Transitional dynamics and convergence

In what follows we shall assume, following the literature, that the propensity to save $s < s^*$ of gloden rule.

This amounts to ruling out overaccumulation, to the effect that a higher propensity to save has a persistent positive level effect on (steady-state) consumption per-capita.

Let us introduce the standard definitions introduced with the Solow model..

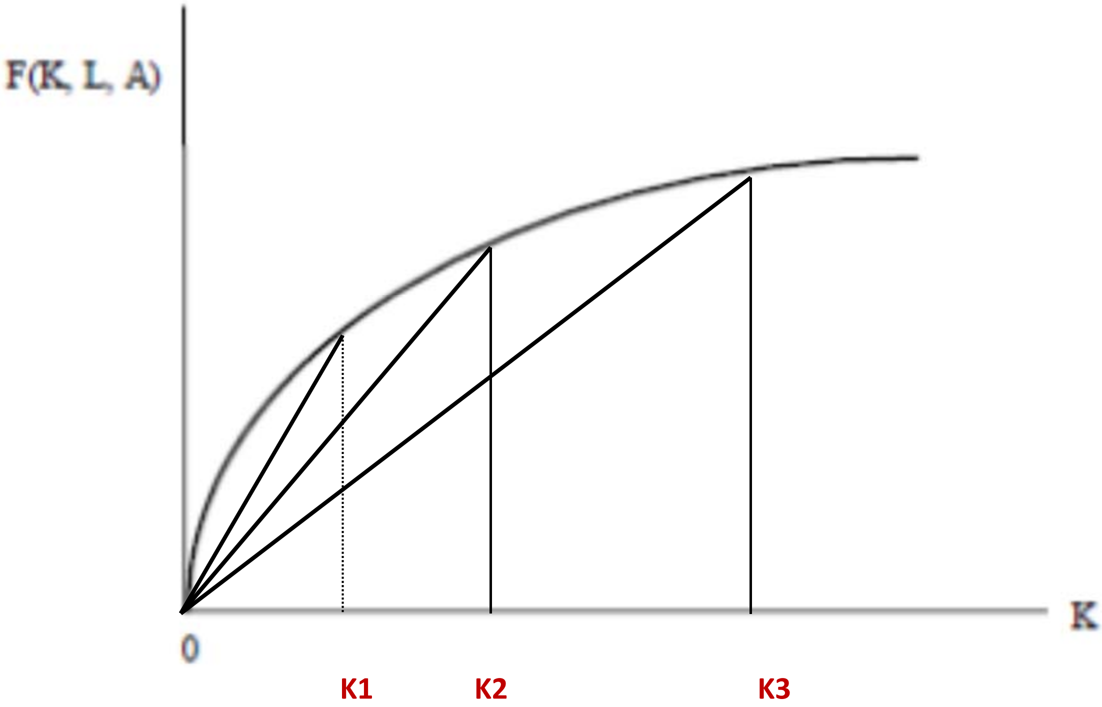
$k = \frac{K}{AL}$ *capital in efficiency units*

$f(k) = \frac{F(K,AL)}{AL}$ *output per efficient worker*

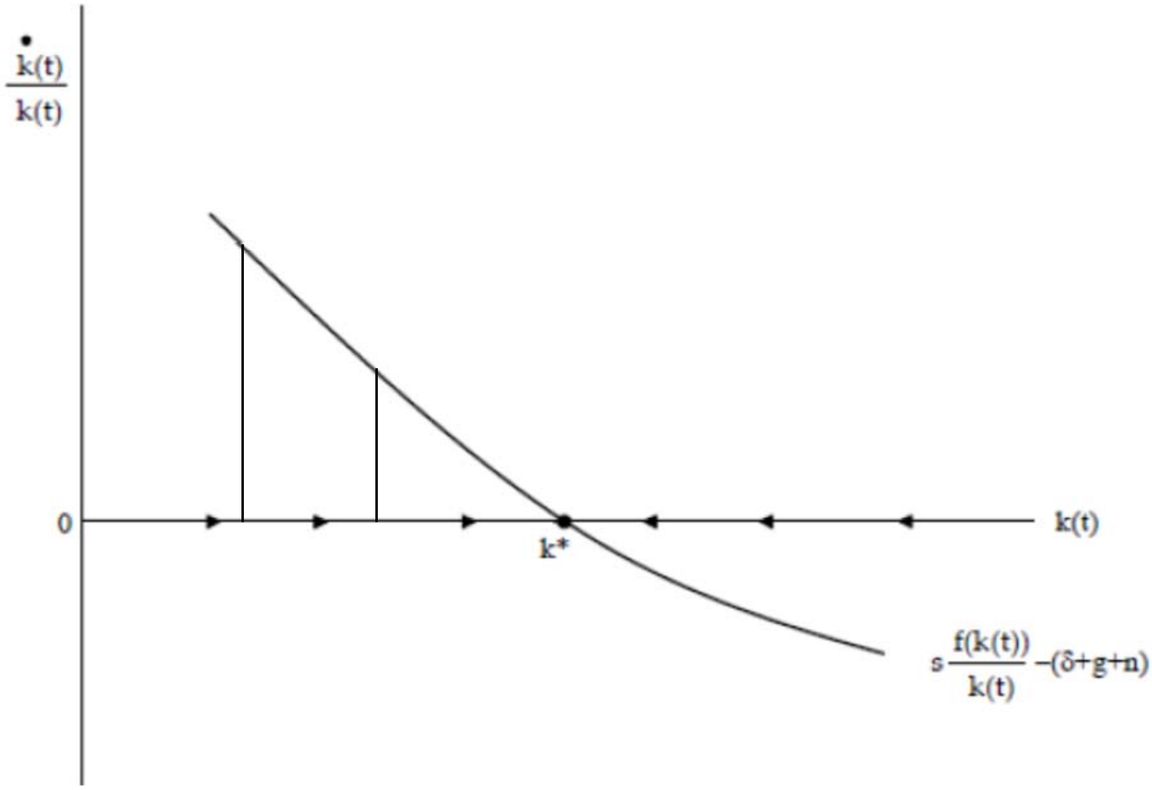
$$\frac{\dot{k}_t}{k_t} = \frac{sf(k_t)}{k_t} - (n + g + \delta)$$

