The neoclassical model of economic growth

- Robert Solow (1956)
- Trevor Swan (1956)

  Give rise to the 'Solow-Swan model'

premises

- Closed economy with 1 final output Y
- Exogenous labor supply L
- Initial physical capital stock K_0

- At every date t, equilibrium in all markets: Y, L, K

Equilibrum may be interpreted as:

- outcome of flexible competitive prices
- outcome of active policy intervention

The neoclassical production function

K = capital L = labor A = technology

$$Y = F(K, L, A)$$

Assumption 1

- $F(0, L, A) = F(K, 0, A) = 0$ K and L are necessary
- $F(\lambda K, \lambda L, A) = \lambda Y$ constant returns to the scale of K, L
- $F_K > 0$ $F_{KK} < 0$ deacreasing marginal product of K
- $F_L > 0$ $F_{LL} < 0$ deacreasing marginal product of L

ASSUMPTION 2. *(Inada Conditions) F satisfies the Inada conditions*

$$\lim_{K\to 0} F_K(K,L,A) = \infty \text{ and } \lim_{K\to\infty} F_K(K,L,A) = 0 \text{ for all } L>0 \text{ and all } A$$
$$\lim_{L\to 0} F_L(K,L,A) = \infty \text{ and } \lim_{L\to\infty} F_L(K,L,A) = 0 \text{ for all } K>0 \text{ and all } A.$$

MPK = F_K = slope of (F(K), K)

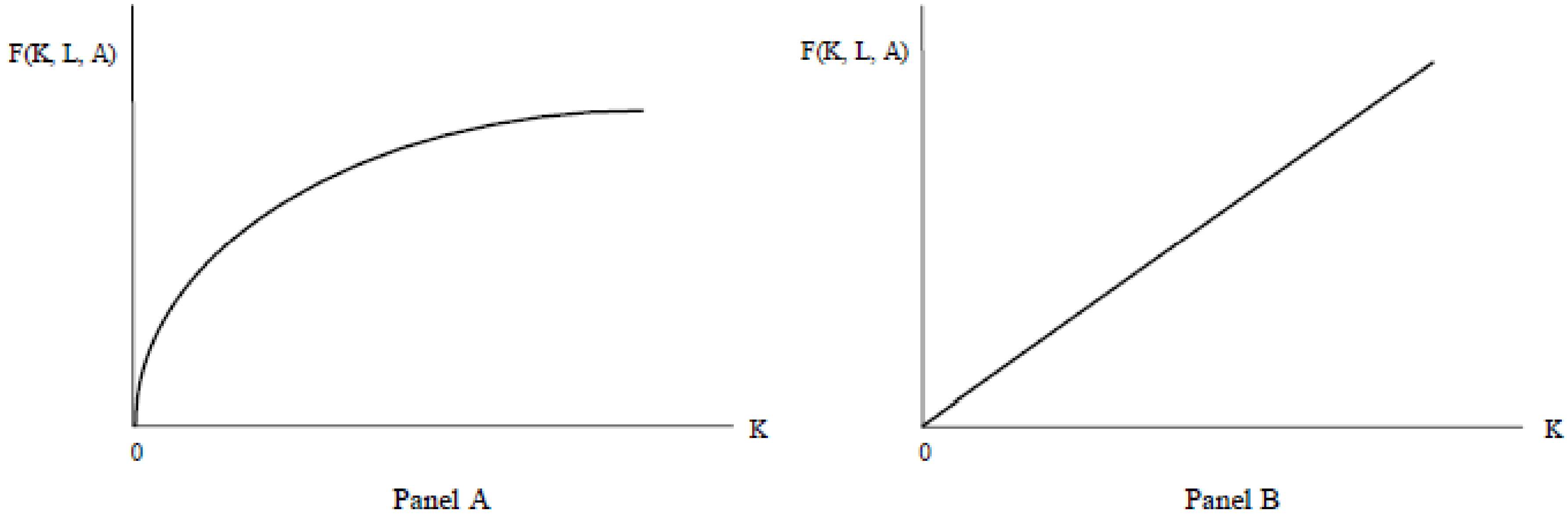


FIGURE 2.1. Production functions and the marginal product of capital. The example in Panel A satisfies the Inada conditions in Assumption 2, while the example in Panel B does not.

Zero profit under competition

Using Euler's theorem, constant returns to the scale of K, L implies:

$$KF_K + L F_L = F(K, L, A)$$

Cost minimization in a competitive economy:

capital rental $R_K = (r + \delta) = F_K$

wage rate $w = F_L$

$$\mathbf{Y = F(K, L, A) = KR_K + Lw}$$

Types of technological progress:

Hicks neutral

$$F\left[K\left(t\right),L\left(t\right),A\left(t\right)\right]=A\left(t\right)\tilde{F}\left[K\left(t\right),L\left(t\right)\right]$$

Solow neutral (capital augmenting)

$$F\left[K\left(t\right),L\left(t\right),A\left(t\right)\right]=\tilde{F}\left[A\left(t\right)K\left(t\right),L\left(t\right)\right]$$

Harrod neutral (labor augmenting)

$$F\left[K\left(t\right),L\left(t\right),A\left(t\right)\right]=\tilde{F}\left[K\left(t\right),A\left(t\right)L\left(t\right)\right]$$

The slope of an isoquant at one point is the ratio MPL/MPK at that point

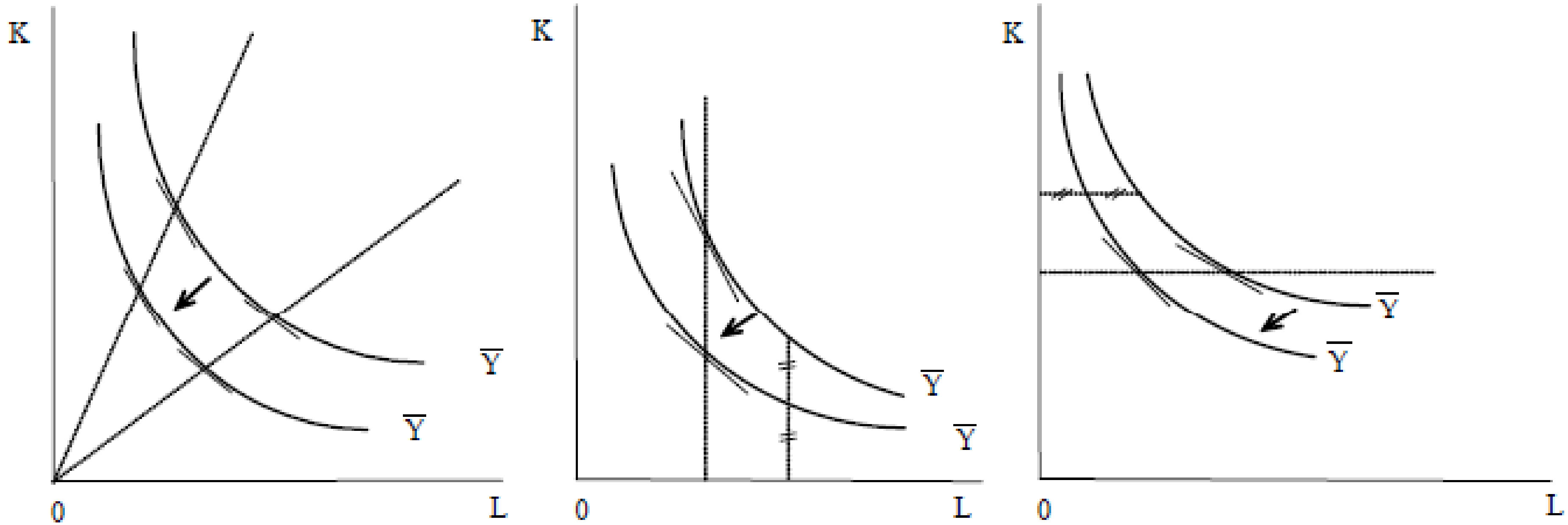


FIGURE 2.12. Hicks-neutral, Solow-neutral and Harrod-neutral shifts in isoquants.

