adverse selection Varian, Microeconomics, 9^{th} ed. ch. 38

Let q be a scalar index signaling the quality of good X, and let x_A and x_B indicate the *quantities* of high quality (q = A), and low quality (q = B) units of good X that are traded in the market (where A > B). The quality q is known to the sellers, but it is not observable by the buyers. The fraction of high quality and low quality goods in the market are π_A, π_B, where $\pi_A + \pi_B = 1$ and $0 < \pi_A < 1$. The distribution (A, B; π_A, π_B) is common information. $p_A = 2000$ is the reservation price of buyers and sellers of X_A. $p_B = 1000$ is the reservation price of buyers and sellers of X_B.
These prices are common information. In a market equilibrium in which each unit of x (regardless of the quality) is sold at a single price p, the quantities x_A, x_B and the price p meet the following.....:

a.	$1000 < p < 2000$	with	$x_A > 0, x_B > 0$
b.	$p = 2000$	with	$x_A > 0, x_B > 0$
c.	$p = 1000$	with	$x_A > 0, x_B > 0$
d.	$1000 < p < 2000$	with	$x_A = 0, x_B > 0$
e.	$p = 1000$	with	$x_A = 0, x_B > 0$

The expected quality of x, for a buyer, is $Eq_x = (A\pi_A + B\pi_B)$ and the demand price for a unit of average quality X is $p_d = (2000\pi_A + 1000\pi_B)$, hence, $1000 < p^d < 2000$.
Since prices p_A, p_B and the distribution (A, B; π_A, π_B) are common information, the price p^d is common information as well.
The sellers of x_A are not willing to offer the high quality good at a price lower than 2000; therefore the good is withdrawn from the market.
This decision is correctly anticipated by buyers, consistently expecting that only quality B goods will be on sale. The reservation price for such goods is $p_B = 1000$. The correct answer is e.

Questions on hidden information, adverse selection, Varian, Microeconomics, 9th ed. ch. 38

1. In the second-hand market for mid-size cars there are high and low quality cars.
This information is known to buyers, but they are not in the position to discriminate, by superficial inspection, a good car from a bad one. Buyers are willing to pay up to 10.000 euro for a good quality car, and up to 4000 for a bad quality car. Sellers are willing to offer a good quality car at supply price €5000 and a bad quality car at supply price €2000. Assess whether good quality cars will be sold in the market, when it is common information the fraction of low quality cars is ½.

a) no, because adverse selection prevents any sale of good quality cars
b) yes, because the average price buyers are willing to pay is higher than 5000
c) no, because the average price buyers are willing to pay is lower than 5000
d) no, because the lowest price buyers are willing to pay is lower than 5000
e) none of the other answers is correct

2. Assume that buyers' reservation price for a high-quality cars is 5.000 euro, and
sellers reservation price for such cars is 4.000 euro. The Reservation price for low-quality cars is 2000 euro for buyers, and 1.800 euro for sellers. Identify the frequency range of high-quality cars causing adverse selection, in the presence of asymmetric information between buyers and sellers.
a) $\pi > 1/2$
b) $\pi > 2/3$
c) $\pi < 2/3$
d) $\pi < 1/2$
e) none of the other answers is correct

3. Suppose a university BA + master degree requires 5 years education *($h = 5$).*
Firms accept the degree as a quality signal and pay a monthly wage 5000€ to workers with the degree, and a monthly wage 2000€ to uneducated workers. Education costs depend on intrinsic skills that are independent of education. Education costs $C_A = 400h$, to agents with high intrinsic skills, and $C_B = 800h$, to agents with low intrinsic skills.
Assess whether the degree will be an efficient signal of agent's intrinsic skills.
a) yes, because unskilled agents have the incentive of getting the degree
b) no, because skilled agents have the incentive of getting the degree
c) yes, because only skilled agents have the incentive of getting the degree.
d) no, because unskilled agents do not have the incentive of getting the degree
e) none of the other answers is correct

4. Suppose a degree requires 3 years education *(h = 3)*. Firms accept the degree as a quality signal and pay wage 2.000 euro to degree holders, and wage 1100 euro to workers without the degree. Education costs are a linear function of h: they are $C_A = \alpha h$, for high-skill agents, and $C_B = \beta h$, for low-skill agents. Indicate which of the pairs (α, β) below meet the self-selection constraint: unskilled agents do not send the quality signal, and skilled agents do.