0utput/capital ratio is a decreasing function of k: $\frac{f(k)}{k} = \frac{F(K,LA)}{K}$

$\rightarrow$ *rate of capital accumulation is decreasing function of k*



If k^* is the same, the poorer country grows faster than the rich <u>during transition</u> (temporary growth component)



Temporary and persistent growth component

$$k = K/AL \qquad \hat{y} = Y/AL$$

$Y = F(K, AL) = K^{\alpha}(AL)^{1-\alpha}$ Cobb-Douglas case

$$\frac{Y}{AL} = \hat{y} = f(k) = k^{\alpha}$$

$$\frac{Y(t)}{L(t)} = y(t) = A_t f(k_t) = A_t k_t^{\alpha}$$

$$\frac{\dot{y}_t}{y_t} = g + \alpha \frac{\dot{k}_t}{k_t}$$

g = **persistent growth component**

$\frac{\dot{k}_t}{k_t}$ = **temporary growth component, is increasing with distance from k***

Take linear approximation of $\frac{\dot{k}_t}{k_t} = sk_t^{\alpha-1} - (n + g + \delta)$

around *k** *recalling that* $\frac{\dot{k}_t}{k_t} = 0$ *at k**

$$\frac{\dot{k}_t}{k_t} = (\alpha - 1)s\frac{{k^*}^{\alpha-1}}{k^*}(k_t - k^*)$$

first order expansion of *log k* around *k**: $log\ k \approx log\ k^* + \frac{1}{k^*}(k - k^*)$

$$\frac{(k_t - k^*)}{k^*} \approx logk_t - logk^*$$

$$s{k^*}^{\alpha-1} = (n + g + \delta)$$

$$\frac{\dot{k}_t}{k_t} \approx (\alpha - 1)(n + g + \delta)(logk_t - logk^*)$$

$$\frac{\dot{k}_t}{k_t} \approx (1-\alpha)(n+g+\delta)(logk^* - logk_t)$$

The equivalent expression for the growth rate of per-capita output near the steady state is :

$$\frac{\dot{y}_t}{y_t} \approx g + (1-\alpha)(n+g+\delta)(logy_t^* - logy_t) \quad (7)$$

Persistent temporary

Determinants of the steady state component *y*(t)*

$sf(k^*) = (\delta + n + g)k^*$ steady state restriction

$sk^{*\alpha} = (\delta + n + g)k^*$ Cobb-Douglas

$$k^* = \left(\frac{s}{(\delta + n + g)}\right)^{1/(1-\alpha)}$$

$$y_t^* = A_t k^{*\alpha} = A_t \left(\frac{s}{\delta+n+g}\right)^{\alpha/(1-\alpha)}$$

$$\log y^*(t) = \log A(t) + [\alpha / (1 - \alpha)] \log [s/(n + g + \delta)]$$

steady state *per capita GDP is explained by:*

- initial technology level A_0
- population growth n
- propensity to save s
- technology growth g
- elasticity of output with respect to capital

If countries A, B are structurally identical: $y_A^*(t) = y_B^*(t)$

from $y_A^*(t) = y_B^*(t)$: $y_A(t) > y_B(t)$ → $g_y^A(t) < g_y^B(t)$

- *persistent growth components are identical in A and B*
- *temporary growth component is lower in rich country A than in poor country B*

→ *growth is faster in B than in A =* *unconditional convergence*

Barro regressions

A discrete appoximation of equation (7) above is:

$$g_{t,\, t-1} \approx g + (n + g + \delta)(1 - \alpha)\, [\log y^*(t-1) - \log y(t-1)]$$

- $g_{t,t-1} \approx g + (n + g + \delta)(1 - \alpha)\,[\log y^*(t-1) - \log y(t-1)]$

$$g_{i,t,t-1} = b^0 + b^1 \log y_{i,t-1} + \varepsilon_{i,t}, \qquad (8)$$

- $\varepsilon_{i,t}$ is a stochastic term capturing all omitted influences.

$$b^0 = g + (n + g + \delta)(1 - \alpha)\log y^*(t-1) \qquad (8.1)$$
$$b^1 = -(n + g + \delta)(1 - \alpha) \qquad (8.2)$$

notice that b_0 captures the permanent growth component g and effect of the steady state level $y^*(t-1)$

here b_0 is not country specific, is uniform across countries: it amounts to assuming that all countries are converging to the same steady state.

