2 The augmented Solow model of Mankiw, Romer, Weil, (QJE, 1992)

General motivation

● Can we explain the cross-country distribution of GDP per capita with a broader theory of capital accumulation?

● The assumption that the quality of labor is homogeneous across countries is too simplistic

● Human knowledge (human competence) accumulates through time, just as any other **producible factor**

● If 'capital' is interpreted as '**physical + human capital**' the output elasticity of 'capital' (the fraction of 'capital' income in GDP) is higher than 0.33. This may reconcile the Solow-model predictions with the empirical facts.

Objection

● Human knowledge is hardly separable from 'technology'. How can the former be 'endogenous' and the latter 'exogenous'?

The model

Assume Cobb Douglas technology for simplicity:

- Aggregate production function is

$$Y(t) = K(t)^{\alpha} H(t)^{\beta} (A(t) L(t))^{1-\alpha-\beta},$$

  where $0 < \alpha < 1$, $0 < \beta < 1$ and $\alpha + \beta < 1$.

- Households save a fraction s_k of their income to invest in physical capital and a fraction s_h to invest in human capital.
- Human capital also depreciates in the same way as physical capital, denote depreciation rates by δ_k and δ_h.
- Dividing both sides of production function by *AL (efficiency units)*

$$\hat{y} = \frac{Y}{AL} = \frac{K^{\alpha}}{(AL)^{\alpha}} \frac{H^{\beta}}{(AL)^{\beta}} \frac{(AL)^{1-\alpha-\beta}}{(AL)^{1-\alpha-\beta}} = k^{\alpha} h^{\beta}$$

where $k = \frac{K}{AL}$ $\qquad h = \frac{H}{AL}$

Remark:

according to MRW the technology to produce H and K are identical. Both forms of capital result directly from investment of final output in the accumulation of stocks.

Net investment in 'education' → $\dot{H} = s_h Y - \delta_h H$

Net investment in 'machinery' → $\dot{K} = s_k Y - \delta_k K$

- Assume constant population growth and a constant rate of labor-augmenting technological progress, i.e.,

$$\frac{\dot{L}(t)}{L(t)} = n \text{ and } \frac{\dot{A}(t)}{A(t)} = g.$$

Assume for simplicity same rate of depreciation: $\delta_h = \delta_k = \delta$

in steady state k^* and h^* are constant, which requires that:

$$s_k {k^*}^{\alpha} {h^*}^{\beta} = (\delta + n + g)k^*$$

$$s_h {k^*}^{\alpha} {h^*}^{\beta} = (\delta + n + g)h^*$$

Dividing side by side: $$\frac{s_k}{s_h} = \frac{k^*}{h^*}$$

Solving for $k*$ and $h*$ we obtain

$$k^* = \left(\frac{s_k^{1-\beta} s_h^{\beta}}{\delta + n + g}\right)^{1/(1-\alpha-\beta)}$$

$$h^* = \left(\frac{s_k^{\alpha} s_h^{1-\alpha}}{\delta + n + g}\right)^{1/(1-\alpha-\beta)}$$

Using $\hat{y} = k^{\alpha} h^{\beta}$, we can write:

$$\hat{y}^* = \left(\frac{s_k^{1-\beta} s_h^{\beta}}{\delta + n + g}\right)^{\alpha/(1-\alpha-\beta)} \left(\frac{s_k^{\alpha} s_h^{1-\alpha}}{\delta + n + g}\right)^{\beta/(1-\alpha-\beta)}$$

Taking logs, some terms cancel out and the expression above simplifies to:

$$log\hat{y}^* = \frac{\alpha}{1-\alpha-\beta} log\left(\frac{s_k}{\delta + n + g}\right) + \frac{\beta}{1-\alpha-\beta} log\left(\frac{s_h}{\delta + n + g}\right)$$

$$logy^*(t) = \ logA(t) + log\hat{y}^*$$

$$\log y^*(t) = \log A(t) + \frac{\alpha}{1-\alpha-\beta}\log s_k + \frac{\beta}{1-\alpha-\beta}\log s_h - \frac{\alpha+\beta}{1-\alpha-\beta}[n+g+\delta]$$

