Prova Intermedia di Matematica Generale 7/11/2022 $\boxed{\text{Compito A2}}$

1)

A B C	$(C \Leftrightarrow A)$	(non A e C)	$\Rightarrow$	(non B o A)
~~1 1 1~~	~~1~~	~~0~~	~~1~~	~~1~~
1 1 0	0	0	1	1
~~1 0 1~~	~~1~~	~~0~~	~~1~~	~~1~~
1 0 0	0	0	1	1
0 1 1	0	1	0	0
~~0 1 0~~	~~1~~	~~0~~	~~1~~	~~0~~
0 0 1	0	1	1	1
~~0 0 0~~	~~1~~	~~0~~	~~1~~	~~1~~

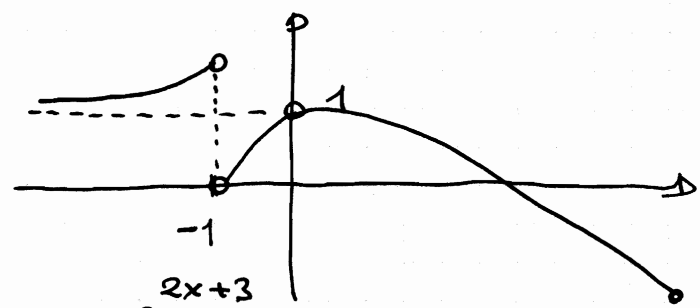
2) $\lim_{x\to 0} \dfrac{\log(1+\operatorname{sen}^2 x)}{1-\cos x} = \lim_{x\to 0} \dfrac{\log(1+\operatorname{sen}^2 x)}{\operatorname{sen}^2 x}\cdot\dfrac{\operatorname{sen}^2 x}{x^2}\cdot\dfrac{x^2}{1-\cos x} = 1\cdot 1\cdot \dfrac{1}{\frac{1}{2}} = 2.$

$\lim_{x\to+\infty}\left(\dfrac{5+3x}{3+2x}\right)^{1-x} = \left(\to \dfrac{3}{2}\right)^{(\to -\infty)} = 0^+.$

3) $\lim_{x\to 0} f(x) = 1$; $\lim_{x\to+\infty} f(x) = -\infty$

b) $\lim_{x\to -\infty} f(x) = 1$;

c) Disc. I Sp. in $x=-1$.



4) $f(x) = \dfrac{2^x}{2^x-2}$; $g(x) = 2x+3$. $f(g(x)) = \dfrac{2^{2x+3}}{2^{2x+3}-2} = y \Rightarrow$

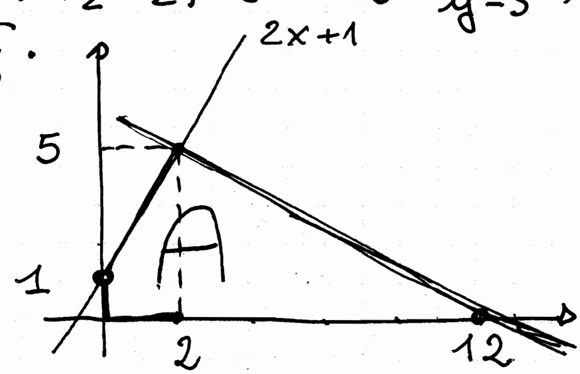
$\Rightarrow 2^{2x+3} = y\cdot 2^{2x+3} - 2y \Rightarrow 2^{2x+3}\cdot(y-1) = 2y \Rightarrow 2^{2x+3} = 2^{2x}\cdot 2^3 = 8\cdot 4^x = \dfrac{2y}{y-1} \Rightarrow$

$4^x = \dfrac{y}{4(y-1)} \Rightarrow x = \log_4 \dfrac{y}{4(y-1)} \Rightarrow f(g(x))^{-1} = \log_4 \dfrac{x}{4x-4}$. $g(f(x)) = 2\cdot\dfrac{2^x}{2^x-2} + 3 = y \Rightarrow$

