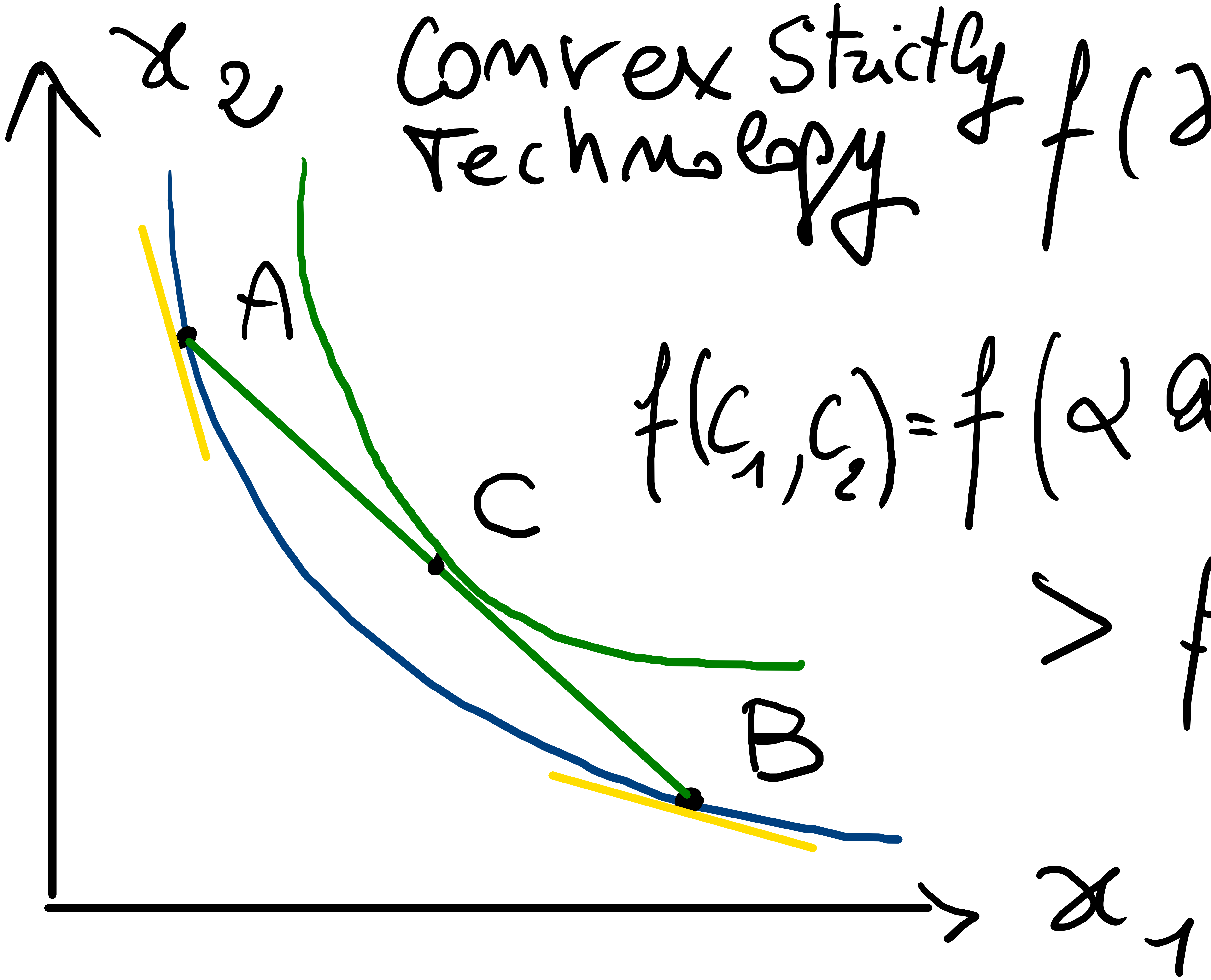
increasing RETURNS to scale:

$$f(\lambda x_1, \lambda x_2) > \lambda f(x_1, x_2) \quad \lambda > 1$$

decreasing RETURNS to scale:

$$f(\lambda x_1, \lambda x_2) < \lambda f(x_1, x_2) \quad \lambda > 1$$

Convex Strictly Technology



$$f(x_1, x_2) = y$$

$$f(c_1, c_2) = f(\alpha a_1 + (1-\alpha) b_1, \alpha a_2 + (1-\alpha) b_2)$$

$$> f(a_1, a_2) = f(b_1, b_2)$$

along 1 isoquant: $\partial y = 0 = \partial x_1 MP_1 + \partial x_2 MP_2$

TECHNICAL RATE OF SUBSTITUTION = $TRS = \dfrac{\partial x_2}{\partial x_1} = -\dfrac{MP_1}{MP_2}$

$|TRS|$ = number of units of x_2 that are replaced by 1 extra unit of input 1, in order that output y is constant.