$\frac{\alpha}{1-\alpha-\beta}$ is the elasticity of $y^*(t)$ relative to Sk

$\frac{\beta}{1-\alpha-\beta}$ is the elasticity of $y^*(t)$ relative to Sh

Remark: the augmented Solow model is a true generalization of Solow 1956: by assuming β = 0, the last term vanishes and we are back in the Solow model.

- Mankiw, Romer and Weil (1992) used regression analysis to take the augmented Solow model, with human capital, to data.
- Use the Cobb-Douglas model and envisage a world consisting of $j = 1, \ldots, N$ countries.
- "Each country is an island": countries do not interact (perhaps except for sharing some common technology growth).
- Country $j = 1, \ldots, N$ has the aggregate production function:

$$Y_j(t) = K_j(t)^{\alpha} H_j(t)^{\beta} (A_j(t) L_j(t))^{1-\alpha-\beta}.$$

- Countries differ in terms of technology $A_j(t)$ saving rates $s_{k,j}$ and $s_{h,j}$, and population growth rates n_j

Technology:

- Mankiw, Romer and Weil (1992) make the following assumption:

$$A_j(t) = \bar{A}_j \exp(gt).$$

- Countries differ according to technology *level*, (initial level $\bar{A}_j$) but they share the same common technology growth rate, g.

$\overline{A_j} = \varepsilon_j A$ $\quad$ ε is an exogenous i. i. d. technology shock

- initial technology $\overline{A_j}$ is **assumed uncorrelated to the exogenous explanatory variables** ***s_k* and *s_h*** .

- $logA_j(t) = logA + log\varepsilon_j + gt$

● Focus on a world in which the effects of transitional dynamics are observable in the data. Countries are in a neighbourhood of their steady state.

- Mankiw, Romer and Weil (1992) take:
  - $\delta_k = \delta_h = \delta$ and $\delta + g = 0.05$.
  - $s_{k,j}$=average investment rates (investments/GDP).
  - $s_{h,j}$=fraction of the school-age population that is enrolled in secondary school.

Omitting the country-subscript j

$$\log y^*(t) = \log A(t) + \frac{\alpha}{1-\alpha-\beta}\log s_k + \frac{\beta}{1-\alpha-\beta}\log s_h - \frac{\alpha+\beta}{1-\alpha-\beta}[n+g+\delta]$$

The dependent variable is country logGDP per-capita $log y^*_{j,t}$ at time t

The explanatory exogenous variables are $logA_{j,t}, logs_{k,j}, logs_{h,j}$

Technology differences across countries reflect accidental i.i.d. shocks

$$logA_j(t) = logA + log\varepsilon_{j,t} + gt$$

- $logy_t^* =$

$$C + \frac{\alpha}{1-\alpha-\beta}\log s_{k,j} + \frac{\beta}{1-\alpha-\beta}\log s_{h,j} - \frac{\alpha+\beta}{1-\alpha-\beta}[n_j + g + \delta] + \log \varepsilon_{j,t}$$

$$\boldsymbol{C = logA + gt}$$

C = regression constant is uniform across countries!

Predictions:

1. regression coefficient of $log s_k$ is positive and $\approx \frac{\alpha}{1-\alpha-\beta}$
2. regression coefficient of $log s_h$ is positive and $\approx \frac{\beta}{1-\alpha-\beta}$
3. regression coefficient of $log(n+g+\delta)$ is $\approx -\frac{\alpha+\beta}{1-\alpha-\beta}$
4. implied $\alpha \approx$ capital share in income
5. implied $\beta \approx$ human-capital share in income

