QUESTIONS for the written test in microeconomics

Only one, of the five answers to each question, is correct.

Externalities and Public Goods (Varian chaps. 34 and 36)

1. Two roommates, A and B, consume hours, x, and a private good, y, (whose price is equal to 1). In the room where they live, five hours are jointly "consumed", either as hours of music, or as hours of silence, without bearing any cost, but listening to music implies giving up to silence. A likes listening to music, B likes silence, as indicated by their respective utility functions: $U_A\ (x_A, y) = 4\ x_A^{½} + y_A$, $U_B\ (x_B, y) = 2x_B^{½} + y_B$, where x_A indicates 'hours of music for A' and x_B indicates 'hours of silence for B'.
How many hours of music are optimal from the social point of view?
a) 1
b) 3
c) 4
d) it depends on which of the two roommates has the right to choose how to spend the hours "consumed" together
e) none of the other answers is correct

2. Two roommates, A and B, consume hours of music, M, and a private good, y, (whose price is equal to 1). The endowments of the private good are $y_A = 10$, $y_B = 10$. In the room where they live, five hours per day are "consumed" either as hours of music, or as hours of silence, but listening to music implies giving up to silence: $S + M = 5$. The right to choose the hours of music belongs to A, who likes music M, at least up to some extent. A wants to be paid, for switching the radio off. B likes silence S:
$U_A\ (M_A, y) = 5M_A - ½\ M_A^2 + y_A$; $U_B\ (S_B, y) = 2S_B + y_B$
How many hours of music are optimal from the social point of view?
a) 0
b) 5
c) 3
d) If B had the right to choose, the optimal M would be lower
e) none of the other answers is correct

3. *(extended solution on p. 5 below)* A steel-producing firm S and a sea bass farm F are carrying on their business around the same lake. Jointly with tons of steel A, firm S produces a variable quantity I of pollutants, damaging the quality of water in the lake, and thereby increasing the cost to firm F producing quantity P of fish.

The steel mill has total revenues and total costs: $R_S(A) = 20A$; $C_S(A, I) = A^2 + 10A - 10I + I^2$.

Firm F has total revenues and total costs: $R_F(P) = 14P$; $C_F\ (P, I) = P^2 + 2P + (1/4)I^2$.
Determine the optimal amount of pollution I, according to S's point of view, and according to the social point of view
a) 5, 0
b) 5, 4
c) 0, 4
d) 4, 4
e) none of the other answers are incorrect

4. With reference to the previous exercise, identify the pollution tax making the production I of pollutant socially optimal.
a) t = 10
b) t = 5
c) t = 2
d) t = 0
e) none of the other answers is correct

5. Firm 1 produces output x with cost function c_1 (x) x^2 + 10. Firm 2 produces output y with cost function $c_2 = (y^2 x)$. Firm's 1 output imposes external costs to firm 2. The two firms operate in perfectly competitive markets and the x good price is €20, the y good price is €40. If only these two companies operate, what is the efficient Pigou tax on good x?
a) 0
b) 1
c) 2
d) 3
e) none of the other answers is correct

6. A producer M of almonds (m) and a producer H of honey (h) operate in neighboring farms, giving rise to positive external effects in production. In particular, the output of each producer exerts external effects on the costs of the other. M has cost function C_M (m, h) = 16m + m^2 – 2h.
H has cost function C_H (m, h) = 8h + h^2 – 2m. The market prices of honey and almonds are given and constant: p_m = 32, p_h = 12. What are the quantities of m and h that are produced, if the two firms decide separately, and if they decide to internalize their external effects?
a) m = 8 h = 2; m = 7 h = 1
b) m = 8 h = 2; m = 8 h = 2
c) m = 8 h = 2; m = 9 h = 3
d) m = 7 h = 1; m = 8 h = 2
e) none of the other answers is correct

6a*. In the previous exercise, assume that the firms fail to reach an agreement to internalize their external effects. In order to encourage M and H to implement the socially efficient levels of m and h, the local government decides to introduce a S_m subsidy to M, for each unit of m produced, and a S_h subsidy to H, for each unit of h produced. Determine the total levels of the subsidies paid, namely mS_m and hS_h , to realize the Pareto optimal productions.

7. In a small country live 500 people who have the same preferences. There is only one private good and one public good. The utility of each agent *i* is $U(x, y_i) = 2x^{1/2} + y_i$, where y_i is the amount of the private good consumed by agent *i*, and *x* is the public good. If the price of the private good is 1 euro and that of the public good 20, then the optimal provision of the public good is:
a) 100
b) 400
c) 625
d) you cannot determine it, if income is unknown
e) none of the answers given is correct

8. In a small private country-road live two individuals, A and B, ensuring road lighting at their own expenses, and at a cost that depends on the number of street lamps L. The total cost is C(L) = 15L. The willingness to pay for the public good are $MRS_A = 30 - 2L$ e $MRS_B = 15 - L$. Determine the number of street lamps it is optimal to buy.
a) zero
b) 7,5
c) 10
d) 30
e) none of the other answers is correct.