Example : Cobb-Douglas with constant returns to scale

$$y = x_1^{\alpha} \cdot x_2^{1-\alpha} = x_1^{c} \cdot x_2^{d} \quad c+d=1$$

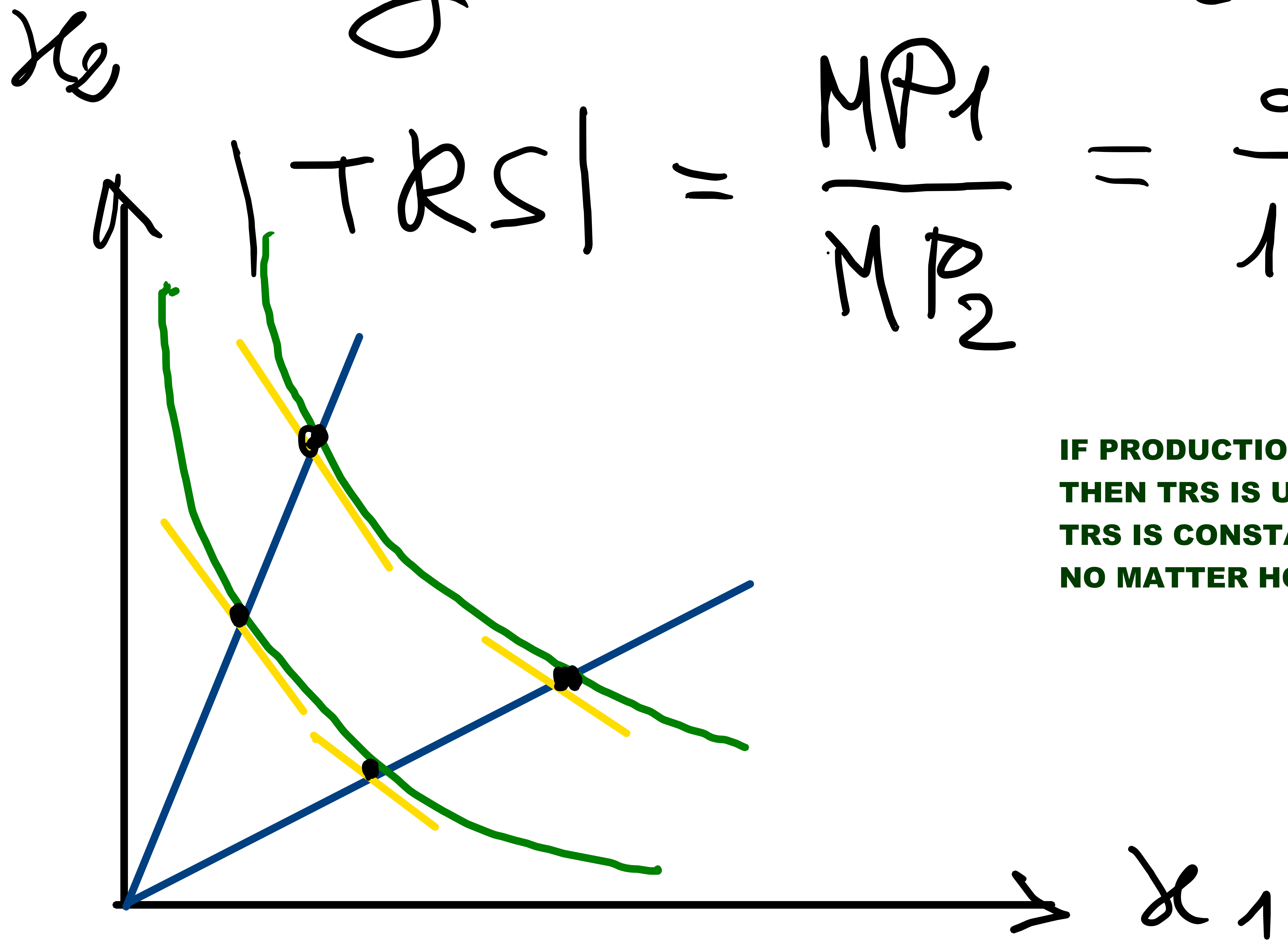
$$\lambda y = (\lambda x_1)^{\alpha} \cdot (\lambda x_2)^{1-\alpha} = \lambda \cdot \left(x_1^{\alpha} \cdot x_2^{1-\alpha}\right)$$