Estimates of the Augmented Solow Model

	MRW	Updated data	
	1985	1985	2000
$\ln(s_k)$	.69	.65	.96
	(.13)	(.11)	(.13)
$\ln(n+g+\delta)$	-1.73	-1.02	-1.06
	(.41)	(.45)	(.33)
$\ln(s_h)$	.66	.47	.70
	(.07)	(.07)	(.13)
Adj R^2	.78	.65	.60
Implied α	.30	.31	.36
Implied β	.28	.22	.26
Observations	98	98	107

Findings on H-augmented Solow

- **the implied α is now consistent with the evidence $\alpha \approx 0.3$**

- If these regression results are reliable, they give a big boost to the augmented Solow model.
  - Adjusted R^2 suggests that three quarters of income per capita differences across countries can be explained by differences in their physical and human capital investment.
- Immediate implication is technology (TFP) differences have a somewhat limited role.
- But this conclusion should not be accepted without further investigation.

- Technology differences across countries are not exogenous, hence are not uncorrelated to the other variables. Equivalently:
- **Technology differences across countries are not orthogonal to all other variables.**
- $\bar{A}_j$ is correlated with measures of s_j^h and s_j^k for two reasons.
  1. *omitted variable bias*: societies with high $\bar{A}_j$ will be those that have invested more in technology for various reasons; same reasons likely to induce greater investment in physical and human capital as well.
  2. *reverse causality*: complementarity between technology and physical or human capital imply that countries with high $\bar{A}_j$ will find it more beneficial to increase their stock of human and physical capital.
- In terms of (19), implies that key right-hand side variables are correlated with the error term, ε_j.
- OLS estimates of α and β and R^2 are biased upwards.

1. *Omitted variable bias* means that **technology is not truly exogenous** but depends on various factors, that are omitted in the model.
   - **The factors explaining investments in R&D, in machinery, in human capital will also affect technology**.
   - **For instance, new technology is mostly embodied in fixed-capital equipment and requires human capital for its use.**

   - **factors affecting investment in physical and human capital**

     $\downarrow$

     Technology adoption

2. *Reverse causality* means there is a channel of causality from

   Technology

   $\downarrow$

   accumulation of physical and human capital

   In countries with a more efficient institutional and technological structure, there are conditions that are more favorable to investment.

estimate of β is too large

- **check predictions following from estimates of α and β against microeconometric evidence of earning and marginal productivity effects of education:**

- If markets are competitive as assumed in the model, the **marginal productivity effects of education should be reflected by the earnings of educated and non-educated workers**:

  What is the extra wage rate that 'on average' the labour market assigns to a worker who has taken one extra year of education? This is an issue addressed by micro-econometric empirical analyses of the US labour market, in the spirit of the 1974 book by Jacob Mincer: *Schooling, experience and wages*.

- This provides a way of estimating the size of β from microeconomic evidence

- **This check suggests that the MRW estimate of β is too large!**

- A related problem is that regression results are very sensitive to the way in which we measure human capital H

Conclusion on Solow with physical and human capital

- It is hard to explain cross-country differences in GDP per capita, only in terms of their differences concerning

$$\frac{\text{physical capital formation}}{GDP} \quad \text{and} \quad \frac{\text{human capital formation}}{GDP}$$

- We have to admit that a fundamental reason why GDP per capita differs across-countries is that there are differences in technology
- We need a theory to explain why

E. Prescott (1998) performs a calibration analysis based on the 'augmented Solow model', and confirms the above conclusion

● Prescott's 'intangible capital'

Human capital is but one form of 'intangible capital' which is largely unmeasured and missing in government statistics. Intangible capital includes not only school training to population in working age, but also on- the-job training, firm specific learning by doing, organization capital, and various forms of unmeasured R&D investment.

Unmeasured investments I in official statistics imply that there is also unmeasured output Y, because Y = C + I. The unconventional part of Prescott's calibration exercise is addressed at dealing with this problem, but we skip this for simplicity.

