origin of Keynesian approach to Growth

can be traced back to an article written after the *General Theory (1936)*:
Roy Harrod, "An Essay in Dynamic Theory", *Economic Journal*, 1939

Theoretical premises of the Keynesian approach

- ***uncertainty and expectations*** play a role in investment decisions
- ***nominal rigidities*** ***(money wage and prices)*** follow from institutional rules forged by social selection*:* more instability with flexible prices and wages
- ***under-employment equilibria:*** capitalistic economies normally operate at less than full employment of labor and at less than full utilization of capacity. Under-employment may be disguised by dualistic labor market.
- ***quantity adjustment*** ***(***Keynesian income multiplier): output is demand-constrained, and an increase in demand is met by a more efficient use of existing resources.

Nominal rigidities and desired K* / Y

- *one good economy: price of output* $P_y = 1$
- *competitive firms minimize costs at* *given money prices* *w, r*
- *constant returns to scale technology*

  L ≡ employment ≤ labor supply at money wage w

  K ≡ existing capital stock from past entrepreneurial decisions

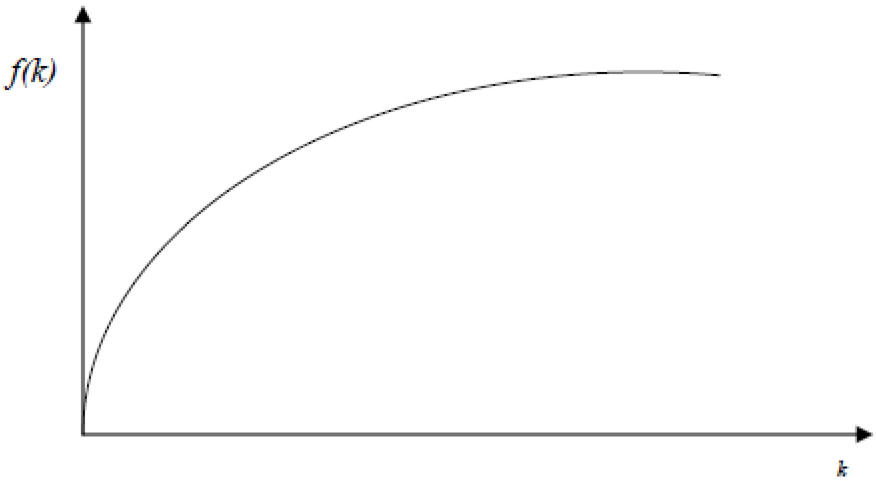
  $Y = F(K, L) \qquad aY = F(aK, aL) \qquad a > 0$

Intensive production function

$$y \equiv Y / Lk \equiv K / L$$

$$y = F(k, 1) \equiv f(k) \qquad f'(k) = \partial F(K, L) / \partial K \quad at \;\; k = K/L$$

$$f'(k) > 0 \qquad f''(k) < 0$$



f′(k) decreasing function of k →   f(k)/k decreasing function of k

Constant returns to scale implies:

$\partial Y / \partial K$ is well defined decreasing function of k

$\partial Y / \partial L$ is well defined increasing function of k

cost minimizing (desired) k

$\rightarrow \partial Y / \partial K = f'(k) = r$ *(assume depreciation δ=0)*

$\rightarrow \partial Y / \partial L = f(k) - k \cdot f'(k) = w$

Price rigidities

Suppose r, w are fixed

→ *cost-minimizing k is fixed*

→ *cost-minimizing y = f(k) is fixed*

→ *k / f(k) ≡ desired capital output ratio is fixed!*

K / Y = k / y → *K / Y is fixed by prices*

K / Y ≡ v ≡ desired ratio between capital stock and output

Alternative interpretation of fixed Capital / Output ratio v

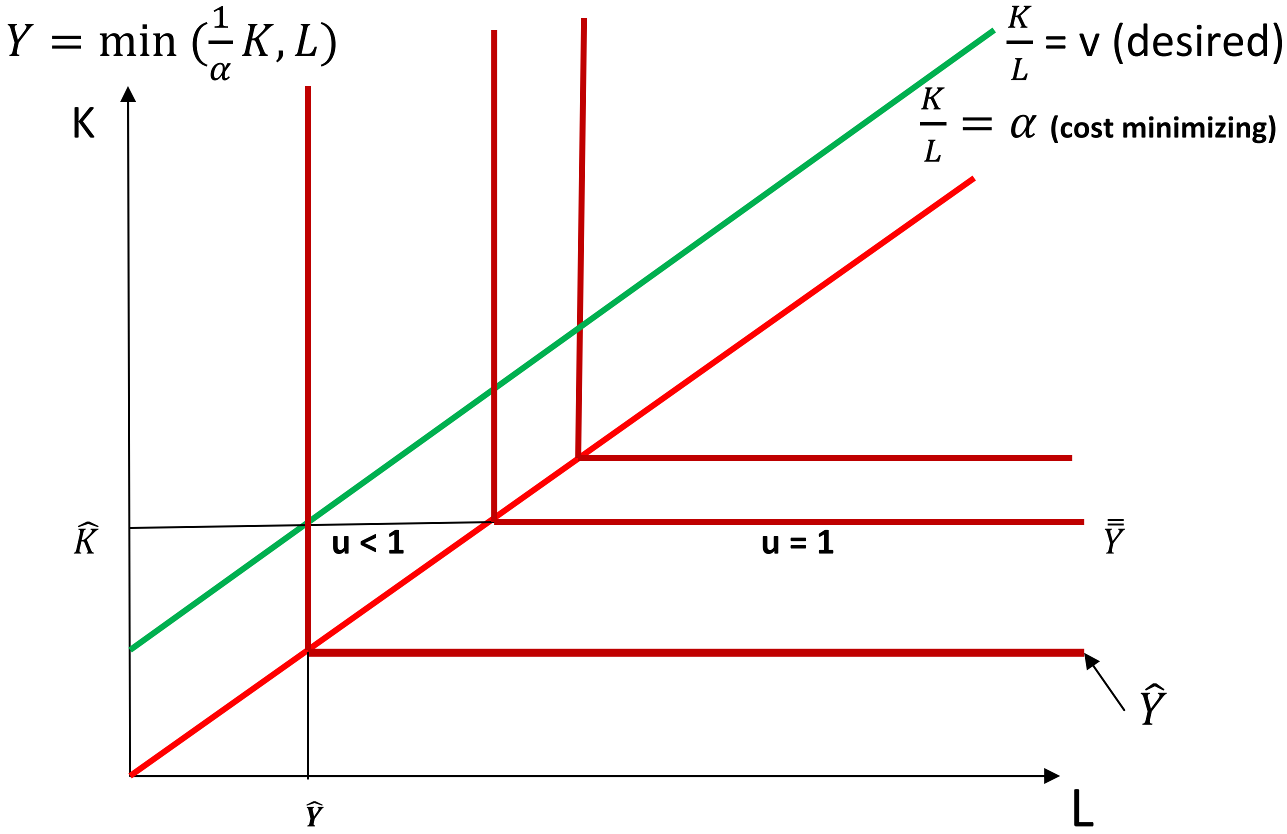
- Modern technologies are Leontief type: $Y \min (\frac{1}{\alpha}K, L)$
- *Static efficiency (cost minimization)* $\rightarrow \frac{K}{L} = \alpha \qquad K = \alpha L$
- $L = \frac{1}{\alpha}K$
- $Y_K = \frac{1}{\alpha}K$ *full capacity output*

  Static efficiency: $K / Y_K = \alpha$ *cost minimizing* K/Y
- *Firms need prompt response to unexpected peaks in demand: they plan* ***desired capacity utilization*** $u_n < 1$
- ***Capacity utilization*** $= \frac{Y}{Y_K} = u \quad u = 1$ *implies* $Y = L = \frac{1}{\alpha}K$
- *Uncertainty + fast adjustment (dynamic efficiency) require:*

  Desired *capacity utilization* $u_n < 1$ *implies*

  Desired $\frac{K}{Y} = \frac{\alpha}{u_n} = v > \alpha$

  Remark: because $Y = L$, *desired* $\frac{K}{L}$ = *desired* $\frac{K}{Y} = v$



If firms expect otput $\hat{Y}$, they wish and plan to have capital sock $\hat{K}$, such that $\frac{K}{L} = \frac{K}{Y} > \alpha$. In this way, the buy a margin of flexibility, and can meet an unexpected peak in output up to $\bar{\bar{Y}}$.

