \Rightarrow e^x(y-1) = 2y+1 \Rightarrow$

$\Rightarrow e^x = \frac{2y+1}{y-1} \Rightarrow x = \log \frac{2y+1}{y-1} \Rightarrow g(x) = \log \frac{2x+1}{x-1} \cdot f(g(x)) = \frac{\log \frac{2x+1}{x-1} - 2}{1 - \log \frac{2x+1}{x-1}}$;

$g(f(x)) = \log \frac{2\cdot\frac{x-2}{1-x}+1}{\frac{x-2}{1-x}-1} = \log \frac{x-3}{2x-3}.$

5) $f(x) = \sqrt{\frac{\log(x+1)}{1-x^2}}$. C.E.: $\begin{cases} x+1>0 \\ \frac{\log(x+1)}{1-x^2} \geq 0 \end{cases} \Rightarrow \begin{cases} x>-1 \\ \log(x+1)\geq 0 \Rightarrow x+1 \geq 1 \Rightarrow x \geq 0 \\ 1-x^2>0 \Rightarrow x^2<1 \Rightarrow -1<x<1 \end{cases}$



C.E.: $0 \leq x < 1. \Rightarrow [0;1[.$

Prova Intermedia di Matematica Generale 7/11/2022 **Compito D1**

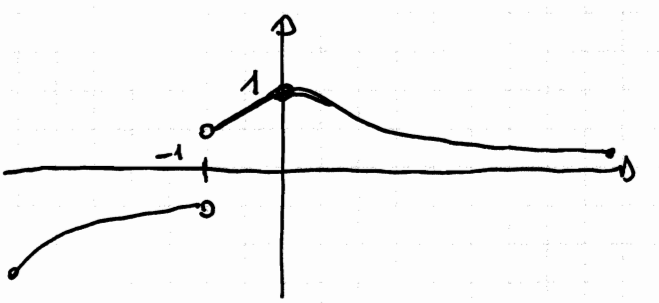
1)

A	B	C	$(B \Leftrightarrow \text{non } A)$	$(A \text{ e non } C) \Rightarrow (B \text{ o non } A)$		
1	1	1	0	0	1	1
1	1	0	0	1	1	1
1	0	1	1	0	1	0
1	0	0	1	1	0	0
0	1	1	1	0	1	1
0	1	0	1	0	1	1
0	0	1	0	0	1	1
0	0	0	0	0	1	1

2) $\lim_{x\to 0} \frac{\log(1+\text{tg}\,2x)}{e^x-1} = \lim_{x\to 0} \frac{\log(1+\text{tg}\,2x)}{\text{tg}\,2x} \cdot \frac{\text{tg}\,2x}{2x} \cdot 2 \cdot \frac{x}{e^x-1} = 1\cdot 1\cdot 2\cdot 1 = 2.$

$\lim_{x\to+\infty} \left(\frac{5+3x}{5+2x}\right)^{1-x} = \left(\to \frac{3}{2}\right)^{(\to -\infty)} = 0^+.$

3) $\lim_{x\to-\infty} f(x) = -\infty$; $\lim_{x\to+\infty} f(x) = 0^+$



b) $\lim_{x\to 0} f(x) = 1$

c) Disc. I Sp. in $x=-1$

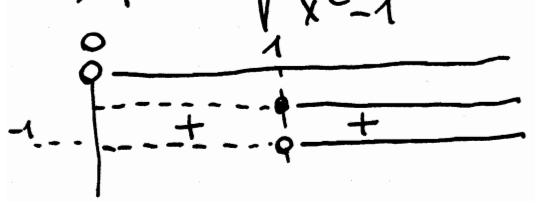
4) $F(x) = \frac{x-2}{x-1} = y \Rightarrow x-2 = xy-y \Rightarrow x(y-1) = y-2 \Rightarrow x = \frac{y-2}{y-1} \Rightarrow$

$f(x) = \frac{x-2}{x-1}$. $G(x) = \frac{\log x+2}{\log x-1} = y \Rightarrow \log x+2 = y\log x - y \Rightarrow \log x\cdot(y-1) = y+2 \Rightarrow$

$\Rightarrow \log x = \frac{y+2}{y-1} \Rightarrow x = e^{\frac{y+2}{y-1}} \Rightarrow g(x) = e^{\frac{x+2}{x-1}}$. $f(g(x)) = \frac{e^{\frac{x+2}{x-1}}-2}{e^{\frac{x+2}{x-1}}-1}$;

$g(f(x)) = e^{\frac{\frac{x-2}{x-1}+2}{\frac{x-2}{x-1}-1}} = e^{-(3x-4)} = e^{4-3x}$.

5) $f(x) = \sqrt{\frac{x\cdot\log x}{x^2-1}}$. $\mathcal{C.E.}$: $\begin{cases} x>0 \\ \frac{x\cdot\log x}{x^2-1} \geq 0 \end{cases} \Rightarrow \begin{cases} x>0 \\ \log x \geq 0 \Rightarrow x \geq 1 \\ x^2-1>0 \Rightarrow x^2>1 \Rightarrow x<-1 \cup x>1 \end{cases}$



$\mathcal{C.E.}$: $x>0$ con $x\neq 1$: $]0;1[\cup]1;+\infty[$.