Prescott's argument

$$ln\ y_j^* = costant + (1-\alpha-\beta)^{-1}[\alpha ln s_{kj} + \beta ln s_{hj} - (\alpha+\beta)ln(n+g+\delta)] + \varepsilon_j$$

- According to IMF estimates physical capital **investment as a share of GDP is about 20% for rich and poor countries after 1960**. This implies that, countries do not differ much in their investment ratios s_k.

- Thus, the burden of explaining cross country differences in $ln\ y_j^*$ falls largely on the human-capital-investment share s_h.

- If we preserve the share s_h in a plausible range for rich and poor countries, …

- it is required that the elasticity **β** of output with respect to 'intangible capital' H **is very high, indeed higher than is suggested by available evidence**.

Prescott’s Conclusion:

● The burden of explaining per-capita income differences must partly fall on understanding why the *constant* in equation (19) is NOT uniform across countries, that is, **what is needed, is a theory about efficiency**.

● Prescott holds to the basic neoclassical assumption that technical knowledge is transferable across countries at low cost: International differences in total factor productivity must then be explained through institutionally based differences in work practices, not in useful knowledge. These differences affect the level of A…

Introduction to endogenous growth

1. The reason why in neoclassical model with exogenous technological progress capital per worker K/L grows through time in steady state at rate g, but MPK does not fall, is that MPK is just preserved by the exogenous growth of efficiency A_t .
2. This leaves the problem of explaining the determinants of efficiency growth $\frac{\dot{A}_t}{A_t}$.
3. This explanation may be obtained in various ways. The explanation offered by the large family of endogenous growth theories has one property in common: the steady state growth rate of efficiency, that causes the steady state growth rate of per-capita output depends on the savings rate, hence on preferences.
4. This requires assuming that technology embeds a special linearity assumption.

One approach to explaining why technology A differs across countries …   is making A endogenous

As example, consider the argument made by M. Frankel (1962)

$$Y = K^{\alpha} L^{1-\alpha} \hat{A}$$

where $\hat{A}_t = A \left(\frac{K_t}{L_t}\right)^{\beta}$ is an externality effect:

a higher K/L ratio in the economy increases firm level efficiency

$$Y_t = A K_t^{\alpha+\beta} L_t^{1-\alpha-\beta}$$

the special case $\alpha + \beta = 1$ yields

$$Y_t = A K_t$$

$$Y_t = AK_t$$

Inada conditions are not fulfilled!

$$sY_t = sAK_t$$

$$K_{t+1} = sAK_t + (1-\delta)K_t$$

divide by $$L_{t+1} = (1+n)L_t$$

$$k_{t+1} = \frac{sAk_t + (1-\delta)k_t}{1+n}$$

$$k_{t+1} = \frac{(1+sA-\delta)}{1+n}k_t$$

$$k_{t+1} = \frac{(1 + sA - \delta)}{1 + n} k_t$$

$$\frac{y_{t+1}}{y_t} = \frac{k_{t+1}}{k_t} = \frac{(1 + sA - \delta)}{1 + n}$$

persistent growth of per-capita output if

$$\frac{(1 + sA - \delta)}{1 + n} > 1$$

That is *if* $sA > \delta + n$

The growth rate of GDP per capita is:

$$g^* = \frac{y_{t+1}}{y_t} - 1 = \frac{(1 + sA - \delta)}{1 + n} - 1 = \frac{sA - \delta - n}{1 + n}$$