Steady states

- Definition:

  A steady state is a growth path on which every variable grows at a constant rate for ever

- variables that are bounded by definition are constant on a steady state (*their growth rate is zero*)

  example: wage share, profit share, propensity to save

- Analytical motivations for steady states:

  Steady state relations are easier to study

Empirical motivation for steady states

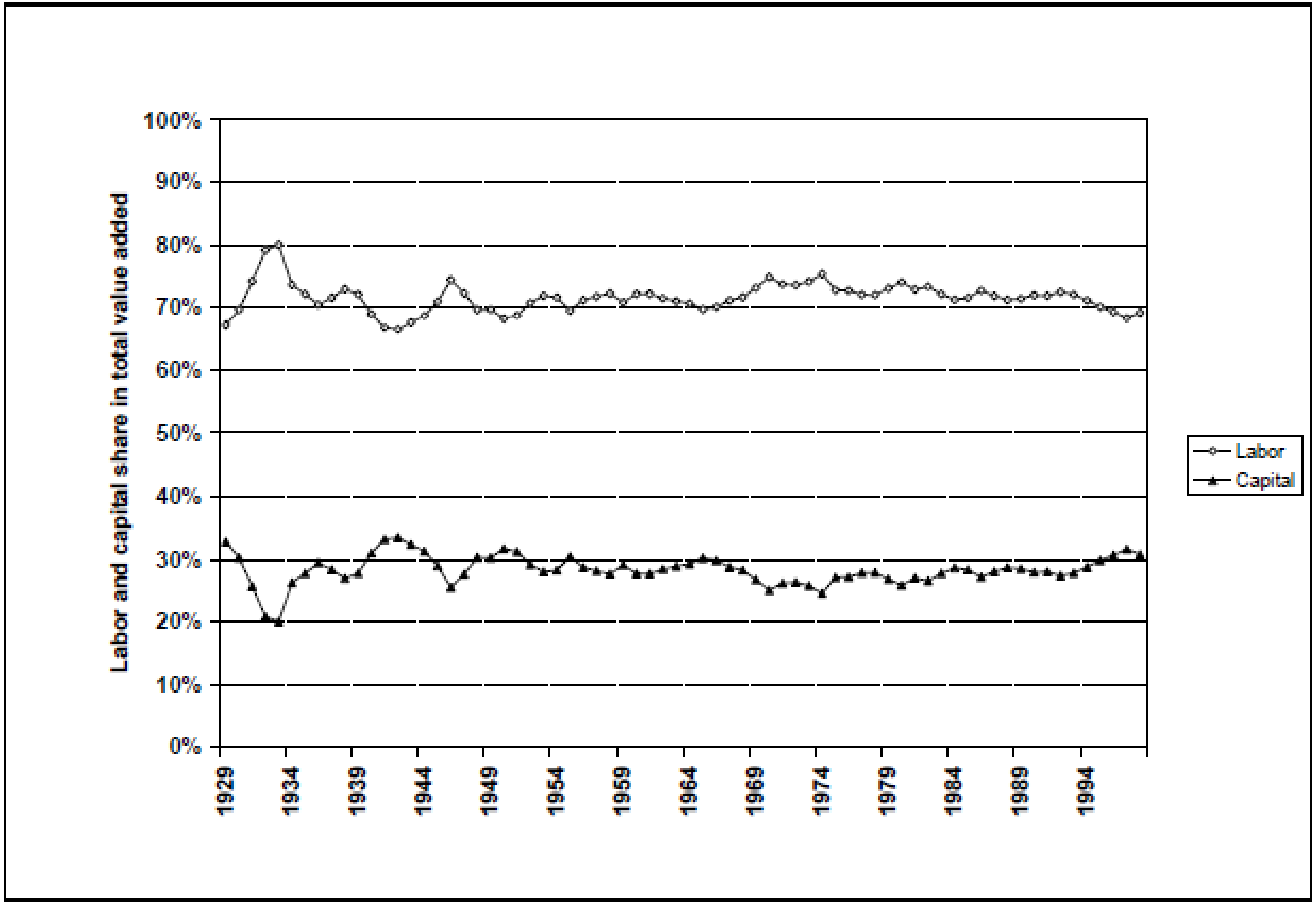


FIGURE 2.11. Capital and Labor Share in the U.S. GDP.

Existence of steady states requires labor augmenting technology

This requires writing **Y = F(K, L, A)**

in the form **Y = F(K, LA)**

LA = efficiency unts of labor

The neoclassical growth model

Robert Solow (1956) Trevor Swan (1956)

- Assume that economic agents save a constant fraction of their gross income:

$$S_t = sY_t$$

Abstrat from changes in s_t through time as resulting from intertemporal utilility maximization

Solow model in discrete time with constant L and A

Let $k = \frac{K}{AL}$ then $\frac{Y_t}{AL} = F\left(\frac{K}{AL}, 1\right) = F(k, 1) = f(k)$

$$K_{t+1} = sF(Kt, AL) + (1 - \delta)K_t$$

$$K_{t+1} = sAL\, F\left(\frac{K_t}{AL}, 1\right) + (1 - \delta)K_t \qquad \text{devide by } AL\ldots$$

$$k_{t+1} = sf(k_t) + (1 - \delta)k_t$$

Steady state

$$k_{t+1} = sf(k_t) + (1 - \delta)k_t$$

steady state k is identified by:*

$$k^* = sf(k^*) + (1 - \delta)k^*$$ hence..

$$sf(k^*) = \delta k^*$$

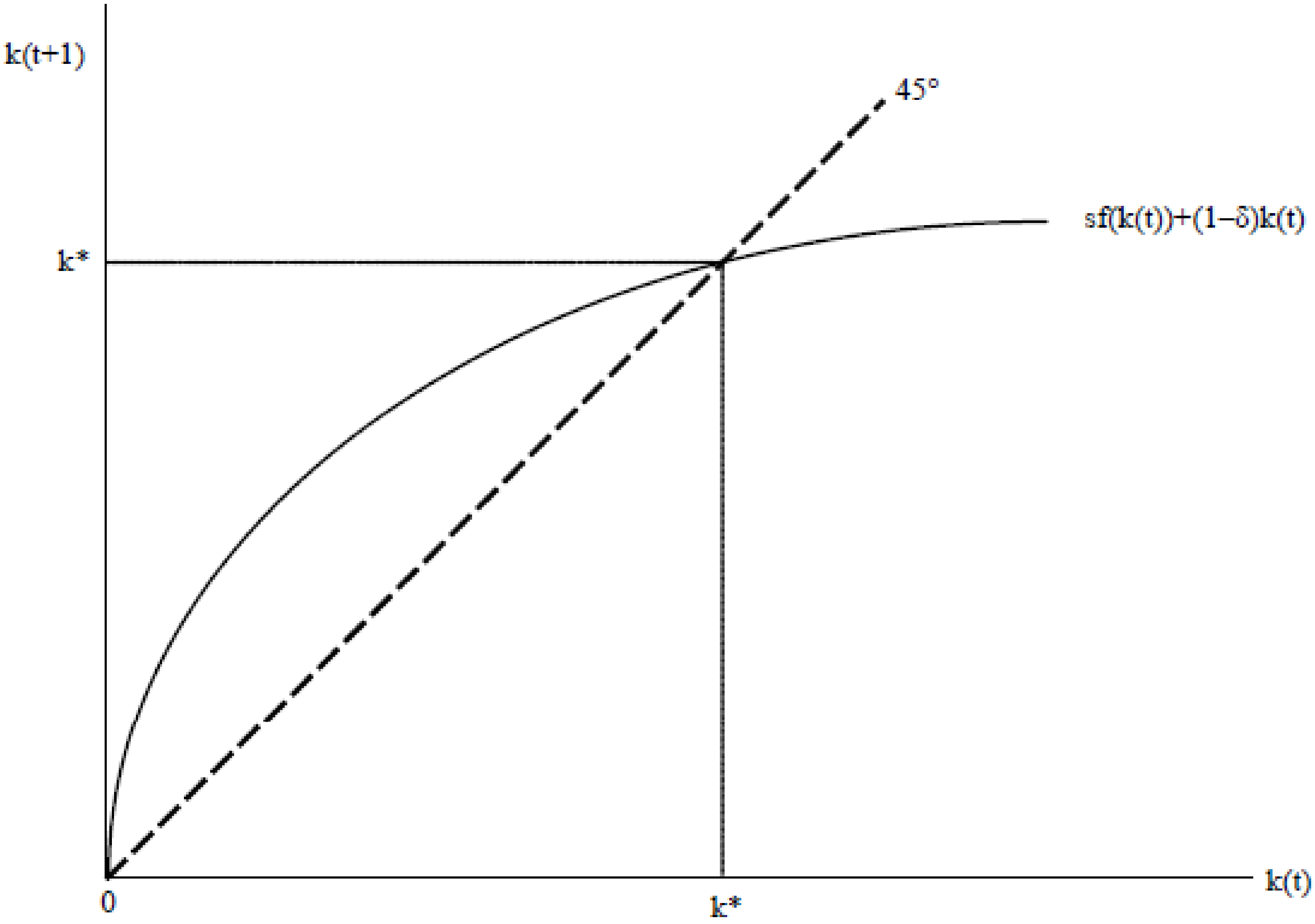


FIGURE 2.2. Determination of the steady-state capital-labor ratio in the Solow model without population growth and technological change.

