Compito di Matematica Generale del 2/9/2020 CMG1

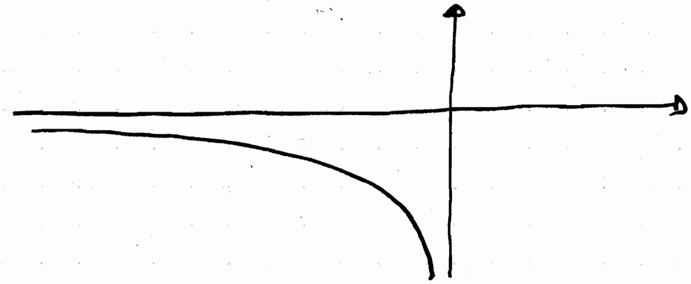
$f(x) = \log(1-e^x)$. C.E.: $1-e^x > 0 \Rightarrow e^x < 1 \Rightarrow x < 0$. $\lim_{x\to-\infty} f(x) = 0^-$; $\lim_{x\to 0^-} f(x) = -\infty$.

$f(x) \geq 0$: $1-e^x \geq 1 \Rightarrow e^x \leq 0$ impossibile.

$f'(x) = \dfrac{-e^x}{1-e^x} < 0 \quad \forall x \in$ C.E.

$f''(x) = -\dfrac{e^x(1-e^x) - e^x(-e^x)}{(1-e^x)^2} = -\dfrac{e^x}{(1-e^x)^2} < 0$

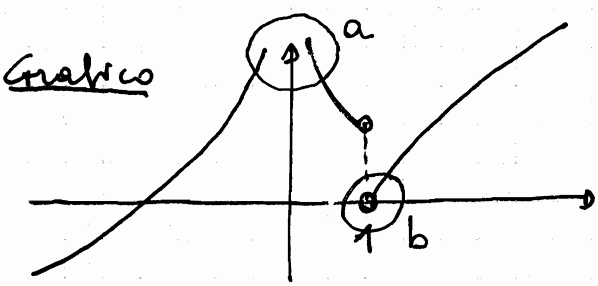
$\forall x \in$ C.E.



2) $\lim_{x\to 1} \dfrac{\operatorname{sen}(x-1)}{1-x} = (\text{posto } x-1=t) = \lim_{t\to 0} \dfrac{\operatorname{sen} t}{-t} = -1$;

$\lim_{x\to+\infty} \left(\dfrac{3x-3}{2x-1}\right)^{1-x} = \left(\to \dfrac{3}{2}\right)^{(\to -\infty)} = 0^+$.

Grafico



3) $\forall \varepsilon \, \exists \delta(\varepsilon): |x| < \delta(\varepsilon) \Rightarrow f(x) > \varepsilon$: $\lim_{x\to 0} f(x) = +\infty$

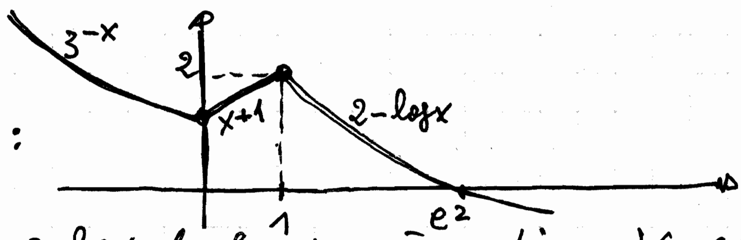
$\forall \varepsilon > 0 \, \exists \delta(\varepsilon): 1 < x < 1+\delta(\varepsilon) \Rightarrow |f(x)| < \varepsilon$: $\lim_{x\to 1^+} f(x) = 0$

4) $f(x) = \dfrac{2x-1}{3x} = y \Rightarrow 2x-1 = 3xy \Rightarrow x(2-3y) = 1 \Rightarrow x = \dfrac{1}{2-3y}$: $f^{-1}(x) = \dfrac{1}{2-3x} \Rightarrow$

$f^{-1}(g(x)) = \dfrac{1}{2-3g(x)} = 2x-3 \Rightarrow 2-3g(x) = \dfrac{1}{2x-3} \Rightarrow 3g(x) = 2 - \dfrac{1}{2x-3} = \dfrac{4x-7}{2x-3} \Rightarrow$

$\Rightarrow g(x) = \dfrac{4x-7}{3(2x-3)}$.

5) $f(x) = \begin{cases} 3^{-x} & : x \leq 0 \\ x+1 & : 0 < x < 1 \\ 2-\log x & : 1 \leq x \end{cases}$ Grafico:



da $\lim_{x\to 0^-} 3^{-x} = 1 = \lim_{x\to 0^+} x+1$ e $\lim_{x\to 1^-} x+1 = 2 = \lim_{x\to 1^+} 2-\log x$ la funzione è continua $\forall x \in \mathbb{R}$.