Persistent growth in the Y = AK model

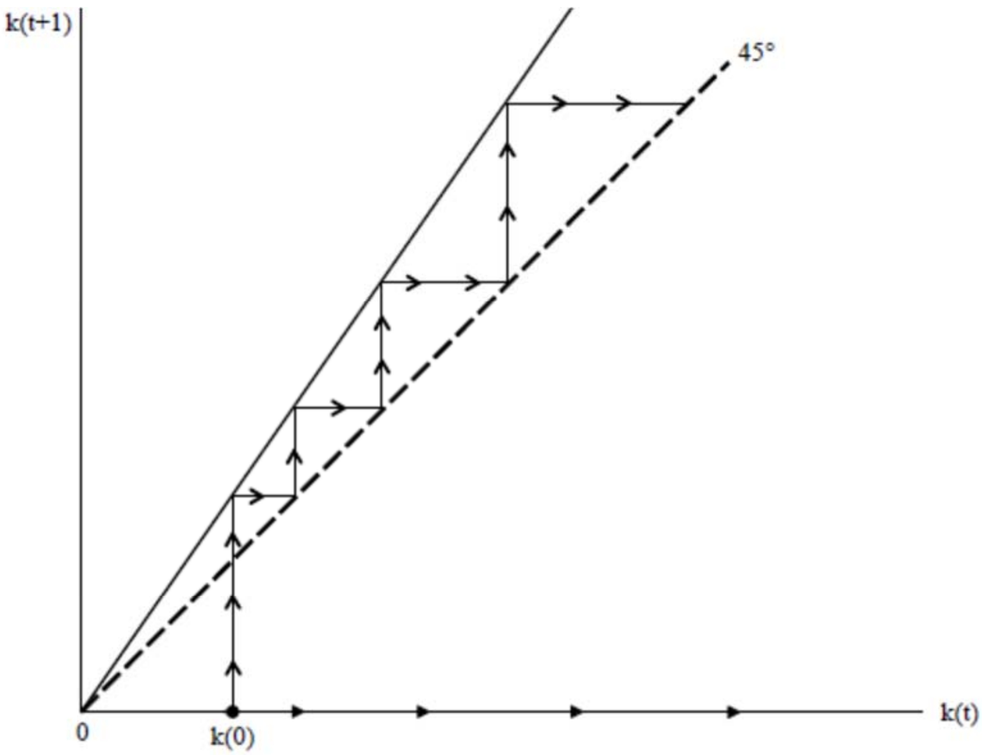

FIGURE 2.10. Sustained growth with the linear AK technology with $sA - \delta - n > 0$.

Transitional growth in the Solow model with constant L and A

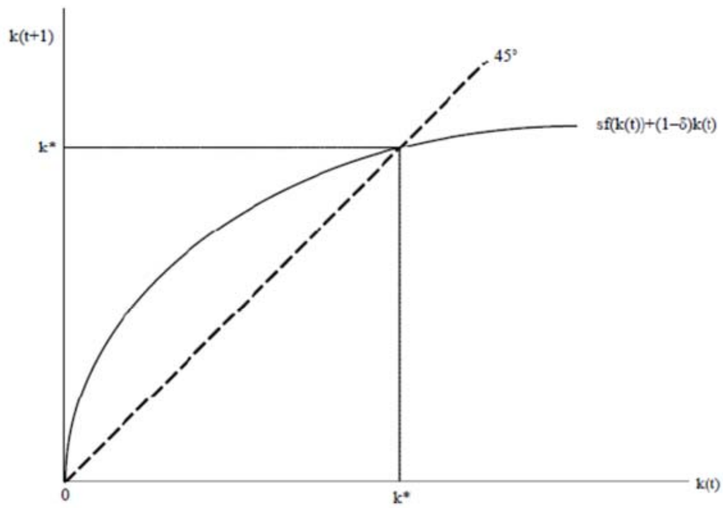

FIGURE 2.2. Determination of the steady-state capital-labor ratio in the Solow model without population growth and technological change.

Why doesn't capital [and human capital] flow from rich to poor countries? (R. Lucas, 1990)

● Unlike MRW, countries are not islands

● Financial capital and human capital flow across countries.

● If we assume that financial and human capital flow where returns are higher, we should expect a tendency towards the equalization of the rates on returns on both forms of capital.

● Neoclassical model with financial flows assumes *perfect* mobility of financial capital → <u>equalization of rates of returns is instantaneous.</u> Any initial difference in initial conditions concerning capital per worker would be instantaneously equalized!