→ equation (8) tests for *unconditional convergence* (absolute convergence)

Unconditional convergence:

- If such an equation is estimated in the sample of core OECD countries, b^1 is indeed estimated to be negative.
- But for the whole world, no evidence for a negative b^1. If anything, b^1 would be positive.
- I.e., there is no evidence of world-wide convergence,
- Barro and Sala-i-Martin refer to this as "unconditional convergence."

Barro Results: average per capita GDP growth 1960-1985 positively correlated with GDP per-capita 1960, on a cross section data of 91 countries

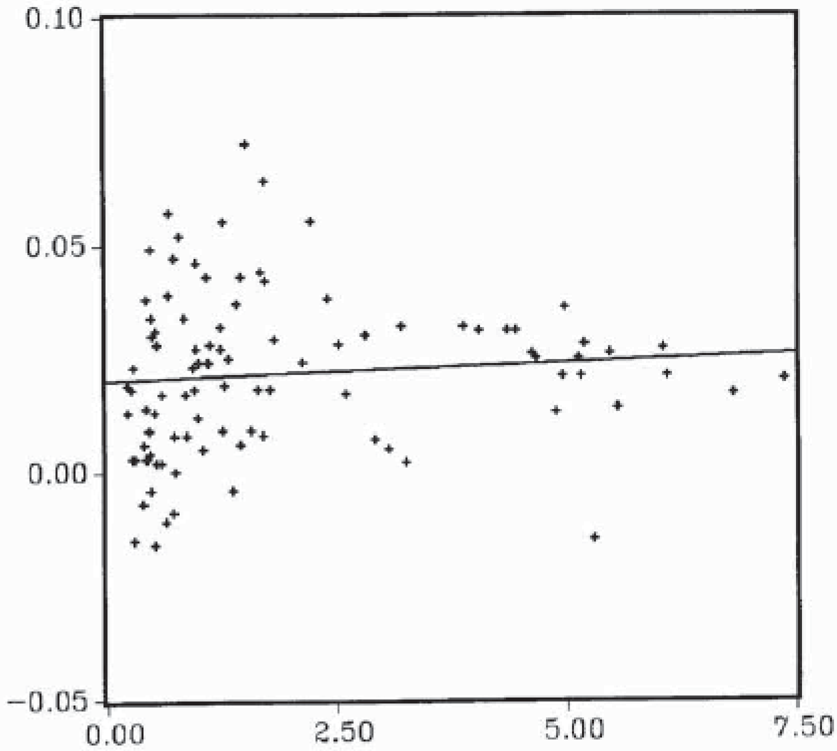


FIGURE I
Per Capita Growth Rate Versus 1960 GDP per Capita

Conditional convergence

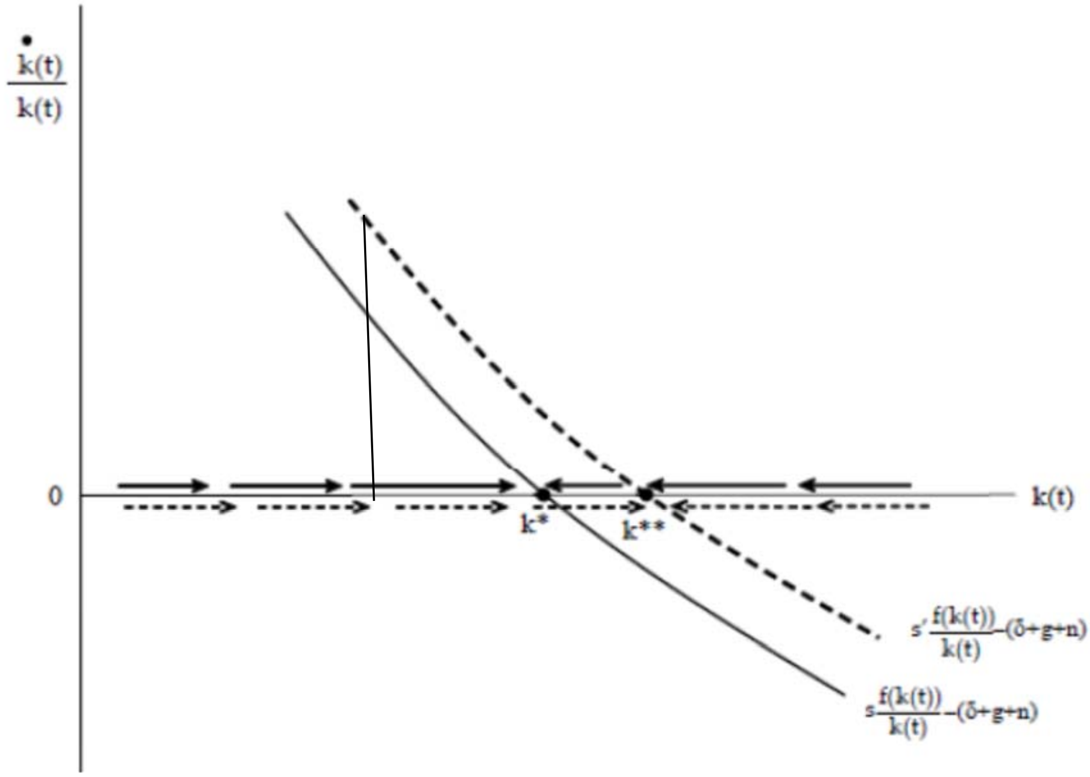
- Unconditional convergence is often too demanding, because it restricts countries to have the same investment ratio (s = I/Y), and the same efficiency of their technology and institutions.
- If countries differ in their characteristics, one has to admit that the constant term b^0 in equation (8) has to be country-specific, because it captures the position of a country steady state. This yields:

$$g_{i,t,t-1} = b_i^0 + b^1 \log y_{i,t-1} + \varepsilon_{i,t}, \qquad (9)$$

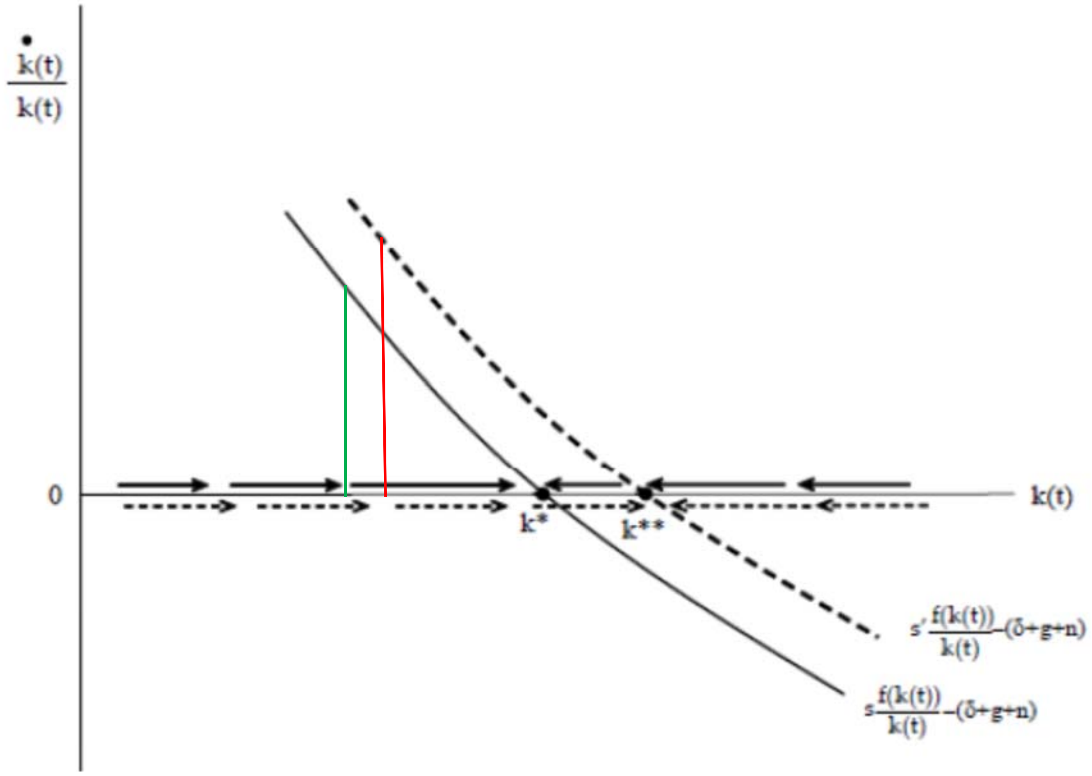
- The regression value of the constant term will then reflect the country characteristics that affect the position of the steady state.
- The slope term b^1 is the *convergence coefficient*, measuring the speed of convergence to the steady state. From equation (8.2) above we have that $b^1 = (n + g + \delta)(1 - \alpha)$. In equation (8) and (9) the convergence coefficient is assumed uniform across countries. The implicit restriction is that countries have same population growth, and same production function, in particular, same elasticity α of output with respect to physical capital.

Effects of different s across countries:

If initial condition is the same but savings rate is higher, the temporary growth component is also higher



If savings rate is higher, the richer country may grow faster than the poor



Conditional convergence:

$$g_{i,t,t-1} = b_i^0 + b^1 \log y_{i,t-1} + \varepsilon_{i,t},$$

recall: $b^0_i = g + (n + g + \delta)(1 - \alpha)\log y_i^*(t-1)$

b^0_i captures the effect of all the country-i varaibles affecting the steady state y_i^*, in particular initial technology $A_i\ (t-1)$ and country i propensity to save s_i.

if the saving rate s_i or productivity A_i are high in a rich country i, its b^0_i is large, and the country may well grow faster than a poor country j, with a low b^0_j.

Barro (1991) and Barro and Sala-i-Martin (2004) adopt a somewhat ‘loose’ interpretation of ‘technology’ A(t) and define:

$$b^0_i = qX_i$$

where q is a row vector of coefficients and X_i is a column vector of <u>all</u> country specific variables, affecting i’s steady state.