increasing RETURNS to Scale : $c + d > 1$

decreasing " " " : $c + d < 1$

$$y = x_1^{\alpha} \cdot x_2^{1-\alpha}$$

$$|TRS| = \frac{MP_1}{MP_2} = \frac{\alpha}{1-\alpha} \cdot \frac{x_2}{x_1}$$



IF PRODUCTION FUNCTION f(x1, x2) HAS CONSTANT RETURNS TO SCALE, THEN TRS IS UNIQUELY DETYERMINED BY THE RATIO x2 / x1
TRS IS CONSTANT ON EVERY STRAIGHT LINE THROUGH THE ORIGIN, NO MATTER HOW LARGE IS THE OUTPUT y

Profit Maximisation

WE CONSIDER A FIRM IN A COMPETITIVE INDUSTRY. MORE PRECISELY WE ASSUME

perfect competition. THIS IS CHARACTERISED BY TWO PROPERTIES

1. price taking ON THE OUTPUT AND ON THE INPUTS MARKETS

2. free entry AND FREE EXIT OF FIRMS INTO AND OUT OF THE INDUSTRY

FIRM PROFIT = $\pi = P_y \cdot y - \sum_{i=1}^{i=n} W_i \cdot x_i$

quasi-fixed cost = cost which is fixed as a result of an indivisibility

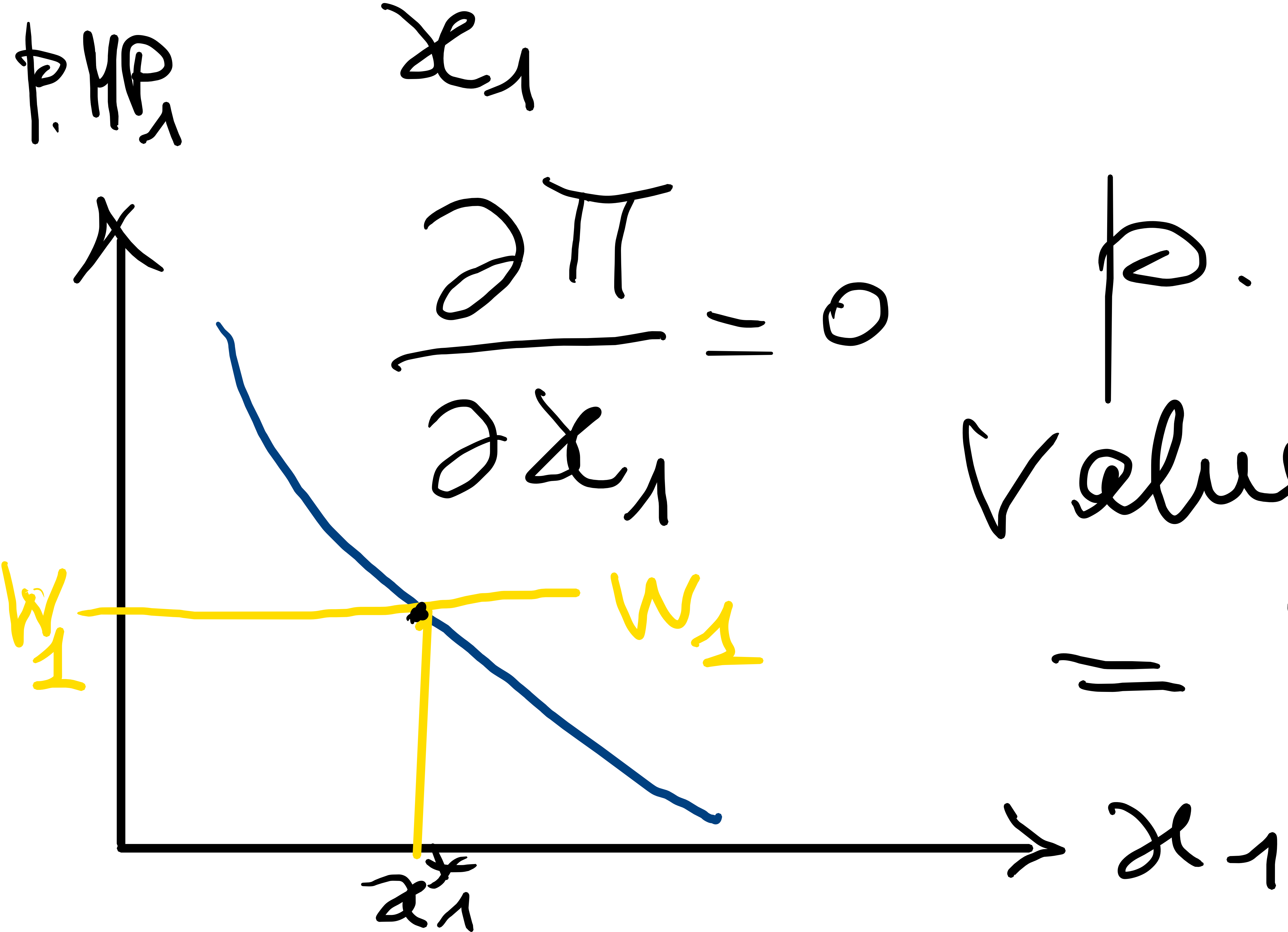
Short-run $\bar{x}_2$ **FIXED FACTOR**

$$\max_{x_1} \pi = p \cdot f(x_1, \bar{x}_2) - w_1 x_1 - w_2 \bar{x}_2$$

$$\frac{\partial \pi}{\partial x_1} = 0 \qquad p \cdot MP_1(x_1, \bar{x}_2) = w_1$$