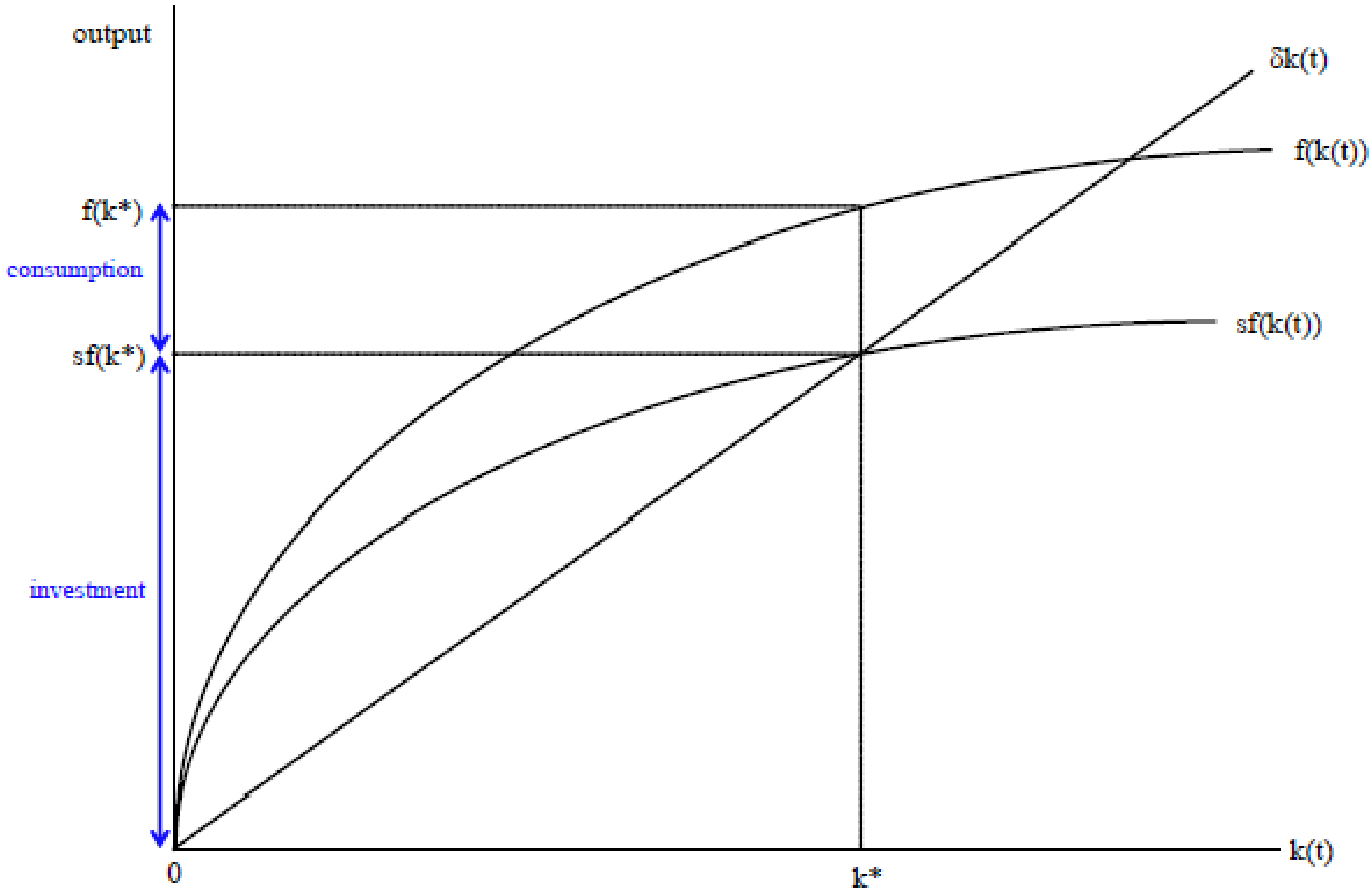
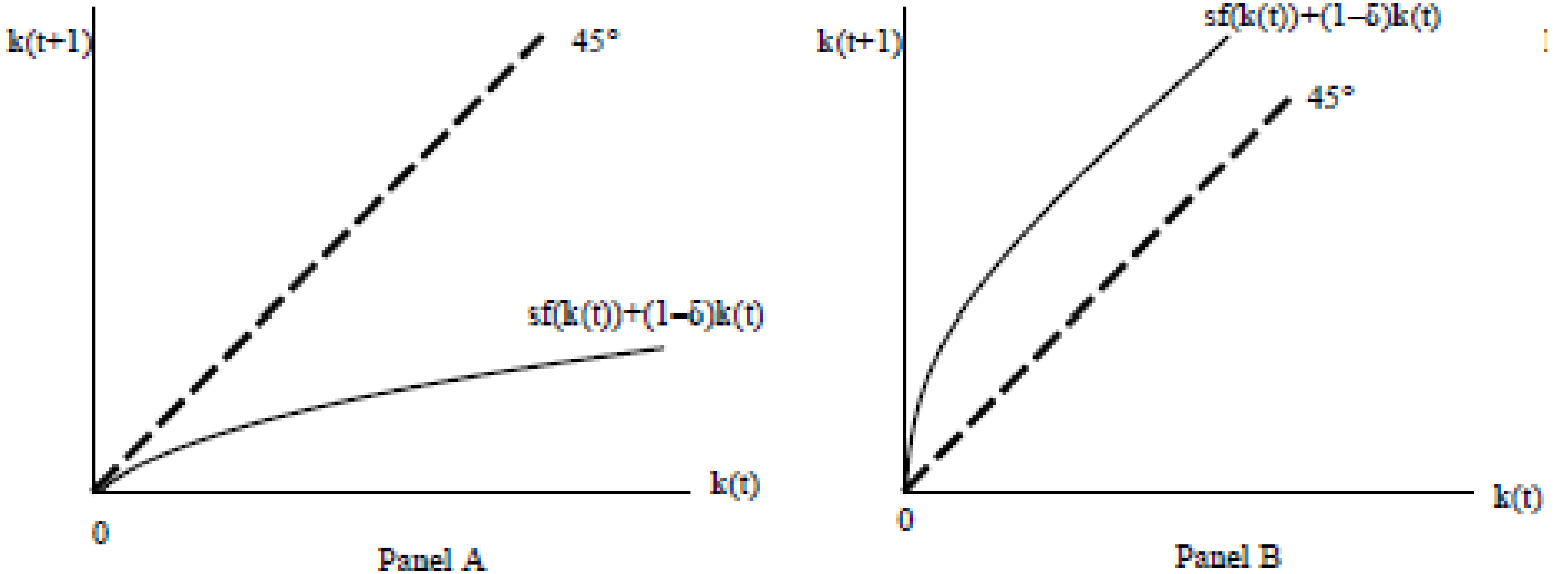
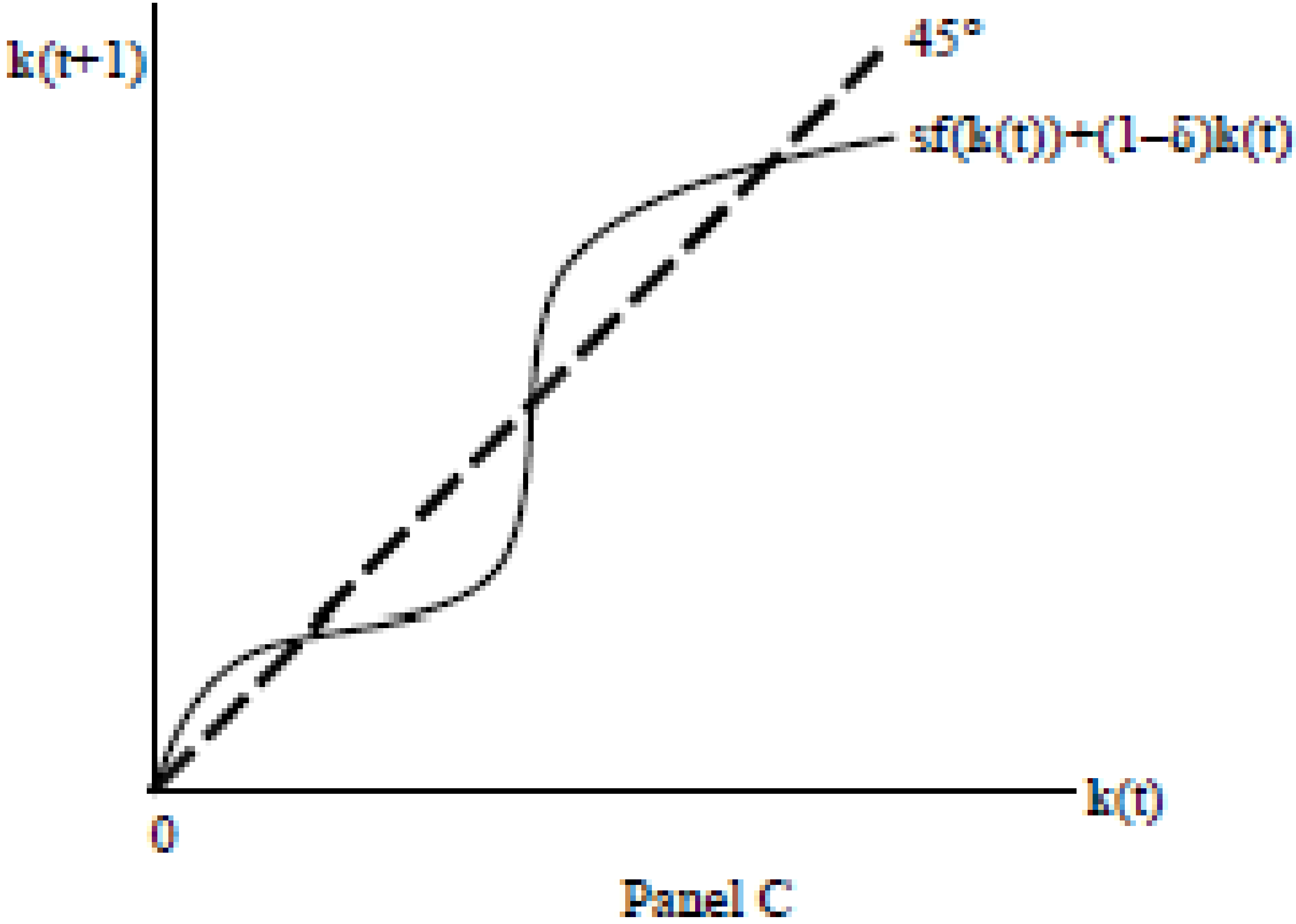