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a) α = 500 and β = 300
b) α = 300 and β = 300
c) α = 300 and β = 500
d) there are no values of α and β that allow you to meet the self-selection constraints
e) none of the other answers is correct

ANSWERS

1: B -

2: C - The constraint for the good quality car market existence is $\pi 5000 + (1 - \pi)2000 \geq 4000$, *which is satisfied for* $\pi \geq 2/3$. *So the market can not be efficient if* $\pi < 2/3$.

3: C - The constraints to be met so to affirm the self-selection ar e: $5000 - 400\,h \geq 2\,000$ *for subjects A and* $5000 - 800h \leq 2000$ *for subjects B. So to be convenient for A indicate high quality must apply* $h \leq 7.5$, *while no be not convenient for B indicate high quality must apply* $h \geq 3.75$.
A five years degree satisfies both constraints and ensure the signal equilibrium.

4: C -

Questions on moral hazard. **Varian, Microeconomics, 9th ed. ch. 38**

1.

Field crop y depends on cultivation effort x, according to $y = f(x)$, where $fx)$ is non-deterministic. In particular, effort x affects the chance of getting a high crop 2400, or a low one 1200. The variable x is binary, with x = 1 indicating maximum effort, and x = 0 indicating minimum effort.
If x = 1, then y = 2400 with probability $p_1 = ¾$ and y = 1200 with probability $(1 - p_1) = ¼$.
If x = 0, then y = 2400 with probability $p_0 = ¼$ and y = 1200 with probability $(1 - p_0) = 3/4$.
The owner P (the principal) wants to entrust cultivation to an agent A, and offers him a contract.
A's cultivation cost, in terms of disutility, is $c(x) = 500x$, and his best alternative to signing the contract, is getting a reservation utility $\underline{u} = 200$.
Both the principal P and the agent A are risk neutral. A's utility of a secure payment s is $u(s) = s$. If A's effort x cannot be monitored by P, which type of contract can maximize P's profit, and under what conditions?
a. Land rent
b. Wage labor contract

SOLUTION

Let's begin by observing that the expected crop is $E(f(x))$, where:
$E(f(1)) = ¾\ 2400 + ¼\ 1200 = 2100$
$E(f(0)) = ¼\ 2400 + ¾\ 1200 = 1500$
Let us call $s(y) = s(f(x))$ the payment received by A, when crop is $y = f(x)$.
P's problem is maximizing expected profit $E(\Pi_P) = E(f(x)) - E(s(f(x)))$, subject to A's *__participation constraint__: expected utility of the net payment $s(f(x) - c(x) \geq \underline{u}$. Since A is risk-neutral, we can write the participation constraint as $E(s(f(x))) - 500x \geq \underline{u}$.*
For any given x, profit is maximized when the constraint bites:
If x = 1, the participation constraint is $E(s(f(1))) = 500 + \underline{u} = 700$
If x = 0, the participation constraint is $E(s(f(0))) = \underline{u} = 200$

When A's participation constraint bites, P's expected profit is
$E(\Pi_P) = E(f(x)) - 700 = 2100 - 700 = 1400 \qquad if\ x = 1$
$E(\Pi_P) = E(f(0)) - 200 = 1500 - 200 = 1300 \qquad if\ x = 0$

The principal is better off if x =1, and seeks to provide incentives to x = 1. Let's see if and how P can succeed in this intent, in the two types of contracts mentioned above.

Wage labor:

A wage labor contract fixes an hourly remuneration W (salary), not according to effort x, but according to the amount of time spent at work. This contract is not suitable to supply agent A with the incentives to produce effort x = 1, rather than x = 0.
Wage labor does not meet the __incentive compatibility constraint__ that is necessary when effort is non observable, that is, is not suitable to prevent A's post-contract opportunism.

Rent:

We call R the __fixed__ amount of the rent paid by A to the principal P. Once paid R, the agent is the residual holder of the rights (residual claimant) of any revenue that may result from the crop.
Therefore $E(s(f(x))) = E(f(x)) - R$
The rent R chosen by P must first satisfy the participation constraint with strict equality:
$E(s(f(x))) - 500x = \underline{u}$ *cioè* : $E[f(x) - R] - 500x = \underline{u}$

Because P is interested in the effort x = 1,

R = $E(f(1)) - 500x - \underline{u}$ = 2100 – 500 – 200 = 1400. With rent *R = 1400, principal P secures himself the maximum expected profit he can draw from any contract voluntarily signed by A.*

Once paid R, agent A has to produce maximum effort x = 1, to avoid suffering expected losses.