value marg product of x_1

$= w_1$

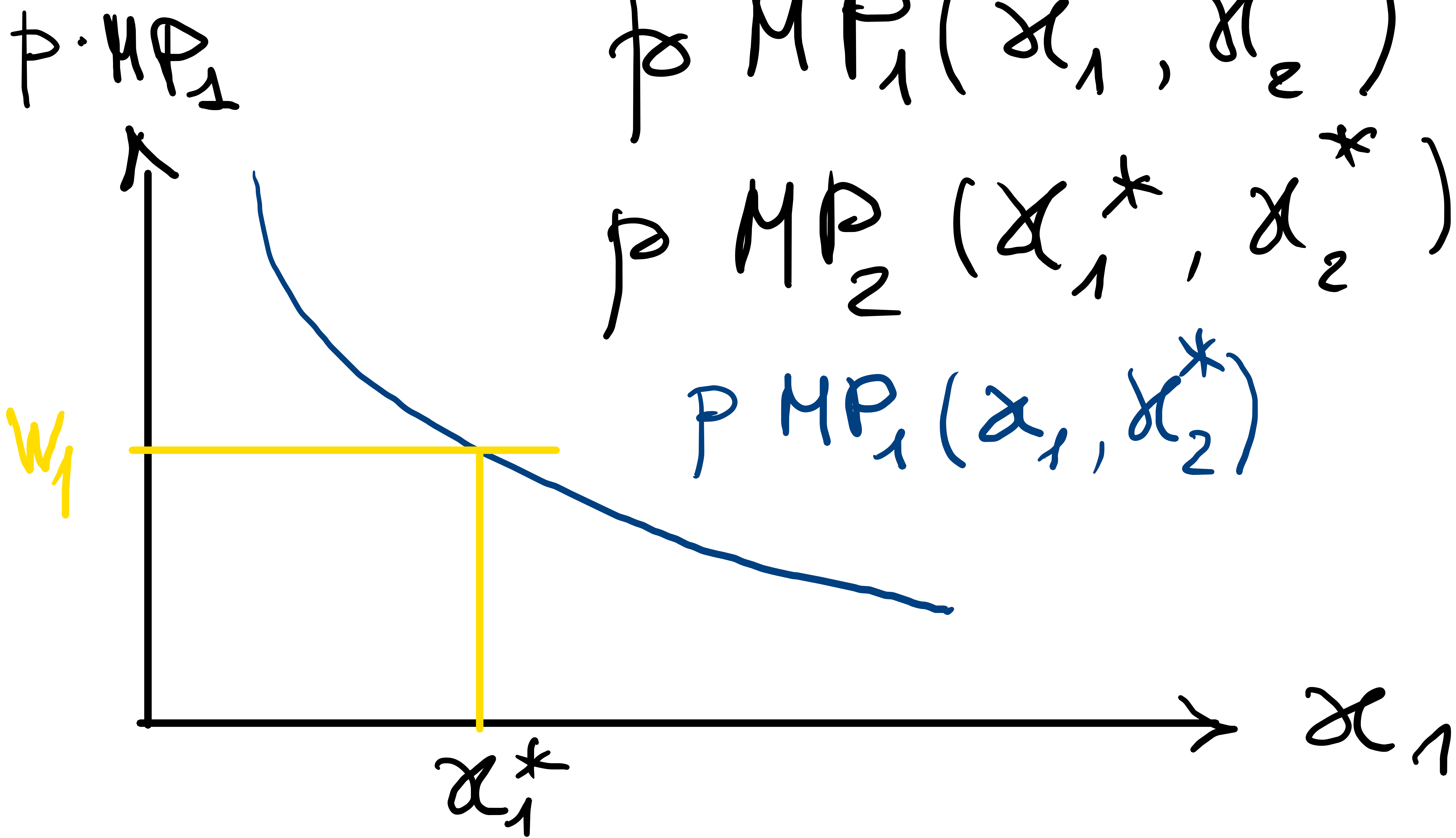


Long-Run

$$\underset{x_1,\, x_2}{\text{Max}}\ \pi : \ p\, f(x_1, x_2) - w_1 x_1 - w_2 x_2$$

$$p\, MP_1(x_1^*, x_2^*) = w_1$$

$$p\, MP_2(x_1^*, x_2^*) = w_2$$



x2* IS THE OPTIMAL QUANTITY OF INPUT 2 AT PRICES p, w1, w2

price taking (perfect competition)

$\lambda > 0 \quad f(\lambda x_1, \lambda x_2) = \lambda f(x_1, x_2)$

CONSTANT RETURNS TO SCALE PRODUCTION FUNCTION

assume (x_1^*, x_2^*) solves $\max \pi^*$

$: p f(x_1, x_2) - w_1 x_1 - w_2 x_2$

and $x_1^* > 0$ finite, $x_2^* > 0$ finite

then $\pi^* = 0$ is THE MAXIMUM PROFIT

if $\pi^* > 0$ ANY FINITE π^* is not maximum

if $\pi^* < 0$ " " " "