FIGURE 2.4. Investment and consumption in the steady-state equilibrium.

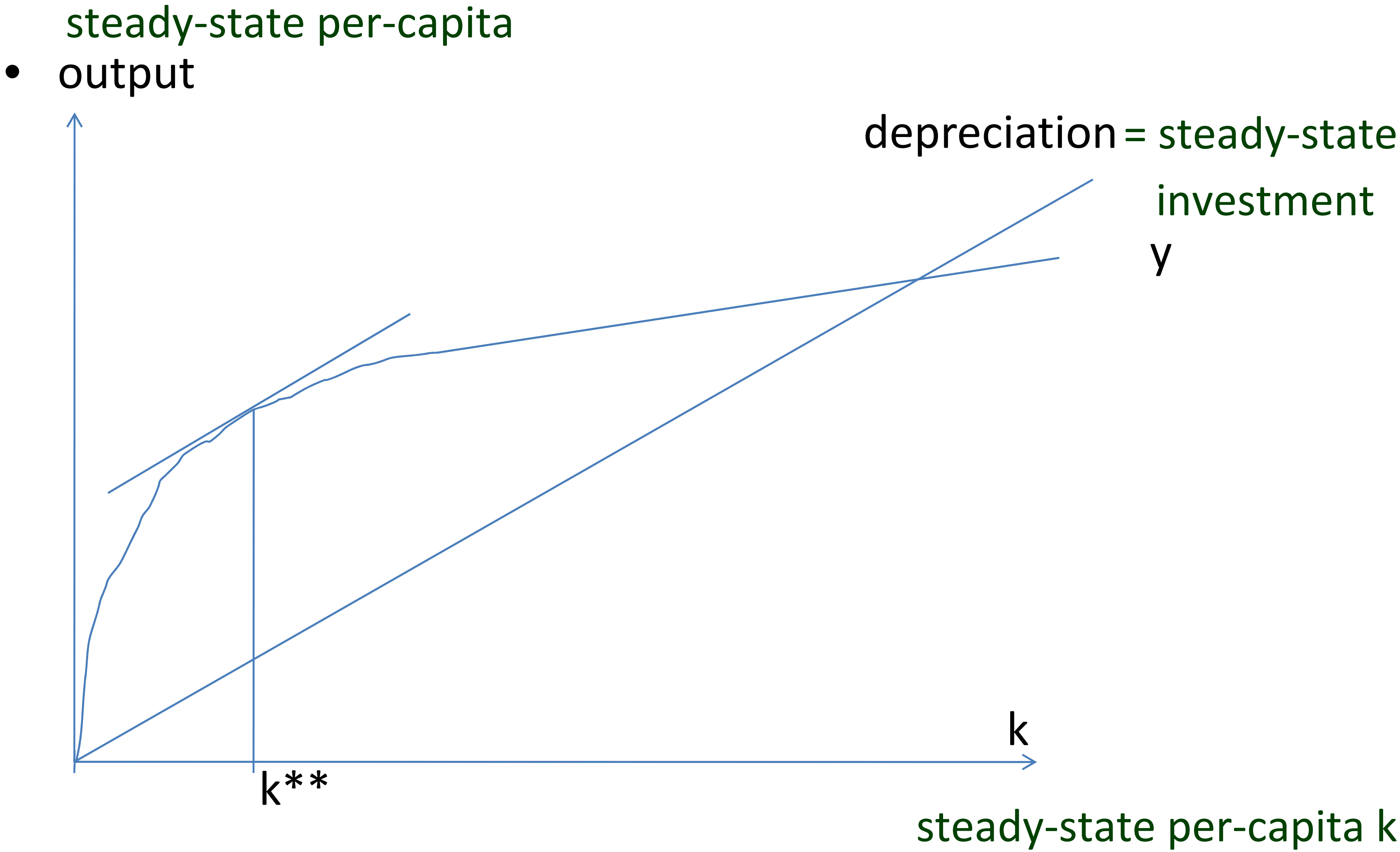
Non existence of a positive stationary state when Inada conditions fail



Multiplicity of stationary states when marginal returns are variable



k** maximizes steady state consumption



The golden-rule savings rate s*

- The value k^{**} maximizing per-capita consumption is defined by:
- Max: $f(k) - \delta k$ hence…
- $f'(k^{**}) = \delta$ this defines k^{**}
- $s^* f(k^{**}) = \delta k^{**}$
- $s^* = \delta k^{**} / f(k^{**})$ this defines s^*