● Lucas' 1990 reply to the question in the title draws upon his 1988 two sector AK model with human capital: perfectly competitive economy, with externality effects.

Lucas 1988:

- Final output Y produced by human and physical capital (no raw labour)
- no exogenous technological progress
- human capital accumulation equation different from MRW
- economy is perfectly competitive, but may embed externalities. The model without externalities is:

$$Y_t = K_t^{\alpha}\left[u_t h_t L_t\right]^{1-\alpha} \tag{1}$$

$$\dot{h}_t = \theta(1 - u_t)h_t \tag{2}$$

- θ is an exogenous constants expressing the efficiency in the process of knowledge accumulation (human capital accumulation).
- *h* is per-capita human capital
- *u* is fraction of time spent by *h* in final-output sector
- *L* is population growing at the exponential rate *n*

Divide equation (1) by L, and define y = Y/L, *k = K/L* *(capital per worker)*

$$y = k^{\alpha} (uh)^{(1-\alpha)} \tag{3}$$

$$\dot{h}_t = \theta(1-u_t)h_t \tag{3'}$$

endogenous growth is related to the linear structure of system (3), (3')

- In sector y there are constant returns to the scale to factors *k* and *h.*
- *The production of new human capital us linear to the input h.*

In particular: $g_h{}^* = \theta(1-u^*)$

steady state growth rate *g** is determined by steady state fraction of time (1– *u**) spent accumulating *h*.

u* is an endogenous decision variable, that depends on preferences!!

$y = k^{\alpha} (uh)^{(1-\alpha)}$

$g_y = \alpha g_k + (1-\alpha)(g_u + g_h)$ (4)

in steady state: $g_y^* = g_k^*$ and $g_u^* = 0$ $u(t) = u^* = \text{constant}$ (5)

$(1-\alpha)\, g_y^* = (1-\alpha)\, g_h^* \rightarrow \quad g_y^* = g_h^*$

Growth of output per capita is explained by growth of human capital per person

$g_y^* = g_h^*$

$g_h^* = \theta(1 - u^*)$

In a optimizing framework, u(t) and u* depend on preferences:

Other things equal, lower u(t) today implies:

- higher future human capital stock h(t + dt)
- lower per-capita output y(t) today, and lower per-capita consumption c(t) today

$$y_t = k_t^{\alpha}\left[u_t h_t\right]^{1-\alpha}$$ Lucas (1)

$$y_t = k_t^{\alpha}\left[A_t\right]^{1-\alpha}$$ Solow

Notice that k is here capital per worker!

now compare output per worker in USA and India,

$y_{\text{USA\_1980}} / y_{\text{India\_1980}} \approx 15$ (Summers and Heston 1988, pp 18-21)

Solow model: suppose A_t is uniform in USA and India

we can normalize $A_t = 1$.

$$y_t = k_t^{\alpha} \qquad\qquad k_t = y_t^{1/\alpha}$$

The gross rate of return to K is

$$r = \alpha k^{\alpha - 1} = \alpha y^{(\alpha - 1)/\alpha}$$

Lucas takes α = 0.4 (average of US and India capital shares)

$$\frac{r_{India}}{r_{USA}} = \left[\frac{y_{India}}{y_{USA}}\right]^{(\alpha-1)/\alpha} = \left[\frac{1}{15}\right]^{(\alpha-1)/\alpha} \approx 58$$

Huge differences in rates of return to K are implied by the differences in GDP/L

So what stopped K from flowing from USA to India???

