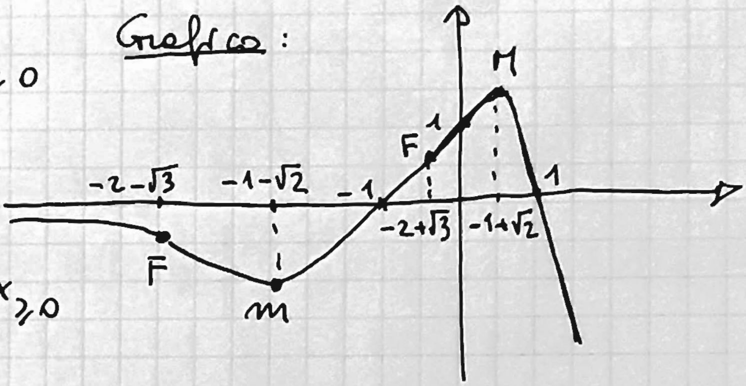
Compito di Matematica Generale del 14/3/2022 $\boxed{\text{CMG1}}$

1) $f(x) = (1-x^2)\cdot e^x$. C.E.: $\mathbb{R}$. $\lim_{x\to -\infty} f(x) = 0^-$; $\lim_{x\to +\infty} f(x) = -\infty$.

$f(x) \geq 0$ per $x^2 \leq 1 \Rightarrow -1 \leq x \leq 1$. $f(0) = 1$.



$f'(x) = (-2x)e^x + (1-x^2)e^x = (1-2x-x^2)\cdot e^x \geq 0$

per $x^2+2x-1 \leq 0$: $-1-\sqrt{2} \leq x \leq -1+\sqrt{2}$

$f''(x) = (-2-2x)e^x + (1-2x-x^2)e^x = (-x^2-4x-1)e^x \geq 0$

per $x^2+4x+1 \leq 0$: $-2-\sqrt{3} \leq x \leq -2+\sqrt{3}$

2) $\lim_{x\to 0} \frac{\log(1-\operatorname{sen}^2 x)}{1-\cos x} = \lim_{x\to 0} \frac{\log(1-\operatorname{sen}^2 x)}{-\operatorname{sen}^2 x} \cdot \frac{-\operatorname{sen}^2 x}{x^2} \cdot \frac{x^2}{1-\cos x} = 1\cdot(-1)\cdot 2 = -2.$

$\lim_{x\to +\infty} \left(\frac{4+3^x}{3+3^x}\right)^{2-3^x} = \lim_{x\to +\infty} \left(\frac{3+3^x+1}{3+3^x}\right)^{2-3^x} = \lim_{x\to +\infty} \left[\left(1+\frac{1}{3+3^x}\right)^{3+3^x}\right]^{\frac{2-3^x}{3+3^x}} = e^{-1} = \frac{1}{e}.$

3) $f(x) = \begin{cases} mx+q & : -1 \leq x \leq 1 \\ e^{x^2}+1 & : \text{altrimenti} \end{cases}$. $\lim_{x\to -1^+} f(x) = -m+q = \lim_{x\to -1^-} f(x) = e+1$;

$\lim_{x\to 1^+} f(x) = e+1 = \lim_{x\to 1^-} f(x) = m+q \Rightarrow \begin{cases} -m+q = e+1 \\ m+q = e+1 \end{cases} \Rightarrow \begin{cases} m = 0 \\ q = e+1 \end{cases}.$

4) $f(x) = e^x - 1$; $g^{-1}(x) = e^x+1 = y \Rightarrow e^x = y-1 \Rightarrow x = \log(y-1)$ e quindi

$g(x) = \log(x-1)$ per cui $F(x) = f(g(x)) = f(\log(x-1)) = e^{\log(x-1)} - 1 = x-1-1 = x-2$.

Se $F(x) = x-2 = y \Rightarrow x = y+2 \Rightarrow F^{-1}(x) = x+2$. Da $x>1 \Rightarrow y+2>1 \Rightarrow y>-1 \Rightarrow$ C.E. $F^{-1}(x)$: $x>-1$.