Because the coefficient b^0_i is allowed to differ across countries, they expect a negative estimate of the convergence coefficient b^1

GDP-per capita growth 1960-1985 net of all explanatory variables of steady state 1960 versus GDP per capita

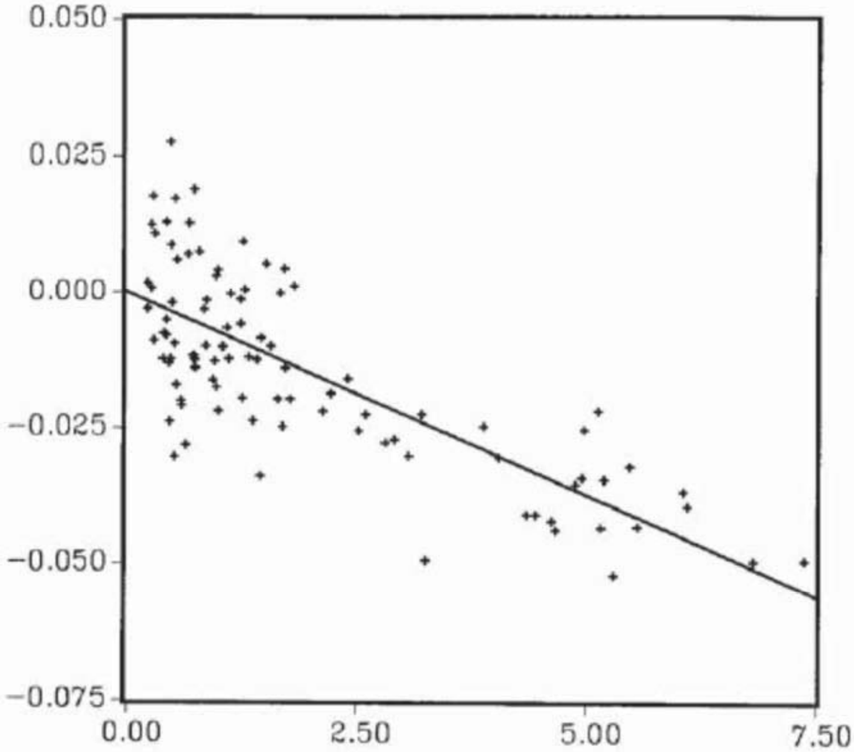


FIGURE II
Partial Association Between per Capita Growth and 1960 GDP per Capita (from regression 1 of Table I)

Evidence of β-conditional convergence across countries is a point of strength of the neoclassical model:

● β (conditional) convergence = convergence of per-capita income in each country or region towards its steady-state

● β (conditional) convergence is however a very weak prediction consistent with a very wide range of growth paths, if steady states are allowed to differ across countries and regions.

- The previous remark can be restated by saying:

β convergence does not imply σ convergence of income per-capita.

<u>σ convergence</u> of income per-capita:

consider a sample of N regional income per-capita y_{it} at date t, $i = 1, \dots, N$
the sample exhibits σ convergence if per-capita income if the variance σ^2 of the cross-country (or region) distribution of log y_{it} $i = 1, \dots, N$ decreases over time

The sample variance of log income in t is given by

$$\sigma_t^2 = \left(\frac{1}{N}\right)\sum_{i=1}^{N}\left[ln(y_{it}) - \mu_t\right]^2,$$

μ_t = sample average of $ln(y_{it})$

.

● In fact, β convergence may <u>cause</u> σ divergence:

Example:

Consider two economies A and B, where $y_{A0} = y_{B0}$, that is,

where both economies begin at the same level of income. However, assume that B begins on its balanced growth path while A begins far below its balanced growth path, and assume that β-convergence holds. The initial variance (σ_0^2) will be zero, but σ_t^2 will grow over time as A grows faster than B and approaches a higher balanced growth path. Indeed, β-convergence is the reason for the increasing variance.[6]

