Solution example of the written exam of 08/01/2020

whatever, will not be considered. **For the 2nd partial test, answer only to questions: 1a, 1b, 2a, 2b, 3a, 3b, 4a, 4b.**

1a. A loan contract of size L = 250 includes a limited-liability clause, and prescribes that the risk neutral borrower invests L in a project A yielding cash-flow 1000 with success probability $\alpha_A = 0.7$ and zero in case of failure. The risk neutral lender wants to avoid that after the contract the borrower invests L in project B yielding cash-flow 1200 with success probability $\alpha_B = 0.5$ and zero in case of failure. Risk-free interest rate is $r = 0.1$. Discuss **i)** lender's participation constraint and **ii)** borrower's incentive-compatibility constraint.

EV_A = EXPECTED CASH FLOW OF A $= 0.7 \cdot 1000 = 700$ $\quad \alpha_A = 0.7 \quad \alpha_B = 0.5$

EV_B = " " " " B $= 0.5 \cdot 1200 = 600$ $\quad r = 0.1$

(i) Lender's Participation Constraint $(1+r)L \leq \alpha_A (1+r_A) L$

$$(1+r_A) \geq \frac{1+r}{\alpha_A} = \frac{1.1}{0.7} \Rightarrow 1.571$$

(ii) Borrower's incentive compatibility constraint: $\max(1+r_A) \leq \dfrac{EV_A - EV_B}{(\alpha_A - \alpha_B) L}$

$$= \frac{700-600}{(0.7-0.5)250} = \frac{100}{(0.2)250} = \frac{100}{50} \Rightarrow 2$$

1b. Anna's wealth consists of a lottery ticket L, paying prize L1 = 1200 with probability 1/2, and prize L2 = 600 with probability 1/2. Her utility function for sure wealth is $U^A = 2c$, and her preference for contingent consumption satisfies the expected utility property. Determine the certainty equivalent of L.

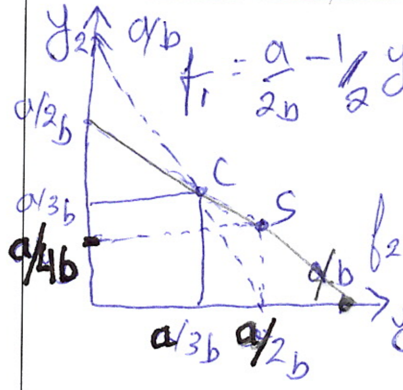
a. 1000
b. 900
c. 700
d. 600
e. every other answer is wrong

$$EU(L) = \tfrac{1}{2} \cdot 2 \cdot 1200 + \tfrac{1}{2} \cdot 2 \cdot 600 = 2E(L) = U(E(L))$$

This is because Anna is risk neutral (she has linear utility function)

$$\rightarrow CE(L) = E(L) = \tfrac{1}{2} \cdot 1200 + \tfrac{1}{2} \cdot 600 = 900$$

2a. In a duopoly with inverse demand $p = a - by$, firms 1 and 2 have zero marginal cost. Explain why Stackelberg equilibrium with quantity leadership yields a higher industry output than Cournot equilibrium.



$f_1 = \frac{a}{2b} - \frac{1}{2} y_2^e$

$f_2 = \frac{a}{2b} - \frac{1}{2} y_1^e$

f_1 and f_2 are the Cournot best reactions of firms 1 and 2

At Cournot equilibrium 1 and 2 best react and $y_1^e = y_1 = y_2^e = y_2$

Cournot: $y_1 = y_2 = \frac{1}{3}\frac{a}{b}$ $\quad y_c = \frac{2}{3}\frac{a}{b}$

stackelberg: leader $y_1 = \frac{1}{2}\frac{a}{b}$

follower $y_2 = f_2(y_1) = \frac{a}{4b}$

$y_S = \frac{3}{4}\frac{a}{b}$

2b. A competitive industry has market demand $y = 3060 - 2p$. Each firm has cost function $C(0) = 0$; $C(y) = 225 + y^2$, if y > 0. Determine the long-period number n of firms in the industry.

a. 200
b. 150
c. 100
d. 85
e. 80
f. Every other answer is wrong

$AC = \frac{225}{y} + y$ $MC = 2y$

$\frac{225}{y} + y = 2y$

$\frac{225}{y} = y \Rightarrow y^2 = 225$

$y = 15$

$p = MC = 15 \cdot 2 = 30$

$y = 3060 - 60 = 3000$

n° of firms $= \frac{3000}{15} = 200$

3a. Explain what is a 'public good'. Consider an economy in which 2 agents A, B consume public good G and/or private good x with market prices $p_G = 1$, $p_x = 2$. A and B have utility $u_A(G, x) = 8logG + x$, $u_B(G, x) = 12logG + x$. Determine contributions g_A, g_B supporting the Pareto efficient quantity G*.