5) $\int_0^1 \frac{1}{x^2+2x+1}\,dx = \int_0^1 \frac{1}{(x+1)^2}\,dx = \left(-\frac{1}{x+1}\right|_0^1 = -\frac{1}{2} + 1 = \frac{1}{2}.$

6) $f(x) = e^{-x}\cdot \log(1+x)$. $f(0) = 1\cdot \log 1 = 0$. $f'(x) = -e^{-x}\cdot\log(1+x) + \frac{e^{-x}}{1+x} \Rightarrow$

$\Rightarrow f'(0) = -1\cdot\log 1 + \frac{1}{1} = 1$. Equazione tangente in $x=0$: $y-0 = 1\cdot(x-0) \Rightarrow$

$\boxed{\text{CMG2}}$

Equazione retta tangente in $x=0$: $y=x$ e quindi

equazione della perpendicolare alla tangente in $x=0$: $y=-x$.

7) $f(x;y)=x+3y-e^{x}-y^{3}$. $\nabla f(x;y)=(0;0) \Rightarrow$

$$\begin{cases} f'_x=1-e^{x}=0 \\ f'_y=3-3y^{2}=3(1-y^{2})=0 \end{cases} \Rightarrow \begin{cases} e^{x}=1 \\ y^{2}=1 \end{cases} \Rightarrow \begin{cases} x=0 \\ y=1 \end{cases} \cup \begin{cases} x=0 \\ y=-1 \end{cases}. \quad H(x;y)=\begin{Vmatrix} -e^{x} & 0 \\ 0 & -6y \end{Vmatrix}.$$

$H(0;1)=\begin{Vmatrix} -1 & 0 \\ 0 & -6 \end{Vmatrix}$: $\begin{cases} |H_1|<0 \\ |H_2|=6>0 \end{cases}$: Punto di MAX; $H(0;-1)=\begin{Vmatrix} -1 & 0 \\ 0 & 6 \end{Vmatrix}$: $|H_2|<0$: P. Sella.

8) $A=\begin{Vmatrix} 1 & 0 & K \\ 0 & K & -1 \\ K-1 & 1 & \end{Vmatrix}$; $X=\begin{Vmatrix} 1 \\ 1 \\ 1 \end{Vmatrix}$. $A\cdot X=\begin{Vmatrix} 1 & 0 & K \\ 0 & K & -1 \\ K & -1 & 1 \end{Vmatrix}\cdot\begin{Vmatrix} 1 \\ 1 \\ 1 \end{Vmatrix}=\begin{Vmatrix} 1+0+K \\ 0+K-1 \\ K-1+1 \end{Vmatrix}=\begin{Vmatrix} K+1 \\ K-1 \\ K \end{Vmatrix}$.

$\|A\cdot X\|=\sqrt{(K+1)^{2}+(K-1)^{2}+K^{2}}=2 \Rightarrow K^{2}+\cancel{2K}+1+K^{2}-\cancel{2K}+1+K^{2}=4 \Rightarrow$

$\Rightarrow 3K^{2}+2=4 \Rightarrow K^{2}=\frac{2}{3} \Rightarrow K=\pm\sqrt{\frac{2}{3}}$.

9) P_2 : $A \Rightarrow (\text{non}(\text{non } B \text{ o non } C)) \not\Leftrightarrow A \Rightarrow (B \text{ e } C)$.

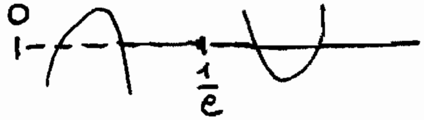
A	B	C	B e C	A ⟺ (B e C)	A ⟹ (B e C)
1	1	1	1	1	1
1	1	0	0	0	0
1	0	1	0	0	0
1	0	0	0	0	0
0	1	1	1	0	1
0	1	0	0	1	1
0	0	1	0	1	1
0	0	0	0	1	1

* Le due proposizioni NON sono logicamente equivalenti: Vedi la V Riga.

10) $f(x)=x\cdot\log^{2}x$. C.E.: $x>0$. $f'(x)=1\cdot\log^{2}x+x\cdot 2\log x\cdot\frac{1}{x}=\log^{2}x+2\log x$.

$f''(x)=2\log x\cdot\frac{1}{x}+2\cdot\frac{1}{x}=2\cdot\frac{1}{x}(\log x+1)$. $f''(x)\geq 0$ per $\log x+1\geq 0 \Rightarrow$

$\Rightarrow \log x\geq -1 \Rightarrow x\geq e^{-1}=\frac{1}{e}$.



f convessa in $]\frac{1}{e};+\infty[$.