Problem: Max Π given p, w_1, w_2

can be split in 2 parts

① $\min_{x_1, x_2} w_1 x_1 + w_2 x_2$ s.t. $f(x_1, x_2) = y$

cost minimisation of producing y

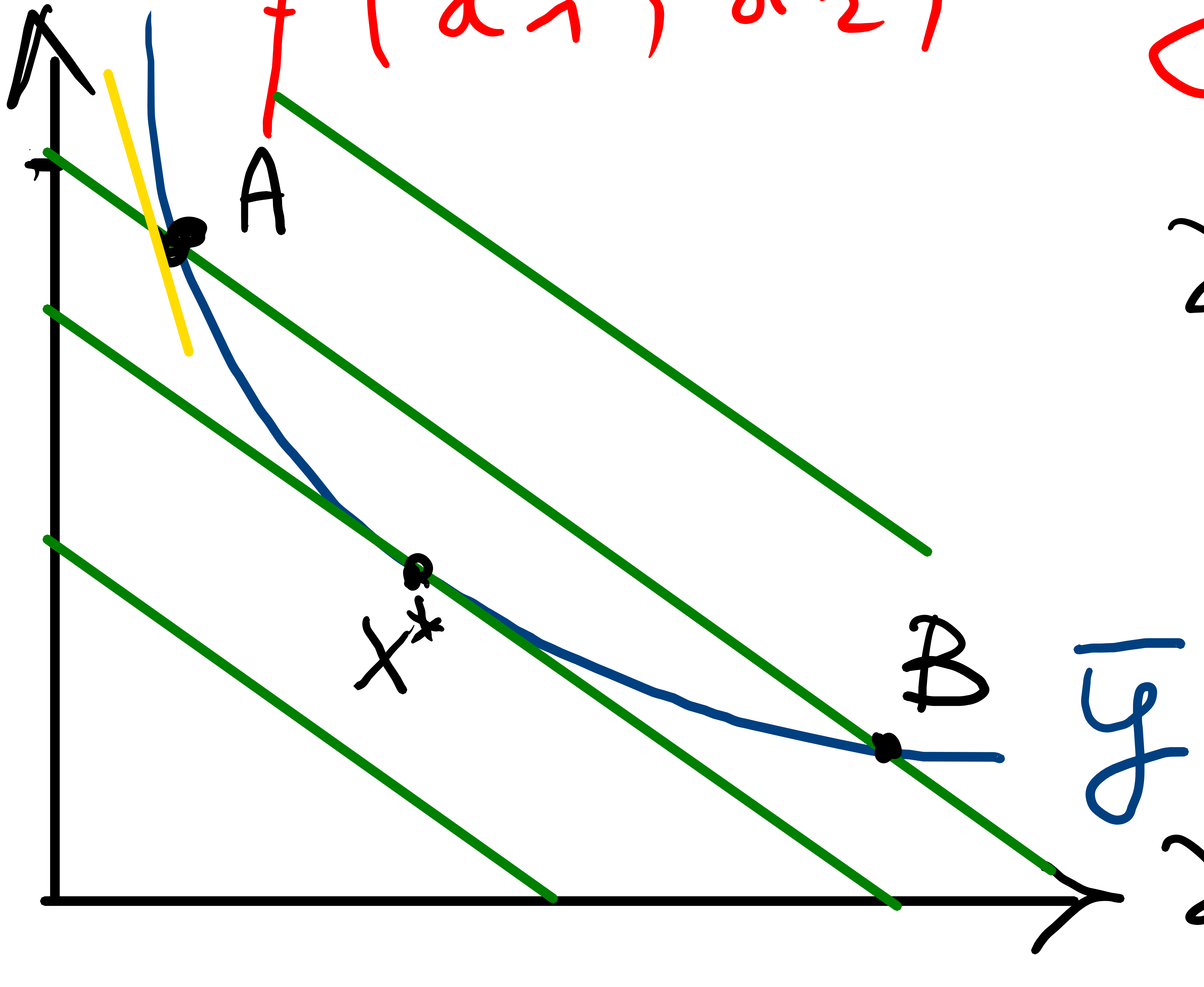
IF WE REPEAT THE PROCEDURE FOR EVERY POSSIBLE OUTPUT y WE OBTAIN THE COST FUNCTION C(y) = min cost to produce y

$C(y) = c(w_1, w_2, y)$ = cost function

② $\max_y : py - c(y) \longrightarrow y^* = \Pi$ maximising output y

COST MINIMISATION: LET US CONSIDER THE PROBLEM OF MINIMISING THE COST OF PRODUCING A GIVEN OUTPUT

$$f(x_1, x_2) = y \qquad \bar{y} \qquad w_1, w_2$$



$$x_1 \cdot w_1 + x_2 w_2 = C$$

$$x_2 = \frac{C}{w_2} - \frac{w_1}{w_2} \cdot x_1$$

equation of constant-cost line

(ISO - COST STRAIGHT LINE)

at A: $x_1 > 0$, $x_2 > 0$, $\frac{MP_1}{MP_2} > \frac{w_1}{w_2}$ $\qquad \frac{1}{MP_1} \cdot w_1 < \frac{1}{MP_2} \cdot w_2$

$$\frac{MP_1(x^*)}{MP_2(x^*)} = \frac{w_1}{w_2}$$

1st order condition for cost minimisation

techology convex

CONVEXITY IMPLIES THAT THE SECOND ORDER CONDITION IS FULFILLED: X* IS A COST-MINIMISING INPUT COMBINATION
IF TECHNOLOGY IS CONCAVE THE 1ST ORDER CONDITION ABOVE IDENTIFIES A COST-MAXIMISING INPUT COMBINATION
IN THIS CASE THE COST-MINIMISING INPUT COMBINATION IS A BOUNDARY SOLUTION, SUCH THAT THE FIRM WANTS TO USE ONLY ONE INPUT