consumption externality by

two Agents A, B

24 hours a day

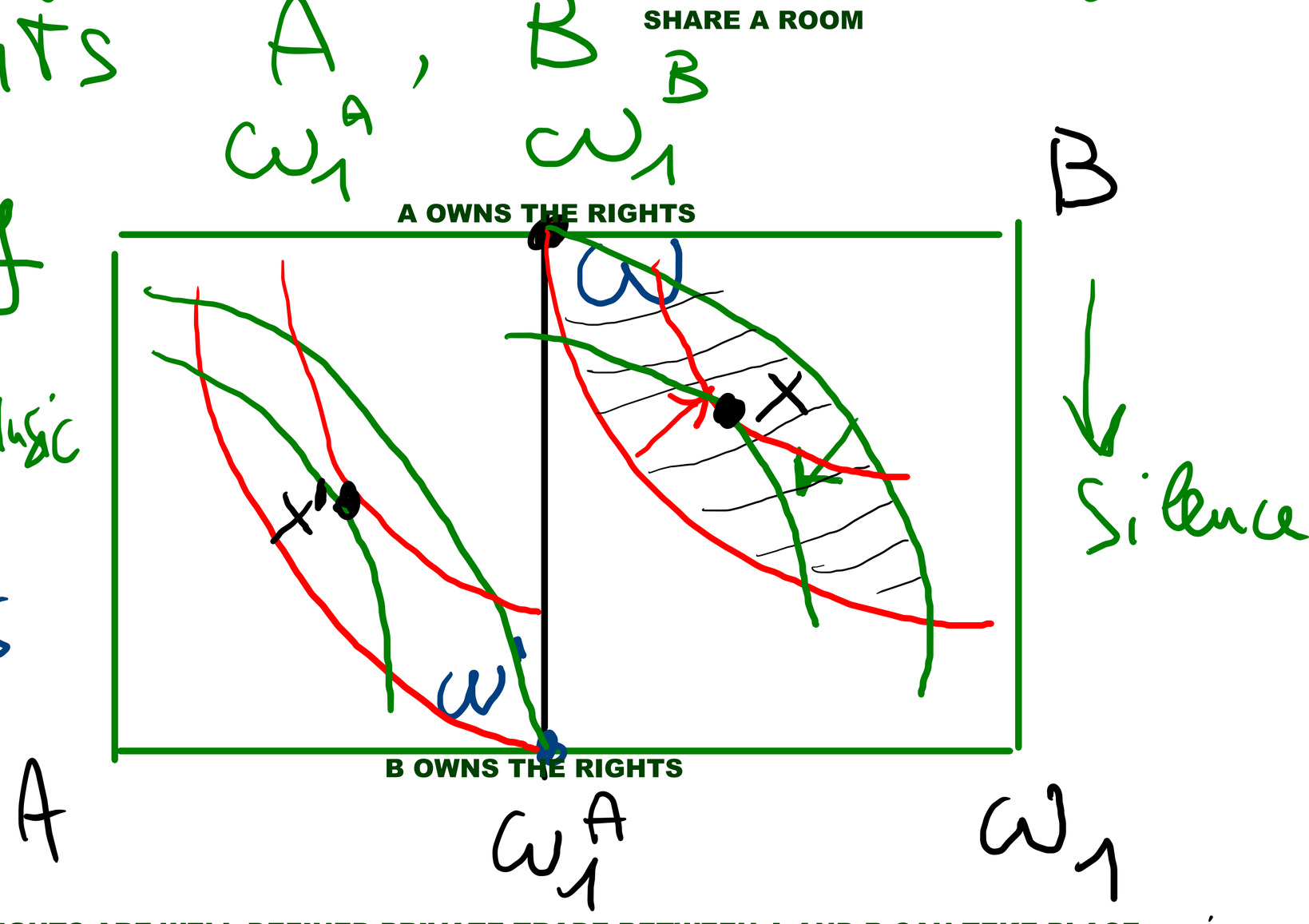
Music / Silence

M / S

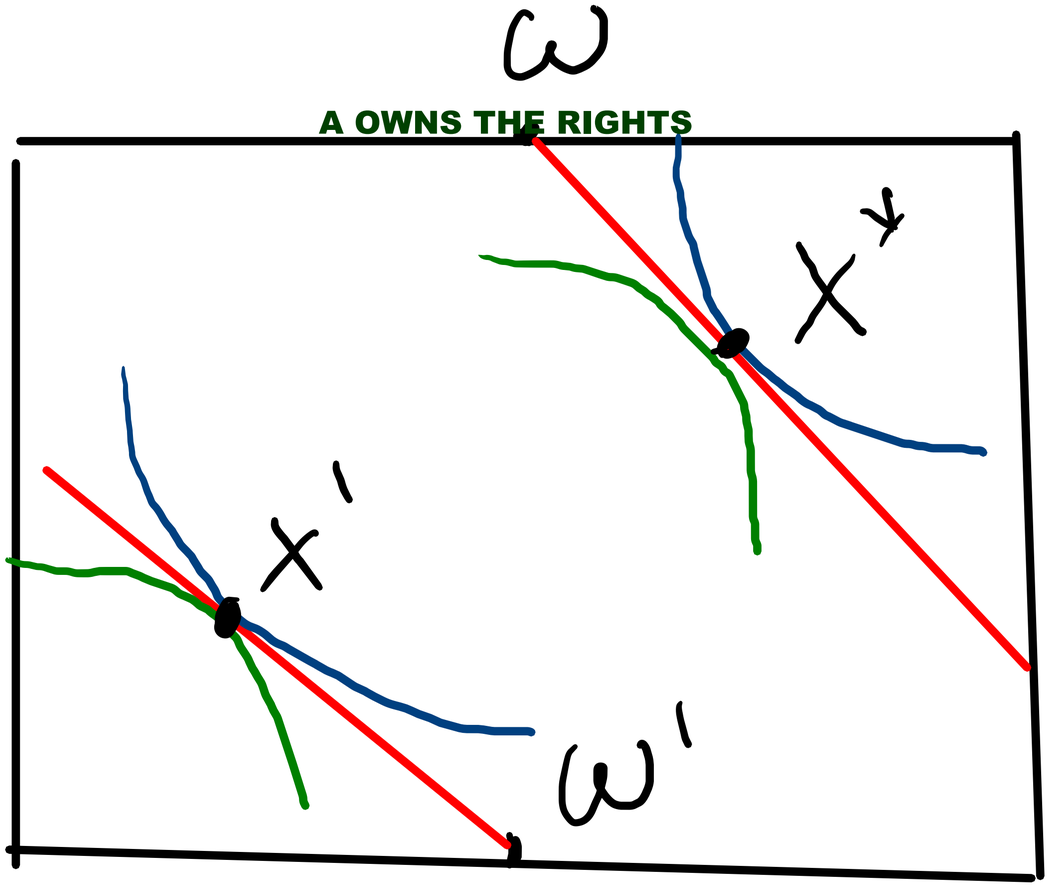
A owns the rights to M/S

$U^A(x_1^A, M)$

$U^B(x_1^B, S)$



AS LONG AS RIGHTS ARE WELL DEFINED PRIVATE TRADE BETWEEN A AND B CAN TEKE PLACE AND BENEFITS FROM TRADE CAN BE FULLY EXPLOITED IF THERE ARE NOT TRANSACTION COSTS



B

$\omega^A = (\omega_1^A, \omega_2^A)$

$\omega^B (\omega_1^B, \omega_2^B)$

Silence

$p_1 x_1^A + p_2 x_2^A = \omega_1^A p_1 + \omega_2^A p_2$

$\omega_1 p_1 x_1^B + p_2 x_2^B = \omega_1^B p_1 + \omega_2^B p_2$

$p_1 = 1$

AS LONG AS RIGHTS ARE WELL DEFINED AN ORGANISED COMPETITIVE MARKET FOR THE RIGHTS TO MUSIC/SILENCE CAN BE CREATED. TRADE BETWEEN A AND B IS MEDIATED BY COMPETITIVE MARKETS