$\Rightarrow \dfrac{2^x}{2^x-2} = \dfrac{1}{2}(y-3) \Rightarrow 2^x = 2^x\cdot\dfrac{1}{2}(y-3) - (y-3) \Rightarrow 2^x\left(\dfrac{y}{2}-\dfrac{5}{2}\right) = y-3 \Rightarrow 2^x = \dfrac{2y-6}{y-5} \Rightarrow$

$\Rightarrow x = \log_2 \dfrac{2y-6}{y-5} \Rightarrow g(f(x))^{-1} = \log_2 \dfrac{2x-6}{x-5}.$

5) $\begin{cases} y = 2x+1 \\ y = -\frac{1}{2}x+6 \end{cases} \Rightarrow \begin{cases} y = 2x+1 \\ 2x+1 = -\frac{1}{2}x+6 \end{cases} \Rightarrow \begin{cases} x = 2 \\ y = 5. \end{cases}$



Area $= \dfrac{1}{2}\cdot 2\cdot(5+1) + \dfrac{1}{2}\cdot 10\cdot 5 = 6+25 = 31.$

Prova Intermedia di Matematica Generale 7/11/2022 $\boxed{\text{Compito B2}}$

1)

A	B	C	(C ⇔ non A)	(A o C)	e	(non B ⇒ A)
1	1	1	0	1	1	1
~~1~~	~~1~~	~~0~~	~~1~~	~~1~~	~~1~~	~~1~~
1	0	1	0	1	1	1
~~1~~	~~0~~	~~0~~	~~1~~	~~1~~	~~1~~	~~1~~
~~0~~	~~1~~	~~1~~	~~1~~	~~1~~	~~1~~	~~1~~
0	1	0	0	0	0	1
~~0~~	~~0~~	~~1~~	~~1~~	~~1~~	~~0~~	~~0~~
0	0	0	0	0	0	0

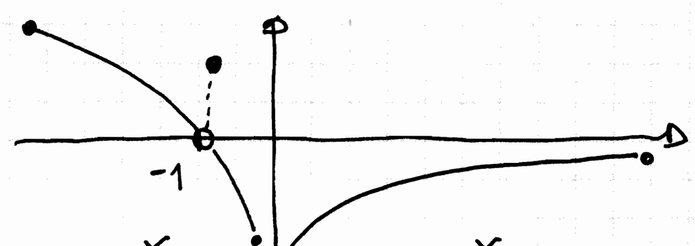
2) $\lim_{x\to 0} \frac{\log(1+\operatorname{sen} 3x)}{\log(1-\operatorname{sen} 2x)} = \lim_{x\to 0} \frac{\log(1+3x)}{3x} \cdot \frac{3x}{(-2x)} \cdot \frac{(-2x)}{\log(1+(-2x))} = 1 \cdot \left(-\frac{3}{2}\right) \cdot 1 = -\frac{3}{2}.$

$\lim_{x\to +\infty} \left(\frac{5+3x}{3+3x}\right)^{2x-1} = \lim_{x\to +\infty} \left(\frac{3+3x+2}{3+3x}\right)^{2x-1} = \lim_{x\to +\infty} \left[\left(1+\frac{2}{3+3x}\right)^{3+3x}\right]^{\frac{2x-1}{3x+3}} = (e^2)^{\frac{2}{3}} = \sqrt[3]{e^4}.$

3) $\lim_{x\to -\infty} f(x) = +\infty$; $\lim_{x\to +\infty} f(x) = 0^-$

b) $\lim_{x\to 0} f(x) = -\infty$

c) Disc. $\boxed{\text{III}}$ sp. in $x=-1$.



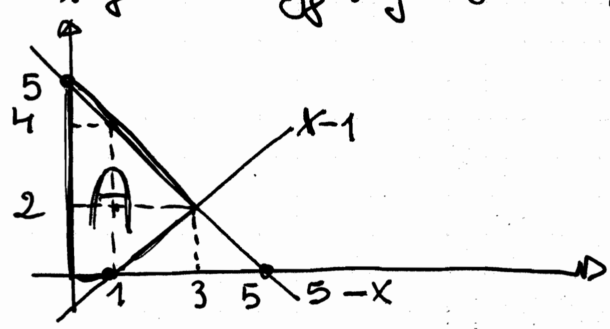
4) $f(x) = 3x-1$, $g(x) = \frac{2^x}{2^x+1}$. $f(g(x)) = 3 \cdot \frac{2^x}{2^x+1} - 1 = y \Rightarrow \frac{2^x}{2^x+1} = \frac{1}{3}(y+1) \Rightarrow$