Examples of σ divergence: Cross-country distribution of log GDP per-capita

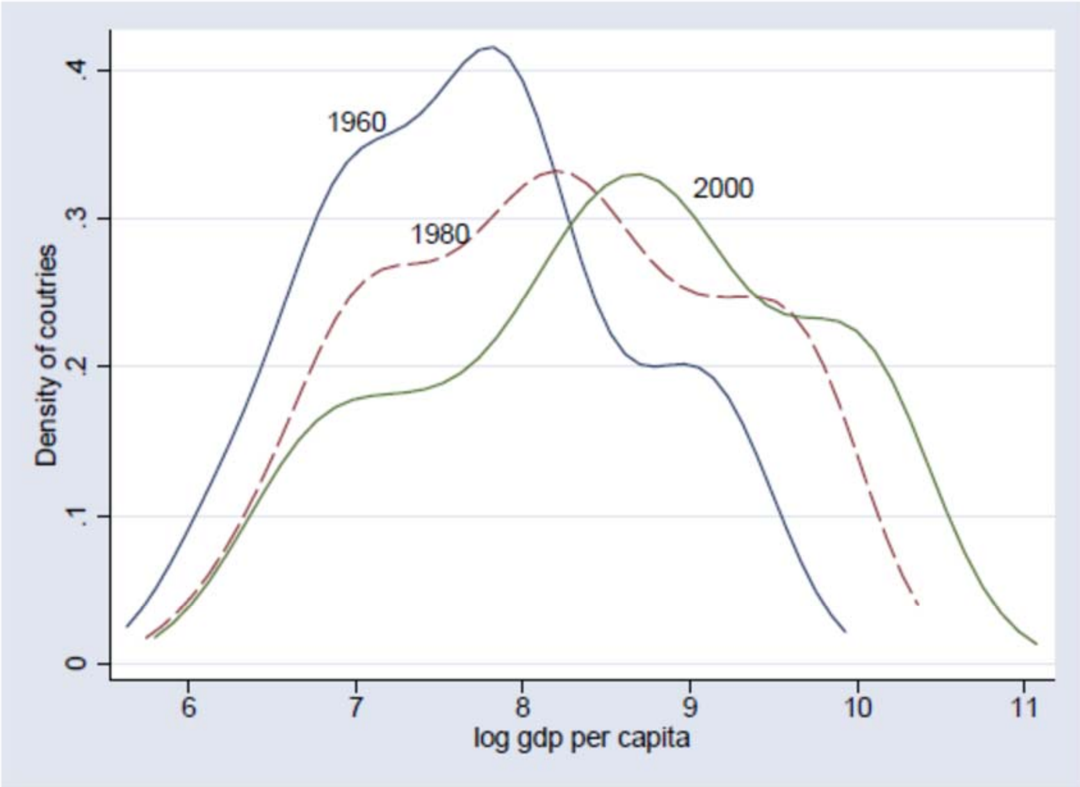


FIGURE 1.2. Estimates of the distribution of countries according to log GDP per capita (PPP-adjusted) in 1960, 1980 and 2000.

- An Example taken from European regions:

Fig. 1 Two spatial regimes in the initial per capita GRP identified by means of the Getis–Ord statistic, $G^*(\delta)$ (with t=1995, δ=350 km)

- The existence of two regimes A and B is suggested by the observation of <u>σ divergence of per-capita income between area A and B</u>.

● introduce the restriction:

regions in the same regime A, B converge to a uniform steady-state y_A*, y_B*

the restriction yields:

- β-convergence results consistent with Solow model (negative convergence coefficient)
- $y_A* > y_B*$ **constant in regression is higher for group A than for B**

Table 1 Two club-convergence testing in a cross-regional [256 regions] context in Europe, 1995–2000

	The *iid* specification with constant error variance (OLS)
Parameter estimates	
(p-values in brackets)	
Constant	
Club *A*	0.580 (0.000)
Club *B*	0.251 (0.000)
Beta	
Club *A*	−0.054 (0.000)
Club *B*	−0.021 (0.000)
Lambda	
The time to convergence	
Annual convergence rate	
(in percent)	
Club *A*	4.8
Club *B*	2.0
Half-distance to the steady-state	
(in years, 95% bounds in brackets)	
Club *A*	14.5 (11.7–19.1)
Club *B*	34.4 (25.4–53.2)

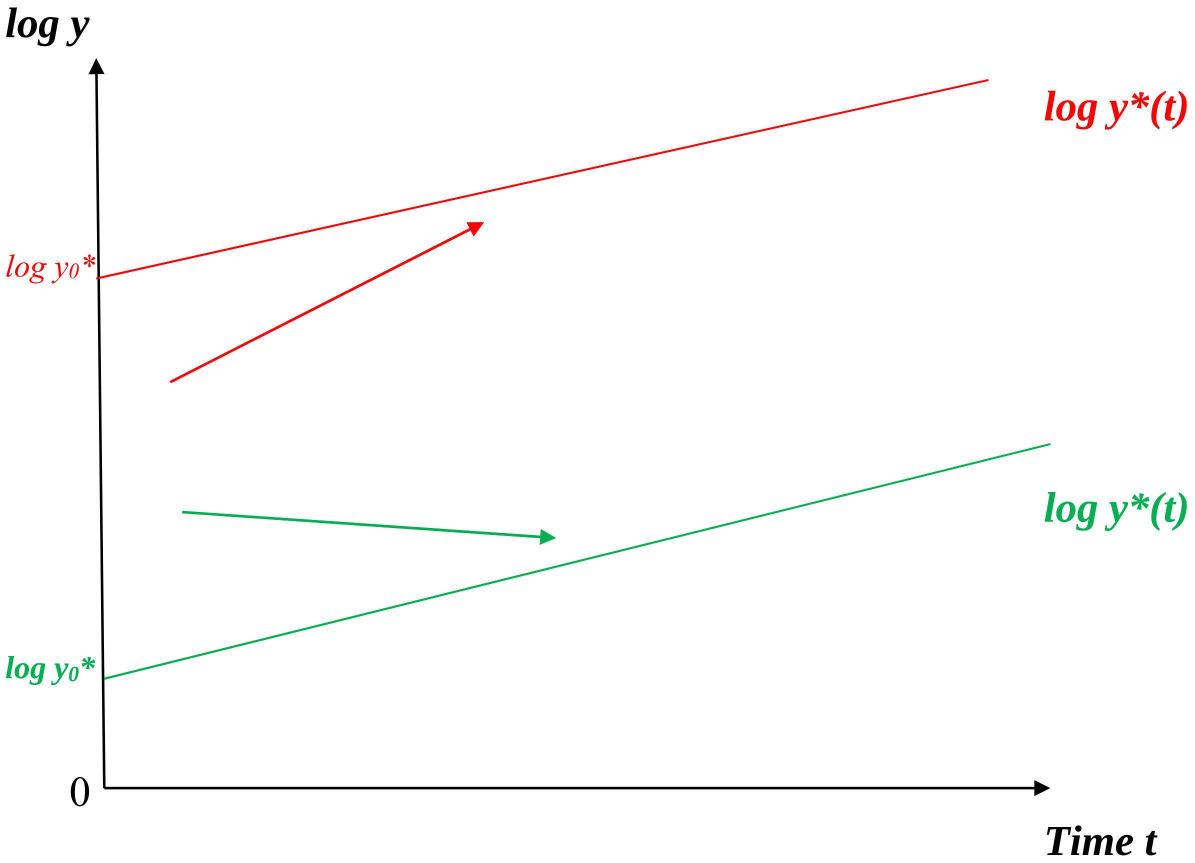
Remark:

What is surprising in the above regression results is that the estimates of b^1

$$b^1 = -(n + g + \delta)(1 - \alpha)$$

is different in the two regimes A, B. Differences in b^1 are larger than cross-regimes differences in population growth n may be able to explain.