Quasi linear preferences

$$u^A(x_1^A, x_2^A) = x_1^A + v(x_2^A) \quad |MRS^A| = \frac{1}{v'(x_2^A)}$$

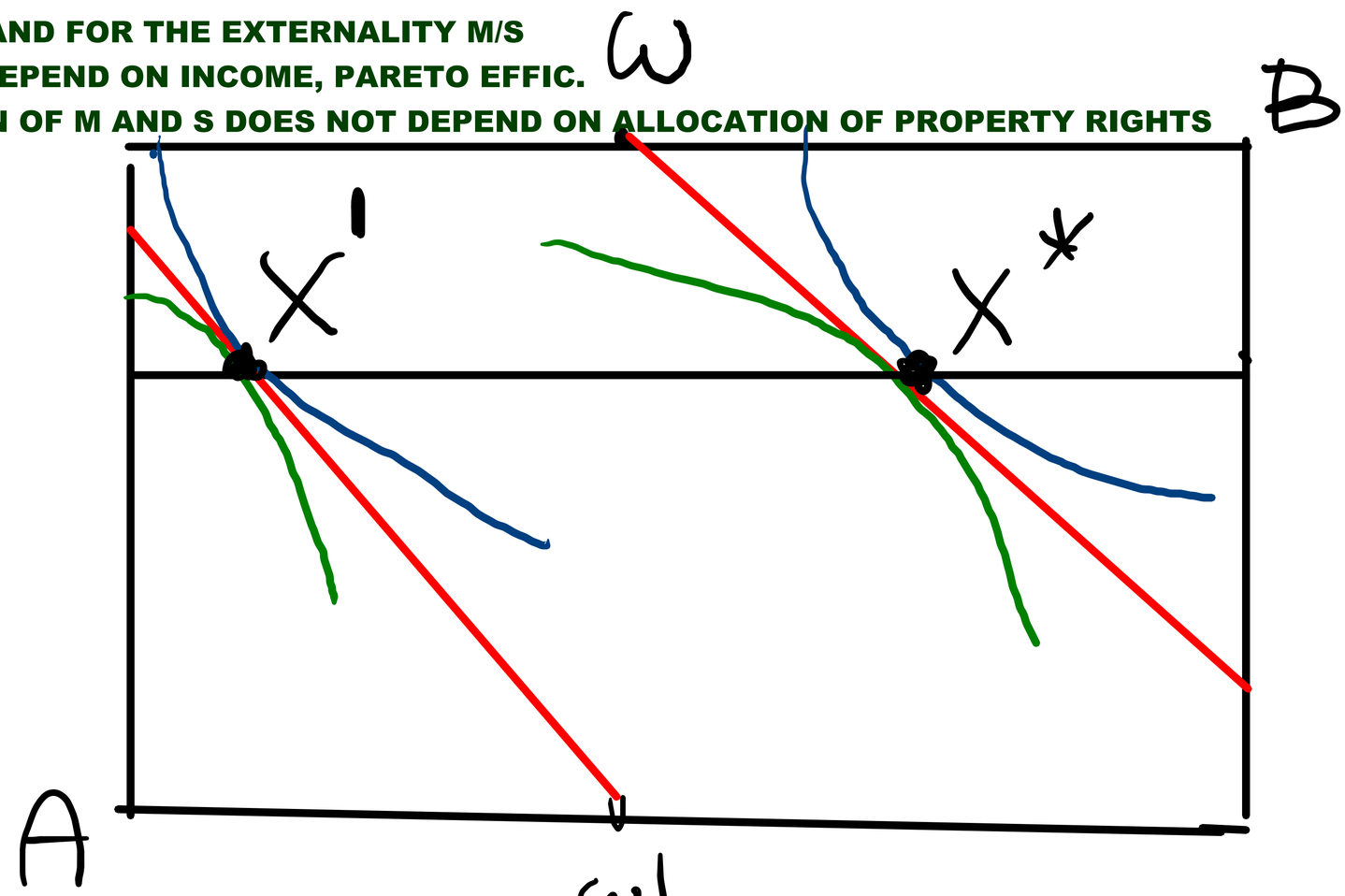
$$u^B(x_1^B, x_2^B) = x_1^B + q(x_2^B) \quad |MRS^B| = \frac{1}{q'(x_2^B)}$$

Pareto efficient interior allocation

$$\frac{1}{v'(x_2^A)} \Leftrightarrow |MRS^A| = |MRS^B| = \frac{1}{q'(x_2^B)}$$

$$x_2^A = 24 - x_2^B$$

IF THE DEMAND FOR THE EXTERNALITY M/S DOES NOT DEPEND ON INCOME, PARETO EFFIC. ALLOCATION OF M AND S DOES NOT DEPEND ON ALLOCATION OF PROPERTY RIGHTS