A public good is a good characterized by the properties of non excludability and non rivalry. ~~it is equally distributed to the~~
It is usually given in the same amount to all users.

$p_G = 1$
$p_x = 2$

$g_A, g_B = ?$

$\frac{8}{G} + \frac{12}{G} = \frac{1}{2} = \frac{p_G}{p_x}$

$(g_A + g_B) = G \cdot p_G$

$(g_A + g_B) = 40$

$\frac{20}{G} = \frac{1}{2}$

$\frac{G}{20} = 2$

$G^* = 40$

$p_x(8 + 12) = p_G \cdot 40 = 40$

$g_A = 16$ $g_B = 24$

3b. Agent A wealth is $W_g = 24000$ if state g occurs, and $W_b = 0$ if state b occurs. State b and g occur with probability ¼ and ¾, respectively. A can ensure wealth K ($0 \leq K \leq 24000$) paying an insurance premium γK, where the premium per unit insurance is $\gamma = 1/3$. A's utility for non-contingent consumption is $u_A(c) = \log c$. A's preferences for contingent consumption satisfy the expected utility property. Explain why the amount of insurance K chosen by A is equal to:

a) K = 0
b) K = 12000
c) K = 18000
d) K = 24000
e) none of the other answers

$W_g = 24.000$ $\pi_g = 1/4$
$W_b = 0$ $\pi_b = 3/4$

damage D = 24000

$\gamma = 1/3$ NOT FAIR PREMIUM

K = ?

$u_A(c) = \log C \rightarrow$ RISK AVERSE because it is a CONCAVE function.

$u'(c)$ is decreasing

Since the premium is not fair,
Since the agent is risk averse, we know he will not buy full insurance. K < D

$\frac{u'_b}{u'_g} \frac{\pi_b}{\pi_g} = \frac{C_g}{C_b}$ $\frac{1/4}{3/4} = \gamma/1-\gamma = 1/2$ $\frac{1/4}{3/4}\frac{C_g}{C_b} = \frac{1}{3}\frac{C_g}{C_b} = \frac{1}{2}$

$C_g = \frac{3}{2} C_b = K$

$W_g \cdot \frac{3}{4} = 18.000$

$C_g = 24000 - \frac{1}{3}K$

$C_b = K - \frac{1}{3}K = \frac{2}{3}K$

24000 = K + (1/3)K = (4/3)K

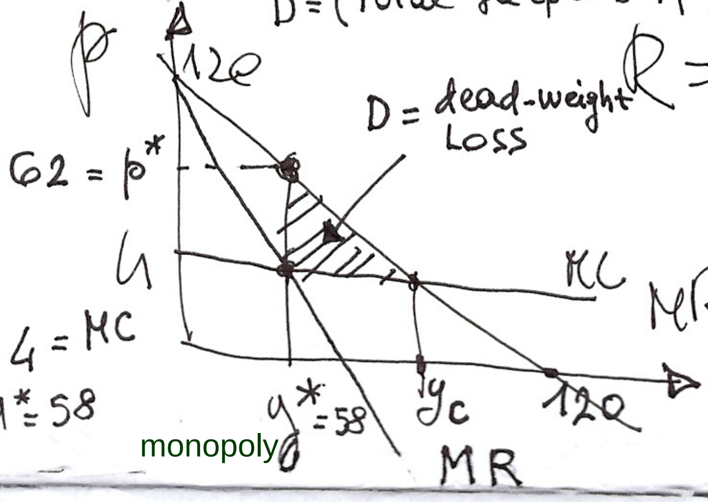
K = 24000 (3/4) = 18000

if $p = MC = 4$ then output is :

$4 = 120 - y_c$ $\quad y_c = 116$ $\quad p = 120 - y$

4a. Discuss the notion of monopoly dead-weight loss and provide a graphical representation referring to a monopolist with cost function $C(y) = 4y$ and market demand $y = 120 - p$.

$D = (\text{Total Surplus if } p = MC) - (\text{Total Surplus if } MR = MC)$



$R = y \cdot p = (120 - y)\, y$

$R = 120y - y^2$

$MR = 120 - 2y$

$\Pi = R - C$

$\Pi = R - 4y$

$MR = 120 - 2y = 4 = MC$

$2y = 116 \quad y^* = 58$

4b. A competitive firm has production function $y = x_1^{1/2} x_2^{1/2}$. Factor prices are $w_1 = 2$, $w_2 = 8$, respectively. Determine the cost function C(y)

a) C(y) = 12y
b) C(y) = 8y
c) C(y) = 4y
d) C(y) = 6y
e) every other answer is wrong

first order condition of cost minimization with interior solution is $|TRS| = w1 / w2$

$|TRS| = \frac{w_1}{w_2} = \frac{\frac{1}{2}}{\frac{1}{2}} \cdot \frac{x_2}{x_1} = \frac{2}{8} \Rightarrow \boxed{x_1 = 4x_2}$