Harrod (1939) Macroeconomic growth model

1 good economy for simplicity

Part 1. explain *equilibrium growth path* of the economy defined as the growth path realizing:

- macroeconomic equilibrium in the good market
  (agg. demand = agg. supply $\leftrightarrow$ I = S) at every $t > 0$
- Capacity produced by investment is utilized at desired rate $\boldsymbol{u_n}$
- Demand expectations supporting investments are fulfilled

Building blocks:

1. Savings

$S_t = s\ Y_t$ s ≡ marginal and average savings rate (1)

2. Keynesian expenditure and income multiplier

$sY_t = I_t \quad It \equiv K_{t+1} - K_t$ = investment demand at time t (2)

K = capital stock

Here causality runs from investent decisions I to Y.
Macroeconomic equilibrium on output market is the outcome of quantity adjustment at rigid prices

3. Investment accelerator

v = desired ratio between capital stock and output planned by firms

Y^e_{t+1} = *expectation on demand at t+1, formulated at t*

$K_{t+1} = vY^e_{t+1}$

$K_t = vY^e_t$

$$I_t = K_{t+1} - K_t = vY^e_{t+1} - vY^e_t = v(Y^e_{t+1} - Y^e_t) \quad (3)$$

Here we abstract from depreciation: investment expenditure = net invetment

4. Endogenous variable: growth expectations g^e_t

$g^e_t \equiv (Y^e_{t+1} - Y^e_t) / Y^e_t$ growth rate of expected demand

'expected growth' is fixed by the state of entrepreneurial long-term expectations

$$Y^e_{t+1} = Y^e_t (1 + g^e_t) \quad (4)$$

$$(Y^e_{t+1} - Y^e_t) = Y^e_t (1 + g^e_t) - Y^e_t = Y^e_t g^e_t \quad (4')$$

Substituting for $(Y^e_{t+1} - Y^e_t)$ in $I_t = v(Y^e_{t+1} - Y^e_t)$ (3)

5. Investment as determined by growth expectations

$$I_t = vY^e_t g^e_t \quad (5)$$

6. Investments as determinants of current effective demand

$$vY^e_t\, g^e_t = I_t = s\, Y_t \qquad (6)$$

expectations at t - 1 concerning Yt produced It - 1, hence Kt .

Interpretation: at time t, K_t is pre-determined and g^e_t is our endogenous variable. For every given g^e_t a different I_t *materializes*

$$K_t\, g^e_t = vY^e_t\, g^e_t \qquad \rightarrow I_t \qquad \rightarrow Y_t$$

- Notice that entrepreneurs take their investment decisions I_t an instant before income Y_t is realized

7. Equilibrium growth: ‘*equilibrium is a state in which agents are not induced to revise their decisions*’.

fulfilled predictions : $Y^e_t = Y_t$ (6)

desired investment $vY^e_t\, g^e_t = I_t = s\, Y_t$ *desired saving* (7)

Substituting from (6) into (7):

$vY_t\, g^e_t = s\, Y_t$

$vg^e_t = s$

$g^e_t = s / v \equiv g^*$ this is Harrod's WARRANTED RATE OF GROWTH: equilibrium on output market + K/Y = v = desired ratio

g^* is the ‘*equilibrium value*’ of g^e_t it is the unique expected growth rate of demand leading to self-fulfilling predictions

Compare Harrod's warranted path with Solow's equilibrium path:

Harrod: $\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{s}{v}$ $\quad$ *v = K/Y = constant,*

$$\frac{s}{v} \neq n \text{ in general, because } v \text{ is fixed}$$

Solow: $\frac{Y}{L} = y = k^{\alpha}$ $\quad$ δ = 0 for simplicity $\quad$ v = k / f(k) is variable depends on K, L

$$\frac{\dot{K}}{K} = \frac{\dot{k}}{k} + n = \left(\frac{sk^{\alpha}}{k} - n\right) + n = \frac{sk^{\alpha}}{k} = \frac{s}{v}$$

in Solow at any date t the growth rate of K is s/v where v is fixed by factor supply K, L