The golden-rule savings rate

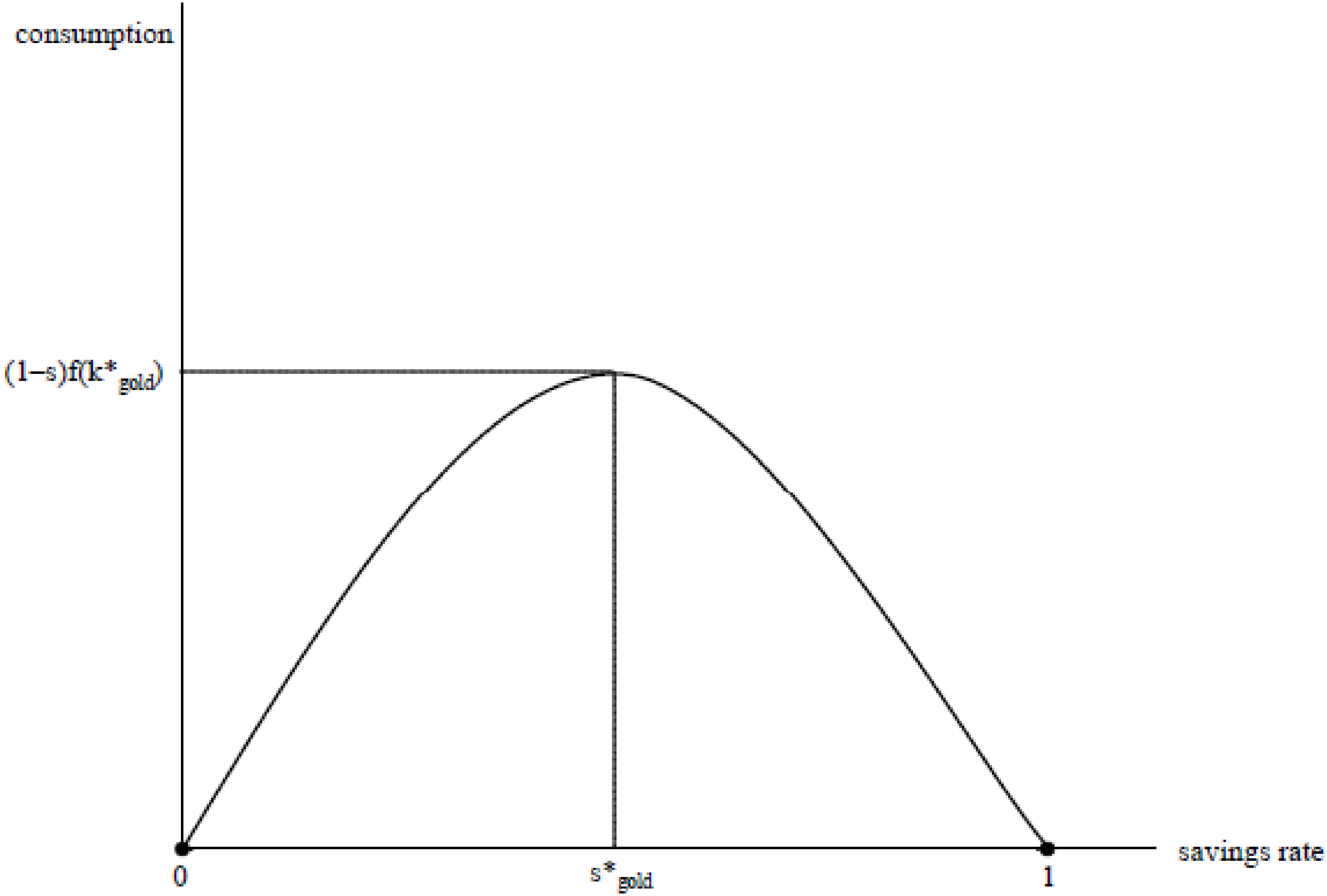


FIGURE 2.6. The "golden rule" level of savings rate, which maximizes steady-state consumption.

Transitional dynamics

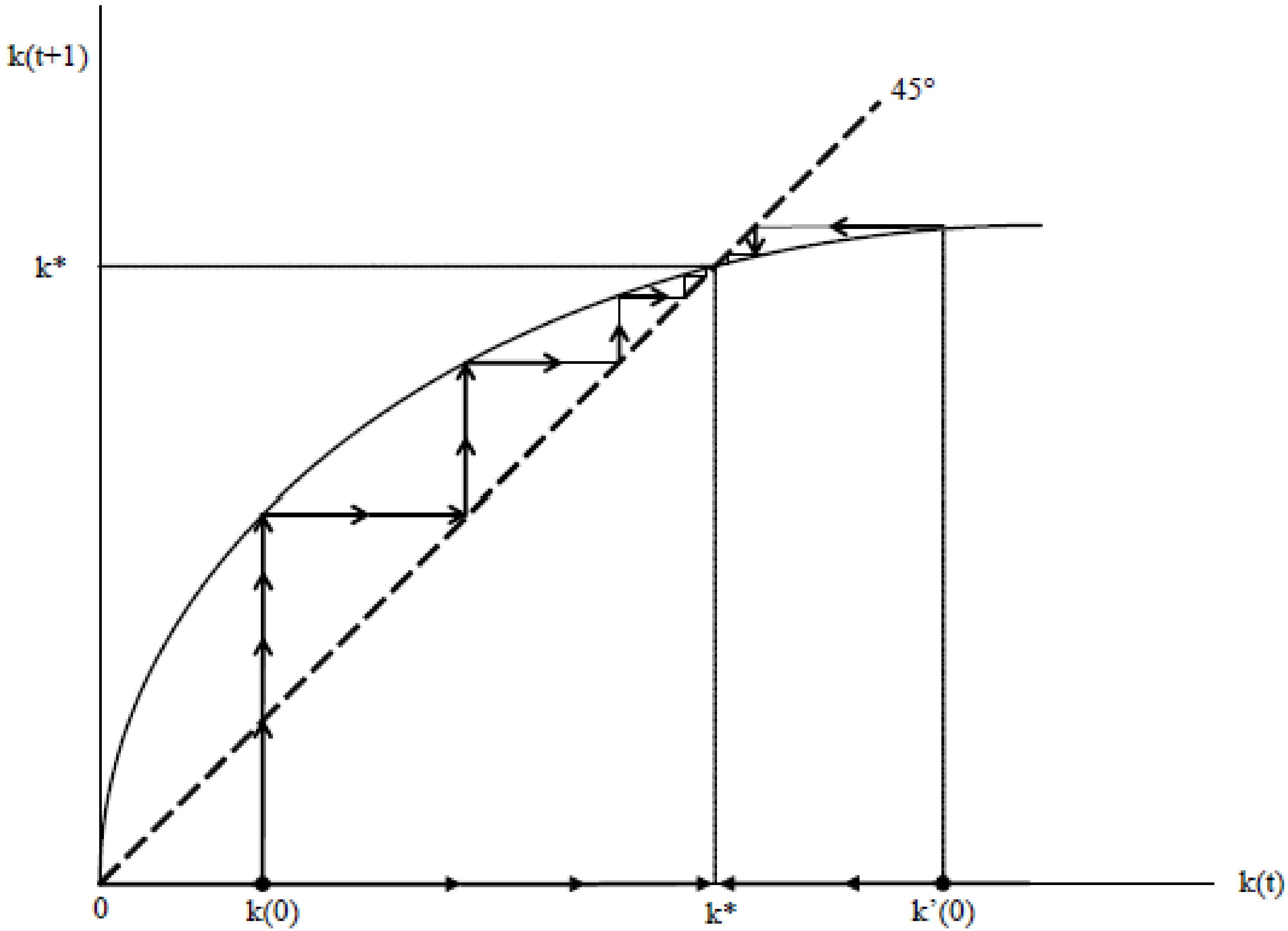


FIGURE 2.7. Transitional dynamics in the basic Solow model.