9. Suppose that in the previous exercise, A and B meet to assess their respective contributions to the purchase of the lamp posts. The proposal is to split equally the overall cost. Determine if, in this hypothesis, the public good will be provided.
a) yes, because the sum of the reservation prices is equal to the cost
b) yes, because the sum of the contributions is equal to the cost
c) no, because the willingness to pay for the public good is greater than the contribution, for both A and B
d) no, because the willingness to pay for the public good is not at least as large as the contribution, for both A and B
e) none of the other answers is correct.

10.* Each of the 100 people living in a village has two opportunities to earn income with his daily work: working in the factory of the neighboring village at the daily wage w = 2, or going to fish. If X is the number of people who decide to go fishing, the amount of fish caught in the village's lake is: $Y = 10X - \frac{1}{2} X^2$. Knowing that the fish market price is p = 1, determine the number of fishermen X , when decisions are taken individually, and when decisions are taken collectively.
a) 0, 0
b) 8, 0
c) 0, 8
d) 16, 8
e) none of the other answers is correct

ANSWERS
1. C
2. C - The exercise is similar to the previous one. Because the hours of music are an externality, any choice of A has adverse effects on B's utility
3. B
4. C - To maximize profits the firm produces not only A but also the level of pollution (I) minimizing costs. The marginal costs compared to I are actually marginal benefits that finish when I=5 (MCI=0). If we take into account the negative external effects on the company that produces P, for example considering a unique property of the two companies, the optimal choice is produced by condition $MC^S_I + MC^F_I = 0$ *(recall that the price of pollution is 0), then I = 4.*
*5. B - The tax suggested by Pigou charges a price on the production of the negative externalities and is equal to the marginal cost of the injured part, so that t = MC2 (x *). Since MC2 is constant and equal to 1, t = 1 no matter what is the value of x *.*
6. C
7. C
8. C

9. D

10. D

The exercise illustrates the tragedy of commons. A non-regulated use of the common property gives to each individual an incentive to go fishing until the average revenue is greater than the average cost . A regulated fishing activity would impose that access to fishing is open until marginal revenue equals marginal cost .

In this exercise, the marginal / average cost is the opportunity cost of not going to work. Then the two values are 16 and 8. The resource lake is overused if everybody has unlimited access.

Private exchange in presence of negative externalities: water pollution.

A steel-producing firm S and a sea bass farm F are carrying on their business around the same lake. Jointly with tons of steel A, firm S produces a variable quantity I of pollutants, damaging the quality of water in the lake, and thereby increasing the cost to firm F producing quantity P of fish.

The steel mill has total revenues and total costs: $R_S(A) = 20A$; $C_S(A, I) = A^2 + 10A - 10I + I^2$.

Firm F has total revenues and total costs: $R_F(P) = 14P$; $C_F(P, I) = P^2 + 2P + (1/4)I^2$.

Determine the optimal amount of pollution I, according to S's point of view, and according to the social point of view.

Without regulation, the unit price of pollution I is zero. The steelworks optimal steel output is such that $MR^S_A = MC^S_A$, hence A = 5. Firm S chooses I such that $MR^S_I = 0 = MC^S_I$ =, hence $-10 + 2I = 0$, I = 5. Profits from pollution amount to 25 (area under the curve of marginal benefits from pollution).
Firm F cannot determine I, but simply observes the choice of the other company and undergoes a total additional cost of 25/2 because of the polluted water.

Suppose it is possible to assign property rights on lake's water use. Identify the amount of I produced in presence of a negotiation mechanism between the parties, with rights assignment to the steelworks S, and, alternatively, to fish company F.

If both firms S and F are are in the possession of a single owner, this would properly consider the external costs imposed on the fishing activity, and his choice would follow from the joint consideration of pollution benefits and costs. The owner has total revenues equal to R(A) + R(P) and total costs equal to C_S (A, I) + C_F (P, I); he chooses simultaneously A, P and I. Maximizing the joint profit function with respect to I, we obtain the condition $MC^S_I + MC^F_I = 0$. So $-10 + 2I + \frac{1}{2} I = 0$, that is, I = 4. This is the socially optimal amount of pollution.
Because no wealth effects are involved in the case of production, the Coase theorem tells us that, in the absence of transaction costs, I = 4 is also the amount of pollution that is determined by free negotiation between the parties, regardless of how the rights are initially allocated.

Production Externalities (with solution)

Firm 1 has total cost function $C^1 = y_1 + E^2 - 24E$, where E is the level of pollutant emissions. These increase the costs of firm 2, which is not in the position to control the level of E. Firm 2 has total cost function $C^2 = y_2 + \frac{1}{2} E^2$
Determine the private and Pareto optimal levels of emissions.