$\Rightarrow 2^x = \frac{1}{3} 2^x (y+1) + \frac{1}{3}(y+1) \Rightarrow 2^x = \frac{\frac{1}{3}(y+1)}{1-\frac{1}{3}(y+1)} = \frac{y+1}{2-y} \Rightarrow x = \log_2\left(\frac{y+1}{2-y}\right) \Rightarrow$

$f(g(x))^{-1} = \log_2\left(\frac{x+1}{2-x}\right)$. $g(f(x)) = \frac{2^{3x-1}}{2^{3x-1}+1} = y \Rightarrow 2^{3x-1} = 2^{3x-1} \cdot y + y \Rightarrow$

$\Rightarrow 2^{3x-1}(1-y) = 2^{3x} \cdot 2^{-1}(1-y) = y \Rightarrow \frac{8^x}{2} = \frac{y}{1-y} \Rightarrow x = \log_8 \frac{2y}{1-y} \Rightarrow g(f(x))^{-1} = \log_8 \frac{2x}{1-x}.$

5) $\begin{cases} y = 5-x \\ y = x-1 \end{cases} \Rightarrow \begin{cases} y = x-1 \\ 5-x = x-1 \end{cases} \Rightarrow \begin{cases} x = 3 \\ y = 2 \end{cases}$



$\text{Area} = \frac{1}{2} \cdot 1 \cdot (5+4) + \frac{1}{2} \cdot (4 \cdot 2) = \frac{9}{2} + 4 =$
$= \frac{17}{2}.$

Prova Intermedia di Matematica Generale 7/11/2022 $\boxed{\text{Compito C2}}$

1)

A B C	$(C \Leftrightarrow B)$	$(A$ o non $C)$	$\Rightarrow$	(non B e $A)$
~~1 1 1~~	~~1~~	~~1~~	~~0~~	~~0~~
1 1 0	0	1	0	0
1 0 1	0	1	1	1
~~1 0 0~~	~~1~~	~~1~~	~~1~~	~~1~~
~~0 1 1~~	~~1~~	~~0~~	~~1~~	~~0~~
0 1 0	0	1	0	0
0 0 1	0	0	1	0
~~0 0 0~~	~~1~~	~~1~~	~~0~~	~~0~~

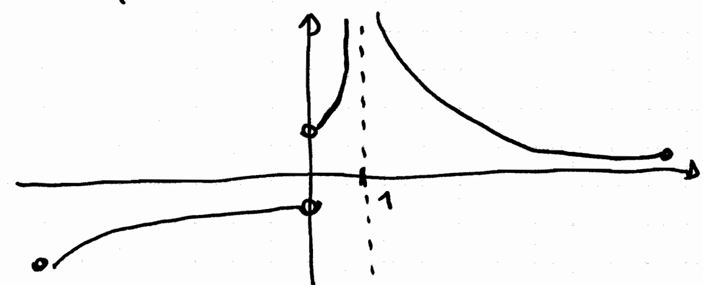
2) $\lim\limits_{x\to 0} \frac{\log(1+tg\,2x)}{sen\,3x} = \lim\limits_{x\to 0} \frac{\log(1+tg\,2x)}{tg\,2x}\cdot\frac{tg\,2x}{2x}\cdot\frac{2x}{3x}\cdot\frac{3x}{sen\,3x} = 1\cdot 1\cdot\frac{2}{3}\cdot 1 = \frac{2}{3}.$

$\lim\limits_{x\to+\infty}\left(\frac{5+2x}{2+3x}\right)^{1-x} = \left(\to\frac{2}{3}\right)^{(\to-\infty)} = +\infty.$

3) $\lim\limits_{x\to-\infty} f(x) = -\infty$ e $\lim\limits_{x\to 1} f(x) = +\infty$

b) $\lim\limits_{x\to+\infty} f(x) = 0^+$

c) Disc. I Sp. in $x=0$.