Solow model in continuous time with growing population and technology

$$\frac{\dot{A}_t}{A_t} = g \qquad \frac{\dot{L}_t}{L_t} = n \qquad k_t = \frac{K_t}{A_t L_t}$$

$$\frac{\dot{k}_t}{k_t} = \frac{\dot{K}_t}{K_t} - (n + g)$$

Rate of depreciation of capital in efficiency units

- if the number of workers is growing with population growth
- If the efficiency of workers is growing with technological progress
- $k = K/AL$ depreciates not only as a result of physical depreciation δ, but also as a result of population growing at rate *n* and of efficiency growing at rate *g*
- *The depreciation rate of k is* $(\delta + n + g)$

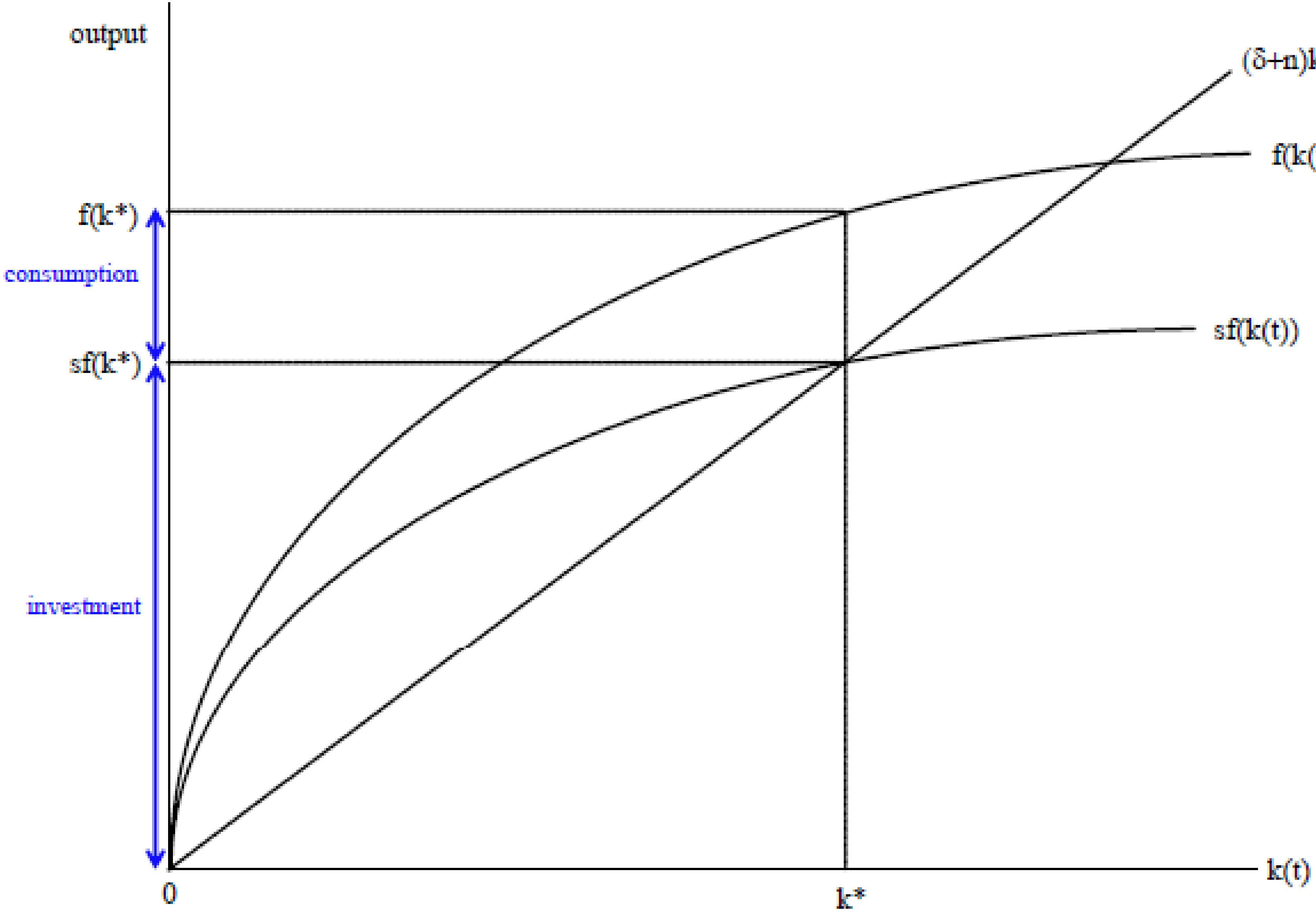
$$\dot{K}_t = sF(K_t, A_t L_t) - \delta K_t$$

$$\frac{\dot{K}_t}{K_t} = \frac{sF(K_t, A_t L_t)}{K_t} - \delta$$

$$\frac{\dot{K}_t}{K_t} = \frac{sA_t L_t f(k_t)}{K_t} - \delta$$

$$\frac{\dot{k}_t}{k_t} = \frac{sf(k_t)}{k_t} - (n + g + \delta)$$

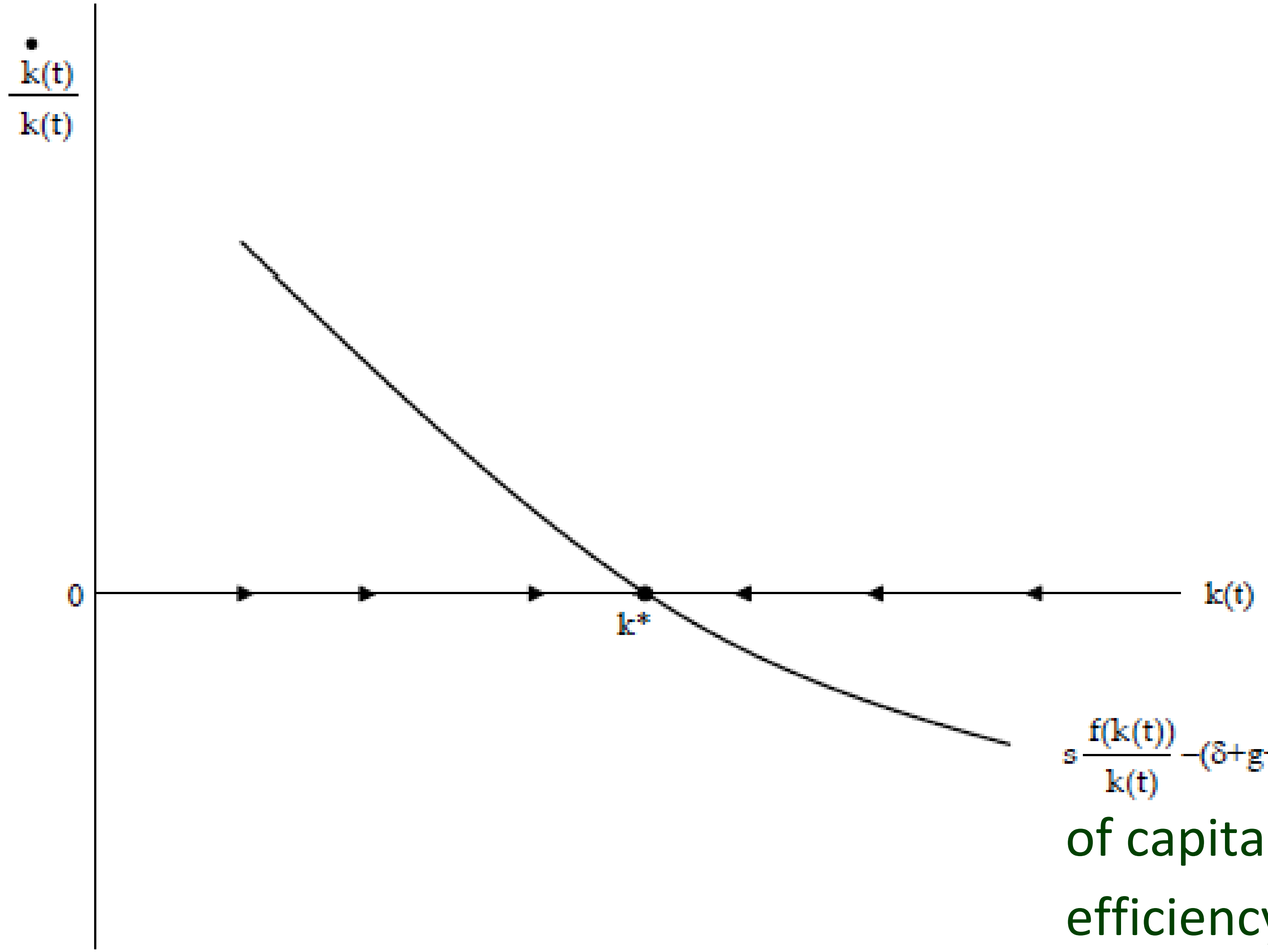
Per-capita consumption and investment with g = 0