The Pareto optimal level of E can be identified through internalization of the externalities, that is, determining the value of E minimizing the total cost of the joint firm 1 + 2. The problem becomes:

Min $C^1 + C^2$ with respect to the variable E.
The 1st order condition for a local minimum is:
$dC^1 / dE + dC^2 / dE = 0$
$2E - 24 + E = 0$ $\quad 3E = 24$ $\quad E = 8$

The privately optimal level of E results from the condition: $dMC^1 / dE = MR^1_E = 0$, where the marginal revenue from pollution is zero, because the price of pollution is zero. This yields $2E - 24 = 0$, $E = 12$.

Consumption Externalities

1. Two consumers A and B are neighbors. Both have an income m = 120, that they spend to buy a good c, with market price is $p_c = 1$. In her spare time, A likes watching the landscape, while B likes hanging the sheets over the edge, but in so doing, she obstructs A's view. Let x^A be the number of hours spent by A watching the landscape, and x^B the number of hours B airs the sheets in the sun, and we assume $x^A + x^B = 12$.

The preferences of A and B are represented by $U^A = c_A + x_A^{1/2}$, $\quad U^B = c_B + 2x_B^{1/2}$.
a. If A and B reach a Pareto efficient solution, how many hours a day A can watch the landscape?

b. If the right to the panorama is assigned to A, what is the price of one hour of panorama in a competitive equilibrium?

c. If the rights to the panorama are assigned to A, but the preferences are:
$U^A = c_A^{1/2} \cdot x_A^{1/2}$, $U^B = c_B^{1/2} \cdot x_B^{1/2}$, what is the equilibrium price of an hour of panorama, and how many hours a day B airs her sheets?

Congestion in the private transport (Varian Ch. 34)

A local community consists of 1000 identical agents, that are interested in two goods, consumption m (with price p = 1) and leisure. To carry out leisure activities every individual moves by car, therefore we measure the consumption of leisure activities through hours of driving, d, and the price of leisure activities through the unit cost of driving hours, € 2 per hour. The weekly income of each individual is M = 500. The utility of each individual is $U(d, m) = 16d - d^2 + m$

How many hours does an agent drive? How many hours are spent driving in local community?

First of all we note that, driving hours, as they have been defined, are also a good, at least up to 8 hours of driving, when the marginal utility falls to zero, before becoming negative (too many leisure activities are tiresome!). Equalling MRS to the price ratio, we get the necessary condition for a non-corner solution to individual utility maximization: $16 - 2d = 2$ implies $d = 7$ (at these prices the optimum is indeed internal).

Individual consumption is $m = 500 - 14 = 486$ and $U = 486 + 112 - 49 = 549$.

In the community, because all individuals have the same preferences in terms of leisure activities and consumption, the total number of driving hours is $h = 1000d = 7000$.

However, excessive collective use of private cars creates disutility, due to congestion created on the roads. In particular, h causes disutility due to congestion resulting from hours of total driving, if all individuals behave in the same way. Taking congestion into account, individual utility is:

$U(d, h, m) = m + 16d - d^2 - 0{,}006h.$

What is the socially optimal number of hours-driving if each individual takes into account the negative externality from traffic congestion?

Because agents are identical, and h depends on d through $h = 1000d$, utility depends only on d and m, also in this case. After substitution for h from $h = 1000d$, individual utility is:

$U(d, m) = m + 16d - d^2 - 6d = m + 10d - d^2$.

Taking congestion into account, the individual choice $d = 7$, implies a lower actual utility: $U = 549 - 42 = 507$. If, on the contrary, individuals consider the congestion effects, the socially optimal choice is such that: $10-2d = 2$ that is $d = 4$.

In this case $U = 492 + 64 - 16 - 24 = 516$.

Individuals, on their own, do not reduce the use of driving hours, either because they do not believe that others will do the same, or because they do not properly perceive the h effects on their utility. As a result, they do not take h into account. The local government decides then to intervene with the aim of inducing individuals to consider congestion costs, and decides to impose a local tax on every hour of driving.

What is the level of a Pigou tax intended to induce individuals to take the negative externality into account?

We note that, if individuals do not correctly perceive congestion effects, their utility function is actually the initial one (otherwise they would drive only 4 hours). Then we have to work out how to impose the tax. If the tax is imposed on d, the price of good d becomes 2 + t. By definition, the Pigou tax must be such as to generate the socially optimal number of hours/drive $d = 4$. It follows that t must be such that the first order condition $16 - 2d = 2 + t$ finds its solution at $d = 4$. This implies $t = 6$. We note that, also in this case $U = 516$.

The system of traffic control called road pricing*, used in northern Europe towns and more recently also in Milan, is based on the same criterion outlined in this exercise: the idea is of indicating through a price increase (given by the tax) that the consumption of a particular good has negative external effects, but of leaving to the individuals the choice of consumption.*