6) $\displaystyle\int_0^1 2e^{3x} - 3x^2 \, dx = \left(\frac{2}{3}e^{3x} - x^3\right|_0^1 = \left(\frac{2}{3}e^3 - 1\right) - \left(\frac{2}{3} - 0\right) = \frac{2}{3}e^3 - \frac{5}{3}$.

7) $A \cdot B \cdot X = \begin{Vmatrix} 1 & 0 \\ 0 & -1 \\ -1 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} K & 1 \\ -1 & K \end{Vmatrix} \cdot \begin{Vmatrix} -1 \\ 1 \end{Vmatrix} = \begin{Vmatrix} 1 & 0 \\ 0 & -1 \\ -1 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} -K+1 \\ 1+K \end{Vmatrix} = \begin{Vmatrix} -K+1+0 \\ 0-1-K \\ K-1+1+K \end{Vmatrix} = \begin{Vmatrix} 1-K \\ -K-1 \\ 2K \end{Vmatrix}$

CMG2

$A \cdot B \cdot X \perp Y \Rightarrow (1-k; -k-1; 2k) \cdot (1; -1; 1) = 1-k+k+1+2k = 2k+2 = 0$

vera per $k=-1$. Quindi $A \cdot B \cdot X = (2; 0; -2)$.

8) $f(x;y) = x^2 y - 2xy + xy^2$. $\nabla f(x;y) = (0;0) \Rightarrow \begin{cases} f'_x = 2xy - 2y + y^2 = 0 \\ f'_y = x^2 - 2x + 2xy = 0 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} y(2x-2+y) = 0 \\ x(x-2+2y) = 0 \end{cases} \Rightarrow \begin{cases} x=0 \\ y=0 \end{cases} \cup \begin{cases} x=2 \\ y=0 \end{cases} \cup \begin{cases} x=0 \\ y=2 \end{cases} \cup \begin{cases} y = 2-2x \\ x-2+4-4x = 2-3x = 0 \end{cases} \Rightarrow \begin{cases} x = \frac{2}{3} \\ y = \frac{2}{3} \end{cases}$.

$H(x;y) = \begin{Vmatrix} 2y & 2x-2+2y \\ 2x-2+2y & 2x \end{Vmatrix}$. $H(0;0) = \begin{Vmatrix} 0 & -2 \\ -2 & 0 \end{Vmatrix} \Rightarrow |H_2| = -4 < 0$: Punto di Sella;

$H(2;0) = \begin{Vmatrix} 0 & 2 \\ 2 & 4 \end{Vmatrix} \Rightarrow |H_2| = -4 < 0$: Punto di Sella; $H(0;2) = \begin{Vmatrix} 4 & 2 \\ 2 & 0 \end{Vmatrix} \Rightarrow |H_2| = -4 < 0$: Punto di Sella;

$H\left(\frac{2}{3}; \frac{2}{3}\right) = \begin{Vmatrix} \frac{4}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{4}{3} \end{Vmatrix} \Rightarrow \begin{cases} |H_1| = \frac{4}{3} > 0 \\ |H_2| = \frac{16}{3} - \frac{4}{3} > 0 \end{cases}$: Punto di Minimo.

9) Essendo A e C logicamente equivalenti sono ambedue vere o ambedue false.

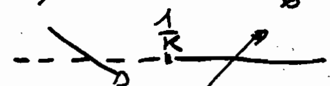
A B C	(non C)	(A ⇒ non C)	(non B)	(non B ⇒ C)	[(A ⇒ non C) ∨ (non B ⇒ C)]
1 1 1	0	0	0	1	1
1 0 1	0	0	1	1	1
0 1 0	1	1	0	1	1
0 0 0	1	1	1	0	1

La proposizione data risulta sempre vera.

10) $f(x) = x \cdot e^{1-kx}$ funzione continua e derivabile $\forall x \in \mathbb{R}$.

$f'(x) = e^{1-kx} - kx\, e^{1-kx} = (1-kx) \cdot e^{1-kx} \geq 0$ per $1-kx \geq 0$.

Se $k > 0$: $1 - kx \geq 0 \Rightarrow kx \leq 1 \Rightarrow x \leq \frac{1}{k}$: 

Se $k < 0$: $1 - kx \geq 0 \Rightarrow kx \leq 1 \Rightarrow x \geq \frac{1}{k}$: 

Se $x = 2$ è punto di Massimo $\Rightarrow k > 0$ e $\frac{1}{k} = 2 \Rightarrow k = \frac{1}{2}$.