The previous slide shows:

- f(k) / k = is a deceasing function of k
- f(k) = output in efficiency unts
- k = capital in efficiency units
- As k gets higher, the marginal product of capital gets lower, causing the fall of f(k)/k

Growth rate of K/AL



$s\frac{f(k(t))}{k(t)} - (\delta+g+n)$ = rate of growth of capital per unit of efficiency

Exogenous growth

$$\frac{\dot{K}_t}{K_t} = \frac{\dot{k}_t}{k_t} + (n + g)$$

In steady state: $\frac{\dot{k}_t}{k_t} = 0$

$$\frac{\dot{K}_t}{K_t} = \frac{\dot{Y}_t}{Y_t} = (n + g)$$

Per capita output $y_t = \frac{Y_t}{L_t}$

Steady-state growth rate of y_t: $\frac{\dot{y}_t}{y_t} = \frac{\dot{Y}_t}{Y_t} - n = g$

Definition of exogenous growth

- Steady state growth rate of <u>per-capita</u> output :

  - does not depend on the savings rate

  - is exogenous (is not explained by the model)

Effects of a higher savings rate s

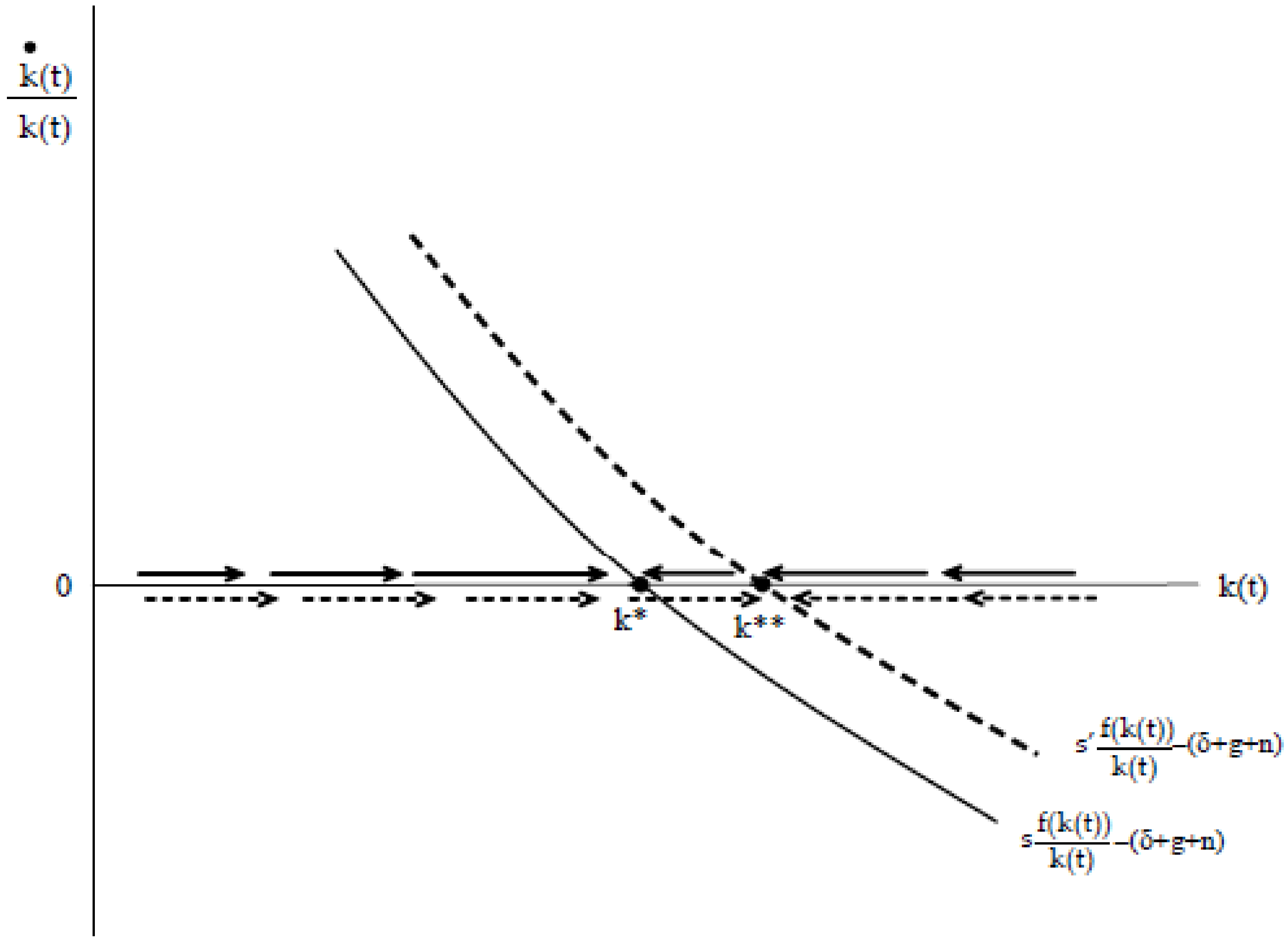


FIGURE 2.13. Dynamics following an increase in the savings rate from s to s'. The solid arrows show the dynamics for the initial steady state, while the dashed arrows show the dynamics for the new steady state.

Effects of a higher savings rate s' > s

$$\frac{\dot{k}_t}{k_t} = \frac{sf(k_t)}{k_t} - (n + g + \delta)$$

1. <u>Transitional growth effects</u>:

   temporary increase of $\frac{\dot{k}_t}{k_t}$

2. Steady-state growth of $\frac{\dot{y}_t}{y_t}$ = g as before

3. <u>Persistent level effects</u>:

   Higher *steady state* $k^{**} > k^*$ *and* $y^{**} > y^*$

4. Consumption per capita may not be higher!

Example: Cobb-Douglas production function

$$Y = F(K, AL) = K^{\alpha}(AL)^{1-\alpha}$$

$$\frac{Y}{AL} = f(k) = k^{\alpha}$$

$$sf(k^*) = (\delta + n + g)k^*$$

$$s{k^*}^{\alpha} = (\delta + n + g)k^*$$

steady-state capital per unit of efficiency

$$k^* = \left(\frac{s}{(\delta + n + g)}\right)^{1/(1-\alpha)}$$

steady-state output per-capita= $y_t^* = A_t {k^*}^{\alpha} = A_t \left(\frac{s}{\delta + n + g}\right)^{\alpha/(1-\alpha)}$

The long-run

- Long-run GDP per capita is

$$y_t^* = A_t {k^*}^{\alpha} = A_0 e^{gt} {k^*}^{\alpha}$$

- If population growth *n* and technological progress *g* are uniform across countries:
- Long-run GDP per capita is higher if
- savings rate *s* is higher, hence k^* is higher
- initial level of efficiency A_0 is higher