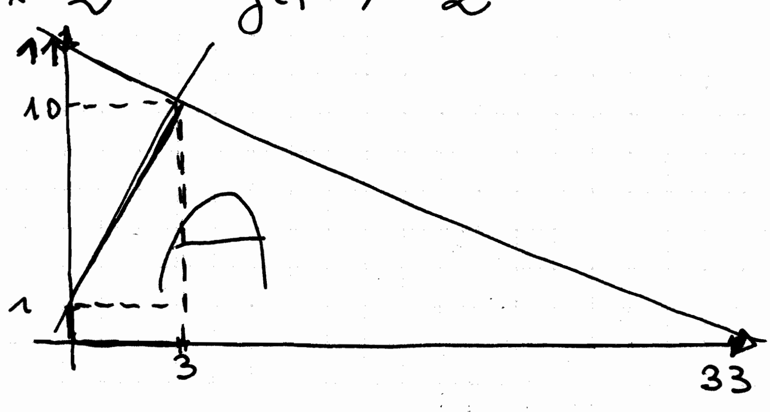
4) $f(x) = \frac{\log_2 x}{\log_2 x - 2}$; $g(x) = 3x+1$. $f(g(x)) = \frac{\log_2(3x+1)}{\log_2(3x+1)-2} = y \Rightarrow \log_2(3x+1) = y\cdot\log_2(3x+1) - 2y$

$\Rightarrow \log_2(3x+1)(y-1) = 2y \Rightarrow \log_2(3x+1) = \frac{2y}{y-1} \Rightarrow 3x+1 = 2^{\frac{2y}{y-1}} \Rightarrow x = \frac{1}{3}\left(2^{\frac{2y}{y-1}} - 1\right).$

$f(g(x))^{-1} = \frac{1}{3}\left(2^{\frac{2x}{x-1}} - 1\right).$ $g(f(x)) = 3\cdot\frac{\log_2 x}{\log_2 x - 2} + 1 = y \Rightarrow 3\log_2 x = (\log_2 x - 2)(y-1) \Rightarrow$

$\Rightarrow \log_2 x\cdot(4-y) = 2-2y \Rightarrow \log_2 x = \frac{2-2y}{4-y} \Rightarrow x = 2^{\frac{2-2y}{4-y}}.$ $g(f(x))^{-1} = 2^{\frac{2-2x}{4-x}}.$

5) $\begin{cases} y = 3x+1 \\ y = -\frac{1}{3}x + 11 \end{cases} \Rightarrow \begin{cases} y = 3x+1 \\ 3x+1 = -\frac{1}{3}x+11 \end{cases} \Rightarrow \begin{cases} x = 3 \\ y = 10 \end{cases}$



Area $= \frac{1}{2}\cdot 3\cdot(1+10) + \frac{1}{2}\cdot 10\cdot 30 =$

$= \frac{33}{2} + \frac{300}{2} = \frac{333}{2}.$

Prova Intermedia di Matematica Generale 7/11/2022 $\boxed{\text{Compito D2}}$

1)

A	B	C	(B <=> A)	(A => non C)	o	(B e non A)
1	1	1	1	0	0	0
1	1	0	1	1	1	0
~~1~~	~~0~~	~~1~~	~~0~~	~~0~~	~~0~~	~~0~~
~~1~~	~~0~~	~~0~~	~~0~~	~~1~~	~~1~~	~~0~~
~~0~~	~~1~~	~~1~~	~~0~~	~~1~~	~~1~~	~~1~~
~~0~~	~~1~~	~~0~~	~~0~~	~~1~~	~~1~~	~~1~~
0	0	1	1	1	1	0
0	0	0	1	1	1	0