Explaining episodes of divergence. The red versus the green country



Some variables in $X_{i,t}$ are hard to measure

By introducing ad hoc large differences in $\log y^*_{j,0}$, the Solow model can explain highly diverging dynamic behavior.

- $\log y^*_{j,0} = \log A_{j,0} + [\alpha / (1 - \alpha)] \log [s_{j,}/(n_j + g + \delta)]$

- s_j and n_j are easy to measure

- ‘technolgy’ A_j is hard to measure

Problem 2. In a theoretical framework broader than the Solow model:

- the separation between 'persistent growth component' and convergence may be questionable. It derives partly from the **assumption that growth is exogenous**.
- If the steady state growth rate depends on the propensity to save (**endogenous growth**), some variables in $X_{i,t}$ may not just determine '*the position of the steady state path*', but also the *growth rate*

- **$X_{i,t}$ may be endogenous : jointly determined with the temporary growth component : OLS estimates of regression coefficients may be biased**

  if variables in vector X are econometrically endogenous causal interpretations of the regression equation may be questionable

Problem 3:

Conditional convergence obtains but is far slower than Solow suggests.

$$\frac{\dot{y}}{y}(t) \approx g-(n+\delta+g)(1-\alpha)[\log y(t)-\log y^*(t)] \qquad (7)$$

Speed of convergence is measured by $\beta = (1-\alpha)(n+\delta+g)$

- speed of convergence is determined by how fast the marginal product of capital falls when *k* increases.
- Convergence is faster if *α* is closer to zero and slower if *α* is closer to 1.

Calibration yields counterfactual predictions about convergence speed:

- Focus on advanced economies
  - $g \simeq 0.02$ for approximately 2% per year output per capita growth,
  - $n \simeq 0.01$ for approximately 1% population growth and
  - $\delta \simeq 0.05$ for about 5% per year depreciation.
  - Share of capital in national income is about 1/3, so $\alpha \simeq 1/3$.

-

-

- $\beta = (1-\alpha)(n+\delta+g)$

- Thus convergence coefficient would be around 0.054 ($\simeq 0.67 \times 0.08$).
- Very rapid rate of convergence:
  - gap of income between two similar countries should be halved in little more than 10 years

-

remark (Cobb-Douglas case): $Y = K^{\alpha}(AL)^{1-\alpha}$ $\quad f(k) = k^{\alpha}$

$$\frac{K \cdot MPK}{Y} = \frac{k \cdot MPK}{f(k)} = \frac{k\alpha k^{\alpha-1}}{k^{\alpha}} = \alpha$$ easy to measure!!

Example 1: R. Barro, X. Sala-I-Martin, O. Blanchard, R. Hall (1991): converence in European regions

Table 6. Regressions for Gross Domestic Product across European Regions, 1950–85

	Basic equation		*Equation with country dummies*		*Equation with country dummies and structural variables*[a]	
Period	$\hat{\beta}$	$R^2[\hat{\sigma}]$	$\hat{\beta}$	$R^2[\hat{\sigma}]$	$\hat{\beta}$	$R^2[\hat{\sigma}]$
1950–60	0.0106 (0.0051)	0.06 [0.0155]	0.0105 (0.0038)	0.78 [0.0077]	0.0206 (0.0078)	0.80 [0.0076]
1960–70[b]	0.0367 (0.0066)	0.39 [0.0149]	0.0279 (0.0036)	0.92 [0.0057]	0.0241 (0.0062)	0.92 [0.0058]
1970–80[b]	0.0035 (0.0035)	0.01 [0.0098]	0.0184 (0.0049)	0.43 [0.0078]	0.0139 (0.0082)	0.44 [0.0078]
1980–85	0.0953 (0.0122)	0.60 [0.0212]	0.0116 (0.0048)	0.95 [0.0077]	0.0111 (0.0060)	0.96 [0.0070]
Four periods combined[c]						
β restricted	0.0183 (0.0029)	. . .	0.0186 (0.0021)	. . .	0.0178 (0.0034)	. . .
Likelihood ratio statistic[d]	70.9	. . .	13.3	. . .	2.6	. . .
P-value	0.000		0.004		0.457	

When β convergence coefficient is restricted to be the same across subperiods $\beta \approx 0.018$, much lower than theoretical value 0.054

Example 1: R. Barro, X. Sala-I-Martin, O. Blanchard, R. Hall (1991): converence across US States