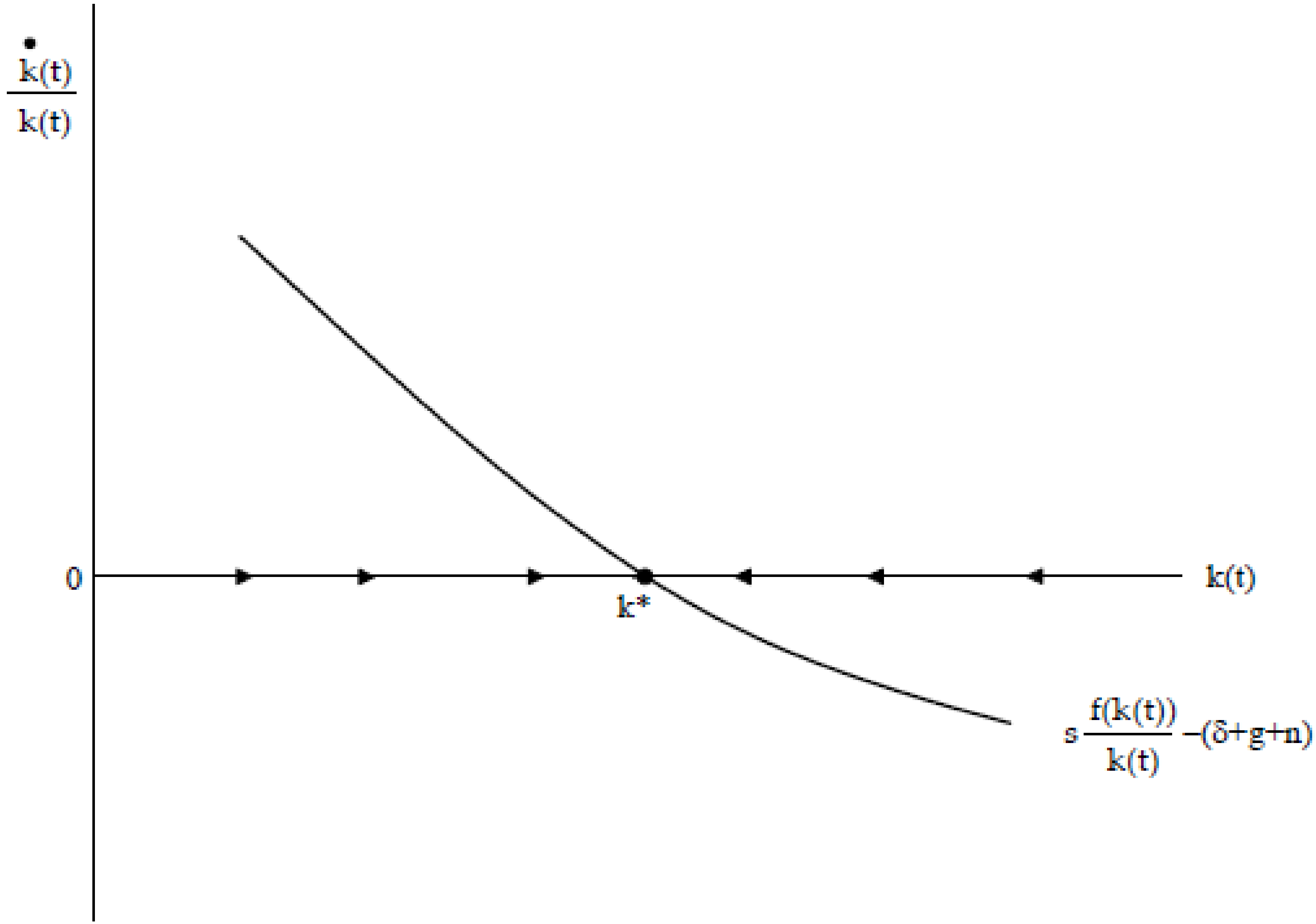
The long-run

- Savings rate *s* does not differ so much across countries to justify the observed large differences in GDP per capita
- As we have seen in the Cobb-Douglas example, steady state k* is uniform, is *s*, *g* and *n* are uniform
- We are still given the possibility of explaining why one country is richer than the other, by invoking the reasons why its initial efficiency A_0 is higher

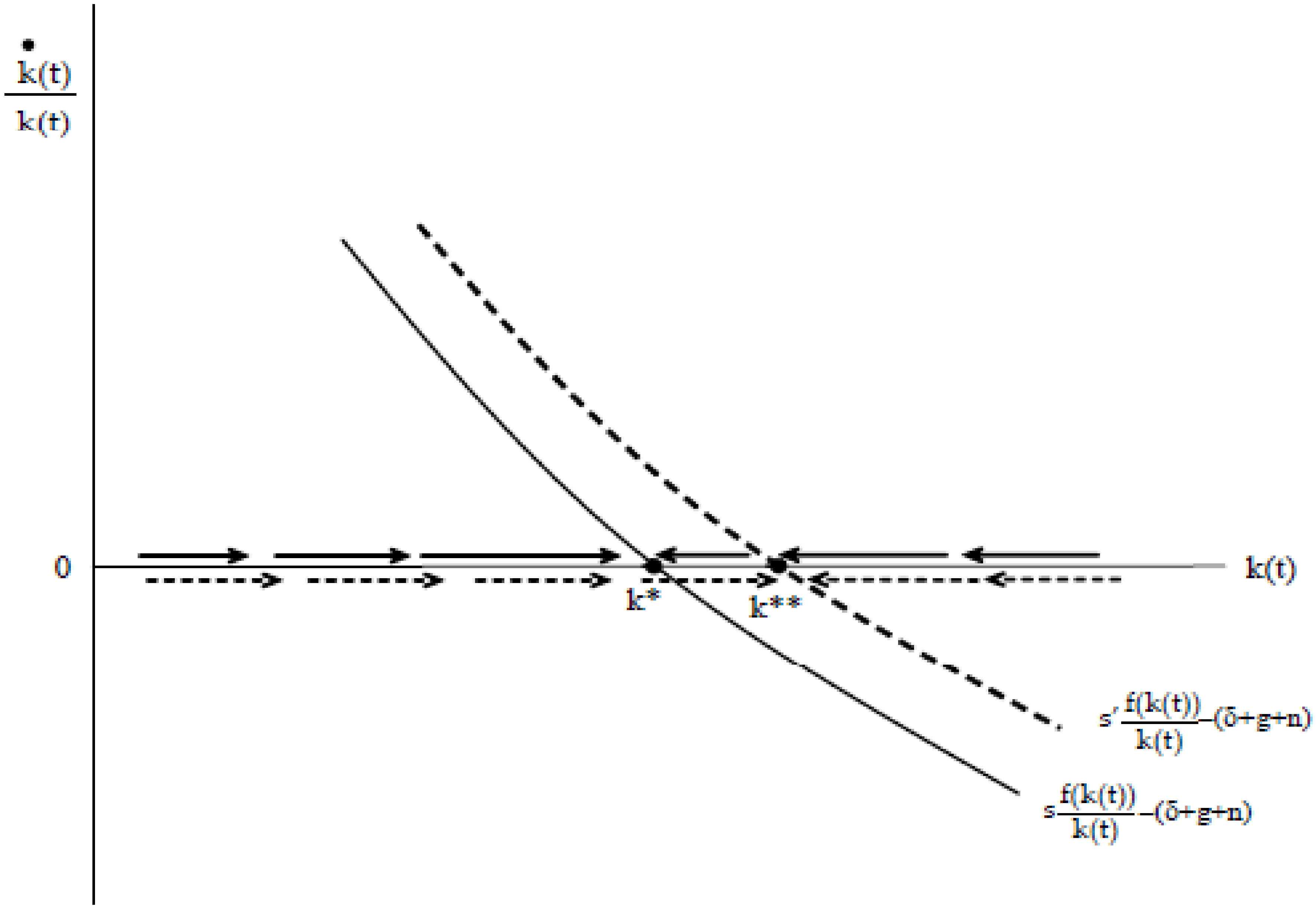
The transition to the long-run

Convergence:
unconditional vs conditional

Unconditional convergence: an economy farther away from the same steady state, grows faster



Conditional convergence: growth rate during transition depends on distance from steady sate. A's saving rate = s; B's savings rate = s'



Conditional convergence

- In the previous slide, if countries A and B have the same initial condition $k_0 < k^* < k^{**}$, then, *during the transition to steady state*, country B grows faster than country A, because it has higher propensity to save.
- The transitional growth component of a country is an increasing funcion of its distance from its steady state $k_t^* - k_0$.
- Other things equal, $k_t^* - k_0$ is larger if s is higher, because $k_0 = K_0 / (L_0 A_0)$

Summing up

- The long-run GDP per capita is an increasing function of the savings rate and of the initial efficiency level of a country
- The persistent growth component of GDP per capita is uniformly equal to g
- The transitional growth component of GDP per capita is an increasing function of initial distance from the country steady state.
- Other things equal, initial distance from steady state is larger if:

  - propenisty to save s is higher

  - initial level of country efficiency A_0 is higher