Lucas model

Dividing equation (1) by the stock $H_Y = uhL$ of human capital in Y sector Output per efficient worker in Y sector is:

$$\hat{y} = \hat{k}^{\alpha}$$

where $\hat{y} = \frac{Y}{uhL}$ output per efficient worker in y sector

$\hat{k} = \frac{K}{uhL}$ capital per efficient worker in y sector

$$\hat{k} = \hat{y}^{1/\alpha}$$

the marginal product of capital is:

$$r = \alpha\hat{k}^{\alpha - 1} = \alpha\hat{y}^{\alpha - 1/\alpha} \qquad (6)$$

To estimate output per efficient worker Lucas has to resort to the 1959 estimates by Ann Krueger:

After taking into account education attainments, the ratio of output per efficient worker in 1959 USA and India is much lower than for GDP/L

$$\frac{\hat{y}_{USA}}{\hat{y}_{India}} \approx \frac{\left(\frac{y}{uh}\right)_{USA}}{\left(\frac{y}{uh}\right)_{India}} \approx 3$$

the ratio was 15 for GDP/L

if we consider output per-capita instead of output per efficient worker, the ratio would be far higher, because average education attainment in 1959 USA is far higher than in 1959 India

With α = 0.4 (average of USA and India capital shares) this gives:

$$\frac{r_{USA}}{r_{India}} = \left(\frac{\hat{y}_{USA}}{\hat{y}_{India}}\right)^{\alpha-1/\alpha} = \left[\frac{\left(\frac{y}{uh}\right)_{USA}}{\left(\frac{y}{uh}\right)_{India}}\right]^{(\alpha-1)/\alpha} \approx 3^{\alpha-1/\alpha} \approx 1/5$$

Lucas observes that the ratio is still too large to be consistent with the evidence on capital flows.

For this reason, he adds an externality in equation (1):

$$Y_t = K_t^{\alpha}\left[u_t h_t L_t\right]^{1-\alpha} h^{\gamma} \tag{1'}$$

$$y_t = k_t^{\alpha}\left[u_t h_t\right]^{1-\alpha} h^{\gamma} \tag{6'}$$

so that output per efficient worker is now:

$$\hat{y} = \hat{k}^{\alpha} h^{\gamma}$$

$$\hat{k} = \hat{y}^{1/\alpha} h^{\gamma/\alpha}$$

the reason for the externality is that **private human capital accumulation augments the socially available useful knowledge**, which increases the productivity of private factors in the output sector.

The productivity of each worker depends **positively** on the average education and productivity of the workers she is working with.

From Denison's USA data concerning 1909-1958, Lucas estimates the external effect $\gamma \approx 0.36$. *"A 10% increase in the average quality of those with whom I work increases my productivity by 3.6%"*.

$$r = \alpha\hat{k}^{\alpha-1}h^{\gamma} \qquad (*)$$

$$\hat{k} = \hat{y}^{1/\alpha}h^{-\gamma/\alpha} \qquad (**)$$

Substitute for $\hat{k}$ in equation * from equation **

$$r = \alpha\hat{y}^{\alpha-1/\alpha}h^{(1-\alpha)\gamma/\alpha}h^{\gamma} = \alpha\hat{y}^{\alpha-1/\alpha}h^{\gamma/\alpha}$$

$$\frac{r_{USA}}{r_{India}} = \left(\frac{\hat{y}_{US}}{\hat{y}_{Ind}}\right)^{\alpha-1/\alpha}\left(\frac{h_{US}}{h_{Ind}}\right)^{\gamma/\alpha} = \left(\frac{\hat{y}_{Ind}}{\hat{y}_{US}}\right)^{1-\alpha/\alpha}\left(\frac{h_{US}}{h_{Ind}}\right)^{\gamma/\alpha}$$

Lucas 1990 indication:

1. On the crucial assumption that **socially available useful knowledge is country specific**, in the sense that **there are not knowledge spillovers across countries**, the estimated γ *"exactly eliminates the [capital] return differential in a 1959 India – U.S. comparison"*.
2. To explain per-capita income differences between the rich countries and those that, unlike modern India, have failed to enter the mechanism of modern economic growth, we must break free of the straight jacket of the convex neoclassical model without externalities.

Problem: As Lucas admits, the weakness in this argument is that it **does not provide an explanation of why the poorest nations are unable to exploit the useful knowledge created outside.**