F(K,L)=Y gives $v = \frac{K}{Y} = \frac{k}{f(k)} = \frac{k}{k^{\alpha}}$ $\quad$ *in steady state,* $\frac{\dot{K}}{K} = \frac{s}{v} = n$

1. **v is flexible and is determined fixed xxxx by factor supply + full employment**
2. **Through capital accumulation, and decreasing MPK, in the long-run v adjusts to the effect that s/v = n**

Harrod's Proposition 1. The unique growth rate of expected demand consistent with equilibrium in the goods market and fulfilled entrepreneurs' expectations is:

$$g^* = s / v$$

if $g^e_t \neq g^*$ *then predictions are not self-fulfilling …*

$$g^e_t \neq g_t$$

Interpretation:

- If the economy is on a warranted 'equilibrium' path, at t entrepreneurs are happy with past investment decisions giving rise to the existing stock K_t.

- This implies that Y_t is such that $v\ Y_t = K_t$

- Because Y_t depends on current investment (through the income multiplier), and investment depends on growth expectations, equilibrium requires that entrepreneurs expectations at time t are such that $g^e{}_t = g^* \equiv s / v$

Part 2. Study the dynamic stability of *warranted* growth path

actual growth rate $g_{t-1} \equiv (Y_t - Y_{t-1}) / Y_{t-1}$ *will materialize at end of t*

Assume the adjustment rule:

$$\begin{aligned} g_{t-1} > g^e_{t-1} &\quad\rightarrow\quad g^e_t > g^e_{t-1} \\ g_{t-1} < g^e_{t-1} &\quad\rightarrow\quad g^e_t < g^e_{t-1} \\ g_{t-1} = g^e_{t-1} &\quad\rightarrow\quad g^e_t = g^e_{t-1} \end{aligned} \tag{8}$$

- **Harrod's expectation formation:**

1.

uncertainty surrounding long-term prospects is very high

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entrepreneurs are forced to consider only immediate prospects

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Immediate prospects are closely linked to recent observations

2.

Harrod claimed that his result did not depend on the 'lag structure'

Remark:

if at *(t – 1)* the economy is on the warranted path (expectations fulfilled)

$$\downarrow$$

$$g_{t-1} = g^e_{t-1} = g^* \quad \rightarrow \quad g^* = g^e_{t-1} = g^e_t = g_t$$

$$\downarrow$$

Without shocks, the economy will be on the warranted path for 'ever'

But what happens if there are 'small' exogenous changes in expectations?

Dynamic-instability: problem set up

- Suppose at time t – 1 the economy is on the warranted path so that

$$Y_{t-1} = Y^e_{t-1} \qquad (9)$$
$$g^e_{t-1} = g^e_{t-1} = g^*$$

- for some reason (a discovery, exogenous change in 'animal spirits'…) at time t entrepreneurs become more optimistic, and expect increase of the growth rate:

$$g^e_t > g^e_{t-1} = g^* \qquad (10)$$

● **Harrod's instability proposition:**

the chain of events following from $g^e_t > g^*$ will induce growth expectations to be revised in the wrong direction

Interpretation:

Initial condition: $g^e_{t-1} = g^* \rightarrow \quad Y_{t-1} = Y^*_{t-1}$

Exogenous shock on expectations at t: $g^e_t > g^*$

$$\downarrow$$

$$I_t > I^*_t$$

$$\downarrow$$

$$Y_t > Y^*_t$$

$$\downarrow$$

$$(Y_t / Y_{t-1}) - 1 \equiv g_t > g^e_t > g^*$$

$$\downarrow$$

$$g^e_{t+1} > g^e_t > g^*$$

g^e_t is revised in the 'wrong' direction

Proof

from 6 $$I_t = vY^e_t\, g^e_t = s\ Y_t$$

from 4 $$Y^e_t = Y^e_{t-1}\,(1+g^e_{t-1}) = Y^e_{t-1}\,(1+g^*)$$

using 9 $$s\ Y_t = v\, g^e_t\, Y^e_{t-1}\,(1+g^*) = v\, g^e_t\, Y_{t-1}\,(1+g^*) \qquad (11)$$

from 11 $$(1+g_t) = Y_t / Y_{t-1} = (v/s)\,(1+g^*)\, g^e_t = (g^e_t + g^e_t\, g^*)\,(v/s) =$$
$$= (g^e_t + g^e_t\, g^*)/g^* = (g^e_t/g^*) + g^e_t \qquad (12)$$

from 12 and 10 $$(1+g_t) = (g^e_t/g^*) + g^e_t > 1 + g^e_t$$

Harrod's Proposition 2:

Under the adjustment rule (8), the warranted growth rate g^* is dynamically unstable: *the warranted path is a 'knife edge'*.