A's expected utility is:

$EU_A(x = 1) = E(f(1)) - R - 500x$ = 2100 – 1400 – 500 = 200 = $\underline{u}$

$EU_A(x = 0) = E(f(0)) - R - 500x$ = 1500 – 1400 = 100 < $\underline{u}$

<u>Therefore the Rent contract is suitable to satisfying the incentive compatibility constraint. Social welfare is also maximized in this sense:</u>

$E(\Pi_P) + EUA = 1400 + \underline{u}$ *is maximized.*

Notice that because of risk neutrality, at R = 1400, the certainty equivalent CE of A's risky net payment s(f (1)) − c (1) is just $\underline{u}$*:* $u(CE) = EU[2400 - 500 - R, 1200 - 500 - R, ¾, ¼] = \underline{u}$.

$EU[2400 - 500 - 1400, 1200 - 500 - 1400, ¾, ¼] = E[500, -700, ¾, ¼] = 200 = \underline{u} = u(\underline{u})$*; hence* $CE = \underline{u}$

2.

Let's now modify one assumption in exercise 1, concerning A's attitude towards risk; it is now assumed that A is risk averse. We call R* the maximum amount of rent compatible with the signing of the contract by A, and with his effort being x = 1. Recalling that, in the case of a risk neutral agent, the amount of Rent R maximizing P's expected profit was R = 1400, you must now have:

a. R* > R

b. R* = R

c. R* < R

d. none possible answer

SOLUTION

A risk neutral agent A is indifferent between receiving a risky net payment s(f (x)) - c (x), or receiving a sure payment equal to the expected value E (s (f (x))) - c (x). In other words, if A is risk-neutral, with u(s) = s, the expected utility EU [s (f (x)) - c (x)] = E [s (f (x)) - c (x)]. If instead A is risk averse, the expected utility of the risky payment (f (x)) - c (x) is: EU [s (f (x)) - c (x)] < E [s (f (x)) - c (x)].

In a rent contract, the risk falls entirely upon the agent. Therefore, after signing the contract, he will certainly have the incentive to produce effort x = 1 rather than x = 0.

<u>The rent contract satisfies the incentive compatibility constraint.</u>

We have to see under what conditions the participation constraint is also satisfied at x = 1.

At x = 1, in case P was to fix an unchanged rent R = R = 1400, we would have:*

At x = 1, A's Expected revenue is : $E[s(f(1)) - c(1)] = 2100 - 1400 - 500 = 200 = \underline{u} >$

$> $ *A's expected utility at x = 1, that is :* $EU[2400 - 500 - R, 1200 - 500 - R, p_1, (1 - p_1)] =$

$¾\, u(500) + ¼\, u(-700)$.

<u>The rent R = 1400 does not satisfy the participation constraint, if A is risk averse</u>

To convince agent A to sign the contract, P must accord him a rent R < R , so that R – R* is just sufficient to cover A's risk. A's expected utility is now*

$EU_A[2400 - 500 - R^*, 1200 - 500 - R^*, ¾, ¼] = \underline{u}$.

Because now A is risk-averse,

$EU[2400 - 500 - R^*, 1200 - 500 - R^*, ¾, ¼] < E[2400 - 500 - R^*, 1200 - 500 - R^*, ¾, ¼]$

Because the participation constraint bites:

$\underline{u} = CE\,[2400 - 500 - R^*,\ 1200 - 500 - R^*,\ ¾,\ ¼]$, *that is*

$$200 = u\,(\underline{u}) = \quad EU\,[2400 - 500 - R^*,\ 1200 - 500 - R^*,\ ¾,\ ¼] <$$
$$< \quad E[2400 - 500 - R^*,\ 1200 - 500 - R^*,\ ¾,\ ¼]$$

At $R^* = R = 1400$, $E[2400 - 500 - R^*,\ 1200 - 500 - R^*,\ ¾,\ ¼] = 200$

Hence, $R^* < R$