$y = (4x_2)^{1/2} (x_2)^{1/2} = 2x_2 \Rightarrow x_2 = \frac{1}{2} y$

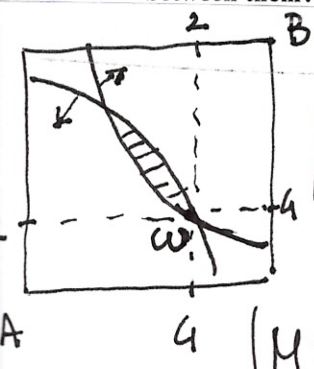
$x_1 = 2y$

Cost $= x_1 w_1 + x_2 w_2 =$

$C(y) = 2y\,(2) + (\tfrac{1}{2}y)(8) = 4y + 4y = 8y$

$\text{Cost} = x_1 W_1 + x_2 W_2 = 2y \cdot 2 + \tfrac{1}{2} y \cdot 8 = 8y$

5a. A's and B's preferences for consumption bundles (x, y) are represented by $u_A = x_A^{1/2} y_A^{1/2}$ and $u_B = x_B^{1/2} y_B^{1/2}$. Their endowments of goods are (4, 2) for A and (2, 4) for B. Do agents A and B have an incentive to exchange goods between them? Describe the situation using Edgeworth's box.



$|MRS_A| = \frac{y_A}{x_A} = \frac{2}{4} = \frac{1}{2}$; $|MRS_B| = \frac{y_B}{x_B} = \frac{4}{2} = 2$

$|MRS_A| \neq |MRS_B| \rightarrow$ The allocation ω is not Pareto effic.

$\omega = \begin{cases} (x_A, y_A) = (4, 2) \\ (x_B, y_2) = (2, 4) \end{cases}$

ω is not Pareto efficient.

Since W is not Pareto efficient, A and B have an incentive to trade at ω.

5b. A consumer has utility function u(x, y) = 6x + 3y. Her income is $m = 90$ and prices are $(p_x, p_y) = [1, 1]$. If price p_x increases at $p_x = 3$ (with price p_y fixed at $p_y = 1$), what is the substitution effect for good x?

a) $\Delta x^s = +60$
b) $\Delta x^s = -60$
c) $\Delta x^s = -90$
d) $\Delta x^s = -45$
e) it is equal to the income effect
f) every other answer is wrong

The utility function shows x and y are perfect substitutes so when price of x increases the consumers substitute it with cheaper good (y).

at initial prices (1, 1) the consumer buys only good x.

$|MRS| = \frac{MU_x}{MU_y} = \frac{6}{3} = 2 > \frac{P_x}{P_y} = \frac{1}{1}$

initial demand $\bar{X} = \frac{m}{P_x} = \frac{90}{1} = 90$

at final prices (3, 1) the consumer buys only good y

with perfect substitutes, the substitution effect explains the entire demand change $\Delta X^S = X - \bar{X} = -90$

$2 = |MRS| < \frac{P_x'}{P_y} = \frac{3}{1} = 3 \rightarrow X = 0$ final demand

6a. Explain why, if consumer's money income is given, a *Giffen good* is necessarily an *inferior good*, but the reverse may not hold.

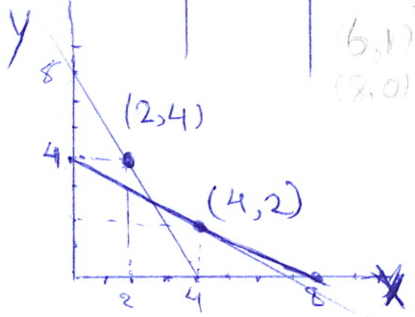
Slutsky equation $\frac{\Delta x_1}{\Delta P_1} = \frac{\Delta x_1^S}{\Delta P_1} - \frac{\Delta x_1^m}{\Delta m} \cdot \bar{x} < 0$ for normal goods.

with given income based on the law of demand we know that when price increases, demand falls. but for giffen goods it doesn't apply (with giffen goods, ~~are~~ when price increases, demand also increases) so ~~giffen~~ giffen goods cant be normal so they are inferior goods. Because the substitution effect is always negative, an inferior good may be an ordinary good and it happens when substitution effect is greater than income effect.

6b. A consumer buys only two goods, x and y. At prices $(p_x, p_y) = [2,1]$ she chooses the basket (2, 4). At prices $(p_x, p_y) = [1, 2]$ she chooses the basket (4, 2). We can say that:

a) consumer's choices violate the weak axiom of revealed preference (WARP)
b) consumer's choices do not violate the weak axiom of revealed preference
c) without knowing preferences we cannot say if consumer's choices violate WARP
d) none of the other statements is correct

	bundle (2,4)	(4,2)
price (2,1)	4+4 8	8+2 10
(1,2)	2+8 10	8



The consumer chose bundle (4,2) with price (1,2) when bundle (2,4) was (more expensive) not affordable. So he didn't violate WARP.