$g^e_t < g^* \rightarrow$ cumulative departure from sustained growth towards depression

$g^e_t > g^* \rightarrow$ cumulative departure from sustained growth towards full employment

working population grows at rate *n*: *natural growth rate*

no endogenous economic force drives *g** towards *n*, or vice versa

Remark: in Solow the equilibrium growth rate g= s/v converges to n (steady state growth without technological progress) through price flexibility and changes in K/L.*

Harrod's Proposition 3: Persistent growth, at a constant rate trough time, can take place only if the following conditions obtain:

$$g^e{}_t = g^* \equiv s / v \qquad\qquad all \quad t$$
$$g^* = n$$

Corollary: As a result of the fact that:

- expectations are made under strong uncertainty
- in general $g^* \neq n$

sustained growth over extended periods of time can only result from appropriate measures of economic policy.

Harrod growth model with autonomous government expenditure

Hicks (1950) adds ‘autonomous’ investment expenditure I^A

Assume I^A growing at exogenous constant rate g^A

$$I(t) = \text{induced investment} + \text{autonomous investment}$$

$$I(t) \quad = \quad vg^e \, Y^e(t) + I^A(t)$$

Remark on autonomous expenditure:

- it does not interfere with firms’ investment decisions: autonomous expenditure is not ‘capacity creating’
- Generalizing Hicks 1950, autonomous expenditure is any source of demand that, <u>unlike induced investment</u>, is <u>relatively independent of short run capacity utilization</u>. Examples are exports, part of R&D expenditure, government expenditure, consumption financed by consumer credit…

induced investment + autonomous investment

Investment : $I(t) = vg^e Y^e(t) + I^A(t)$

Fulfilled predictions on Y : $Y(t) = Y^e(t)$ $g^e = g$

Income multiplier : $sY(t) \equiv Y(t)(1 - c) = I(t) = vg\, Y(t) + I^A(t)$

$$Y(t)(1 - c - vg) = I^A(t)$$

$$Y(t) = \frac{1}{(1-c-vg)} I^A(t)$$

$$(1 - c - vg) > 0$$

on a fulfilled-predictions path:

$$Y(t) = \frac{1}{(1 - c - vg)} I^A(t)$$

On a growth path such that g = constant, $\frac{1}{(1-c-vg)}$ *= constant*

The growth rate of the right-hand side is g^A. Thus:

$$g = \frac{\dot{Y}}{Y} = g^A$$

The exogenous growth rate of autonomous expenditure determines the equilibrium growth rate of the economy.

Conclusions 1.

● *If equilibrium path is dynamically stable, or there are bounded fluctuations around the equilibrium path:*

● *long-run growth determined by growth of autonomous expenditure*

● *in the long run g^A influenced by population growth (infrastructure)*

Harrod (1951) criticized Hick's notion of 'autonomous expenditure'

He argued that in the long run every expenditure is endogenous.

He neglects those sources of expenditure that are scarcely if at all affected by short-run capacity utilization. Examples are basic R&D, household expenditure financed by mortgages (housing investment) and consumer credit, government expenditure.

Conclusions 2.

Boundedness of fluctuations around equilibrium path may result from:

- *income ceiling* of full employment
- *income floor* supported by infrastructure autonomous investment related to population growth and other forms of autonomous expenditure
- *non-linear adjustment behavior*

in the very long run:

- actual growth rate g is influenced by n (natural growth rate)
- but underemployment equilibria may be a normal state of affairs

↓

extensions

- wage and price dynamics may be relevant in the long run

- autonomous expenditure can be made endogenous.

  example: Kaldor Export-led model