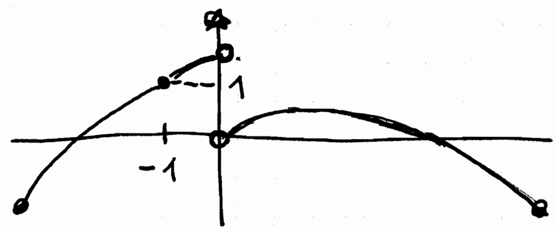
2) $\lim_{x\to 0}\frac{\log(1+3x)}{e^{\operatorname{sen}2x}-1}=\lim_{x\to 0}\frac{\log(1+3x)}{3x}\cdot\frac{3x}{2x}\cdot\frac{2x}{\operatorname{sen}2x}\cdot\frac{\operatorname{sen}2x}{e^{\operatorname{sen}2x}-1}=1\cdot\frac{3}{2}\cdot 1\cdot 1=\frac{3}{2}.$

$\lim_{x\to+\infty}\left(\frac{3+3x}{2+3x}\right)^{1-2x}=\lim_{x\to+\infty}\left(\frac{2+3x+1}{2+3x}\right)^{1-2x}=\lim_{x\to+\infty}\left[\left(1+\frac{1}{2+3x}\right)^{2+3x}\right]^{\frac{1-2x}{2+3x}}=e^{-\frac{2}{3}}=\frac{1}{\sqrt[3]{e^2}}.$

3) $\lim_{x\to-\infty}f(x)=-\infty$ e $\lim_{x\to+\infty}f(x)=-\infty$

b) $\lim_{x\to-1}f(x)=1^-$

c) Disc. I sp. in $x=0$.



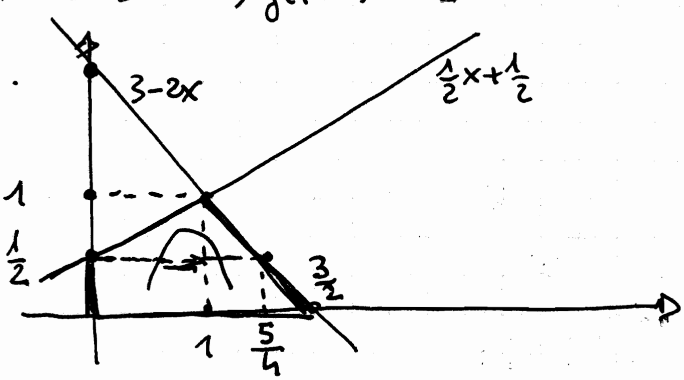
4) $f(x)=x-3$; $g(x)=\frac{\log_3 x-1}{\log_3 x+1}$. $f(g(x))=\frac{\log_3 x-1}{\log_3 x+1}-3=\frac{-2\log_3 x-4}{\log_3 x+1}=y \Rightarrow$

$\Rightarrow -2\log_3 x-4=y\cdot\log_3 x+y \Rightarrow \log_3 x\cdot(y+2)=-y-4 \Rightarrow \log_3 x=\frac{-y-4}{y+2}\Rightarrow x=3^{\frac{-y-4}{y+2}} \Rightarrow$

$f(g(x))^{-1}=3^{\frac{-x-4}{x+2}}$. $g(f(x))=\frac{\log_3(x-3)-1}{\log_3(x-3)+1}=y \Rightarrow \log_3(x-3)-1=y\cdot\log_3(x-3)+y \Rightarrow$

$\Rightarrow \log_3(x-3)\cdot(1-y)=y+1 \Rightarrow \log_3(x-3)=\frac{y+1}{1-y}\Rightarrow x-3=3^{\frac{y+1}{1-y}}\Rightarrow g(f(x))^{-1}=3^{\frac{x+1}{1-x}}+3.$

5) $\begin{cases}y=3-2x\\ y=\frac{1}{2}x+\frac{1}{2}\end{cases}\Rightarrow\begin{cases}y=3-2x\\ 3-2x=\frac{1}{2}x+\frac{1}{2}\end{cases}\Rightarrow\begin{cases}x=1\\ y=1\end{cases}.$



$\text{Area}=\frac{1}{2}\cdot\frac{1}{2}\cdot\left(\frac{3}{2}+\frac{5}{4}\right)+\frac{1}{2}\cdot\frac{5}{4}\cdot\frac{1}{2}=$

$=\text{Area}=\frac{1}{4}\cdot\frac{11}{4}+\frac{5}{16}=\frac{16}{16}=1.$