Table 1. Regressions for Personal Income across U.S. States, 1880–1988

	Basic equation		*Equation with regional dummies*		*Equation with regional dummies and sectoral variables*[a]	
Period	$\hat{\beta}$	$R^2\,[\hat{\sigma}]$	$\hat{\beta}$	$R^2\,[\hat{\sigma}]$	$\hat{\beta}$	$R^2\,[\hat{\sigma}]$
1880–1900	0.0101	0.36	0.0224	0.62	0.0268	0.65
	(0.0022)	[0.0068]	(0.0040)	[0.0054]	(0.0048)	[0.0053]
1900–20	0.0218	0.62	0.0209	0.67	0.0269	0.71
	(0.0032)	[0.0065]	(0.0063)	[0.0062]	(0.0075)	[0.0060]
1920–30	−0.0149	0.14	−0.0122	0.43	0.0218	0.64
	(0.0051)	[0.0132]	(0.0074)	[0.0111]	(0.0112)	[0.0089]
1930–40	0.0141	0.35	0.0127	0.36	0.0119	0.46
	(0.0030)	[0.0073]	(0.0051)	[0.0075]	(0.0072)	[0.0071]
1940–50	0.0431	0.72	0.0373	0.86	0.0236	0.89
	(0.0048)	[0.0078]	(0.0053)	[0.0057]	(0.0060)	[0.0053]
1950–60	0.0190	0.42	0.0202	0.49	0.0305	0.66
	(0.0035)	[0.0050]	(0.0052)	[0.0048]	(0.0054)	[0.0041]
1960–70	0.0246	0.51	0.0135	0.68	0.0173	0.72
	(0.0039)	[0.0045]	(0.0043)	[0.0037]	(0.0053)	[0.0036]
1970–80	0.0198	0.21	0.0119	0.36	0.0042	0.46
	(0.0062)	[0.0060]	(0.0069)	[0.0056]	(0.0070)	[0.0052]
1980–88	−0.0060	0.00	−0.0005	0.51	0.0146	0.76
	(0.0130)	[0.0142]	(0.0114)	[0.0103]	(0.0099)	[0.0075]
Nine periods combined[b]						
β restricted	0.0175	. . .	0.0189	. . .	0.0224	. . .
	(0.0013)		(0.0019)		(0.0022)	
Likelihood-ratio statistic[c]	65.6	. . .	32.1	. . .	12.4	. . .
P-value	0.000		0.000		0.134	

Also in the second example estimated convergence parameter β is lower than predicted by the model

$$\frac{\dot{y}}{y}(t) \approx g - (n+\delta+g)(1-\alpha)[\log y(t) - \log y^*(t)] \qquad (7)$$

$$\beta = (n+\delta+g)(1-\alpha)$$

Hint:

adopting a broader notion of capital (physical + human capital):

- the capital income share would increase
- convergence to steady state would be slower!

Footnote (optional !!)

Step 1. $y^*(t) = A(t)\, f(k^*)$ and take logs

$$\log y^*(t) = \log A(t) + \log f(k^*)$$

Step 2.

consider k function of $\log k$: $k = e^{\log k} \rightarrow \partial k / \partial(\log k) = e^{\log k} = k$

Take Taylor expansion of $\log y$ as function of $\log k$ around $\log k^*$:

$$\log y(t) = \log y^*(t) + \frac{\partial[\log f(k^*)]}{\partial[\log k)]} \cdot [\log k(t) - \log k^*]$$

$$\frac{\partial[\log f(k)]}{\partial[\log k)]} = \frac{\partial[\log f(k)]}{\partial k} \cdot \frac{\partial k}{\partial \log k} = \frac{f_k(k^*)]}{f(k)]} \cdot k = \varepsilon_f(k^*)$$

$$\log y(t) \approx \log y^*(t) + \varepsilon_f(k^*) \cdot [\log k(t) - \log k^*] \qquad (6.2)$$

$$\log k(t) - \log k^* \approx [\varepsilon_f(k^*)]^{-1} [\log y(t) - \log y^*(t)] \qquad (6.2.bis)$$

Step 3.

$$(\partial k / \partial t) = sf(k) - (n + \delta + g)k$$

from first order expansion of *log k* around k^*: $\log k \approx \log k^* + (1/k)(k - k^*)$

taking a first order Taylor expansion of $\dot{k}(t)$ around k^*, using $[\log k(t) - \log k^*] \approx [1 - k^*/k]$ and substituting for s:

$$\frac{\dot{k}(t)}{k(t)} \approx (\partial + n + g)(\varepsilon_f(k^*) - 1) \cdot [\log k(t) - \log k^*] \quad (6.3)$$

Step 4. Use (6.2) together with log $y^*(t) = log\ A(t) + log\ f(k^*)$ to write:

$$\log y(t) \approx \log A(t) + \log f(k^*) + \varepsilon_f(k^*) \cdot [\log k(t) - \log k^*]$$

differentiating $\log y(t)$ with respect to time

$$\frac{\dot{y}(t)}{y(t)} \approx g + \varepsilon_f(k^*)\frac{\dot{k}(t)}{k(t)} \approx g - (\partial + n + g)[1 - \varepsilon_f(k^*)] \cdot \varepsilon_f(k^*)[log\ k(t) - log\ k^*]$$

1. to derive the previous result, substiute for the growth rate of k from 6.3
2. now substitute for $log\ k(t) - log\ k^*$ from 6.2.bis

$$\frac{\dot{y}(t)}{y(t)} \approx g - (\partial + n + g)[1 - \varepsilon_f(k^*)] \cdot [log\ y(t) - log\ y^*(t)]$$