$$MRS^A = -\frac{1}{MU_2^A(x_2^A)}$$

$$MRS^B = -\frac{1}{MU_2^B(x_2^B)}$$

$$-\frac{p_1}{p_2} = MRS^A = MRS^B$$

$p_1 = 1$ $\quad x_2^A(p_2) + x_2^B(p_2) = \omega_2 = 24$

determines p_2^*

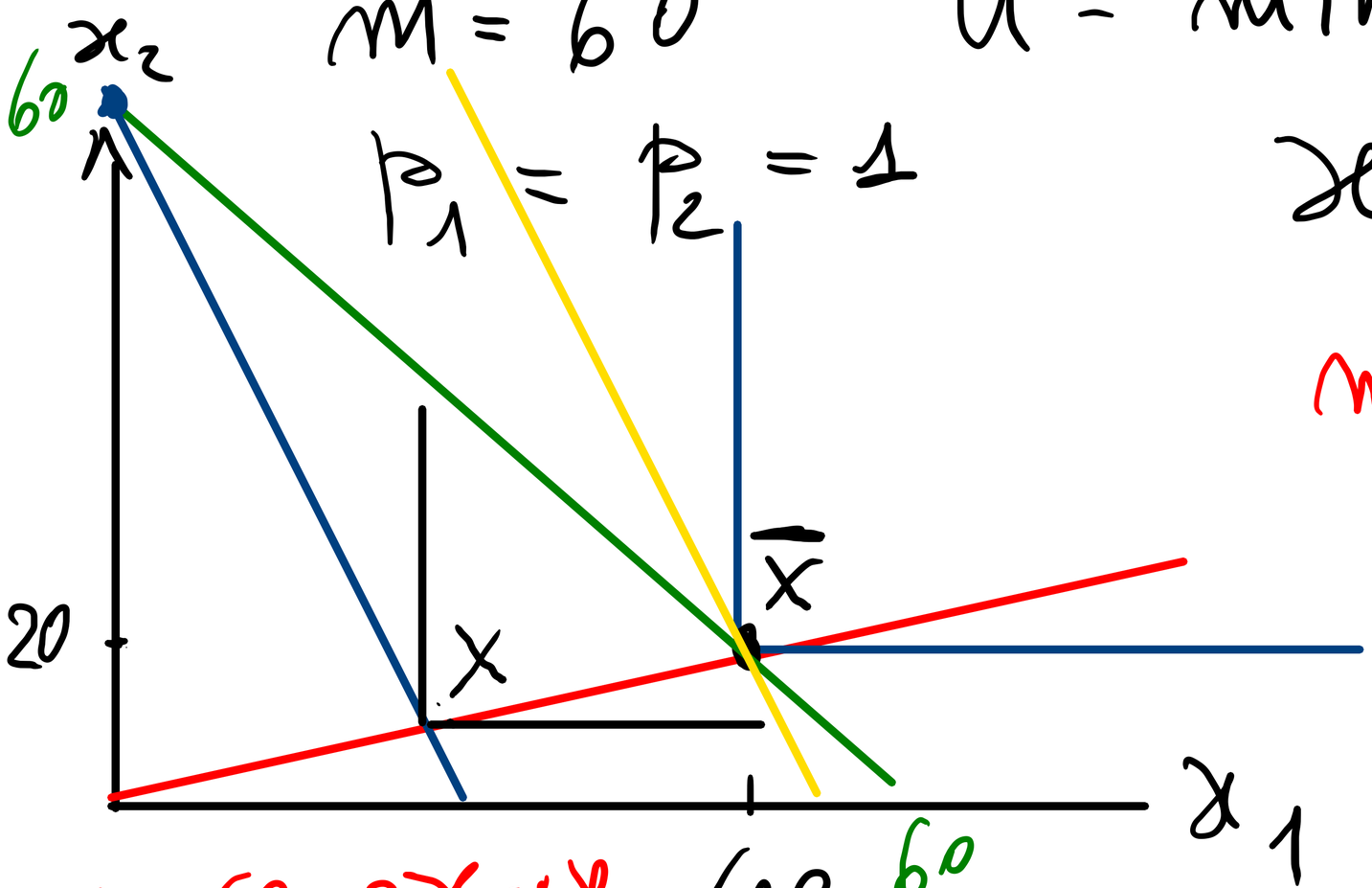
Coase Theorem

① if property rights are well defined and there are no transaction costs voluntary trade leads to Pareto efficient outcomes

② if the demand of the externality does not depend on income, the Pareto efficient quantity of externality does not depend on the allocation of property rights

$m = 60$ $\quad u = \min(x_1, 2x_2)$

$p_1 = p_2 = 1$ $\quad x_1 = 2x_2$ $\quad \frac{x_2}{x_1} = \frac{1}{2}$



$m = 60 = x_1 + x_2$

$60 = 2x_2 + x_2$

$\bar{x}_2 = 20$

$\bar{x}_1 = 40$

$m = 60 = 2x_1 + x_2$ $\quad 60 = 4x_2 + x_2$ $\quad x_2 = 12$ $\quad x_1 = 24$

$p_1' = 2$ $\quad x_1^s = \bar{x}_1$

$m' = 60 + \bar{x}_1 \Delta p_1 = 100$ $\quad \Delta x_1^s = x_1 - \bar{x}_1 = 0$

$x_1 - \bar{x}_1 = \Delta x_1 = \Delta x_1^m = 24 - 40 = -16$