Cournot Duopoly

$MC_1 = MC_2 = c$

$$c_1(y_1) = c_2(y_2) \qquad p(y) = a - by$$

$$y = y_1 + y_2 \qquad p(y_1^e + y_2)\cdot y_2 = R_2^e$$

$$MR_2 = MC = c$$

Firm 2 Revenue given y_1^e (expectation)

$$y_2 = f_2(y_1^e)$$

$$y_1 = f_1(y_2^e)$$

Cournot Equil = Nash Equil $\begin{cases} y_1^* = y_1^e \\ y_2^* = y_2^e \end{cases}$

EXPECTATIONS ARE FULFILLED

Cournot Equil = Nash Equil $\begin{cases} y_1^* = f_1(y_2^*) \\ y_2^* = f_2(y_1^*) \end{cases}$

EAC FIRM IS MAKING AN ACTION WHICH IS A BEST RESPONSE TO THE ACTION MADE BY THE OTHER FIRM

Since $MC_1 = MC_2$

$\to y_1^* = y_2^*$

Symmetric Equil.

$$p = a - by \qquad MC = c$$

$$R_2^e = p \cdot y_2 = \left(a - b(y_1^e + y_2)\right) y_2$$

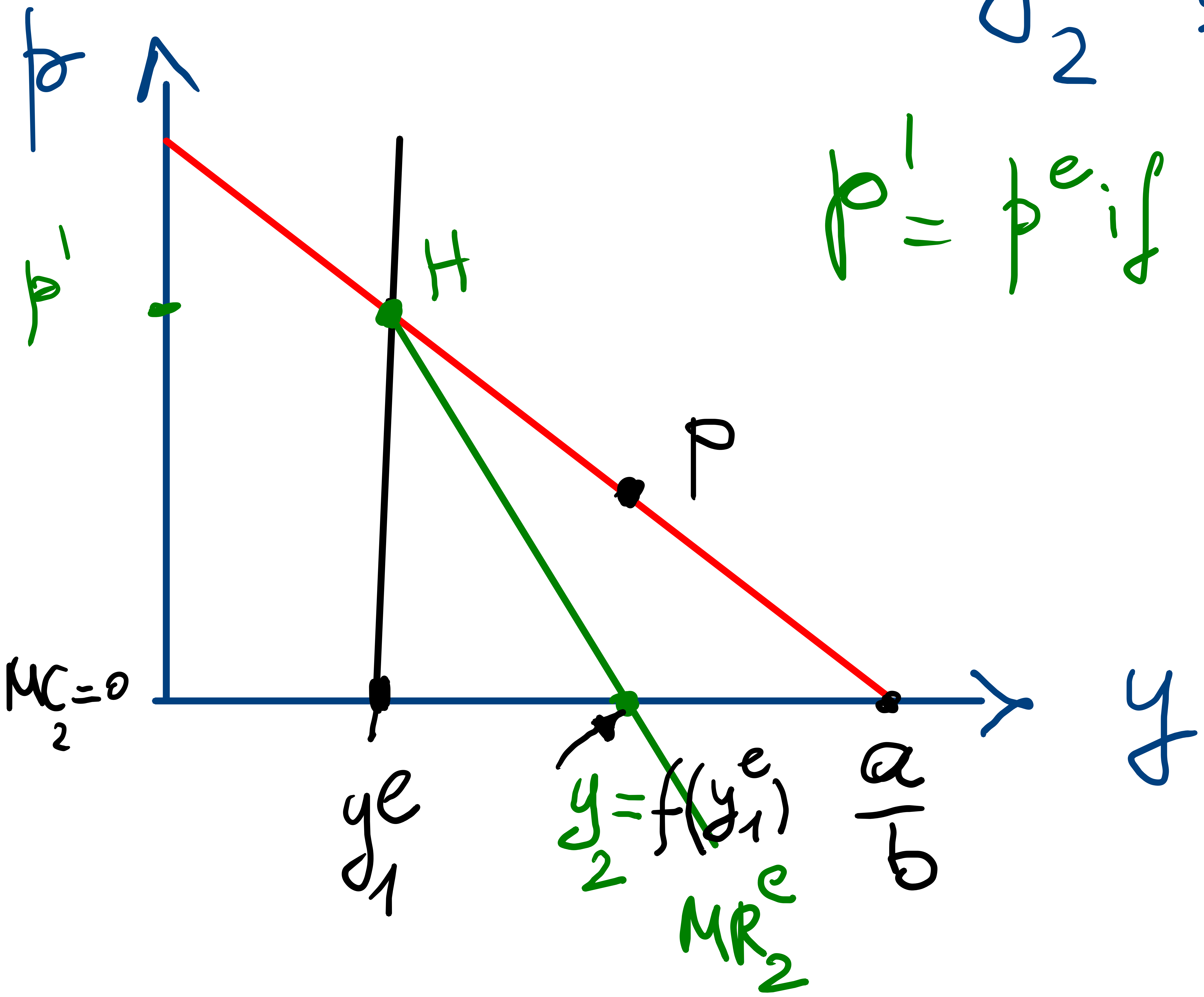
$$= a y_2 - b y_1^e \cdot y_2 - b y_2^2$$

$$MR_2^e = a - b y_1^e - 2 b y_2 = MC = c$$

$$y_2 = \frac{a - c}{2b} - \frac{1}{2} y_1^e$$

$MC = c = 0$

$$y_2 = \frac{a}{2b} - \frac{1}{2} y_1^e = \frac{1}{2}\left(\frac{a}{b} - y_1^e\right)$$

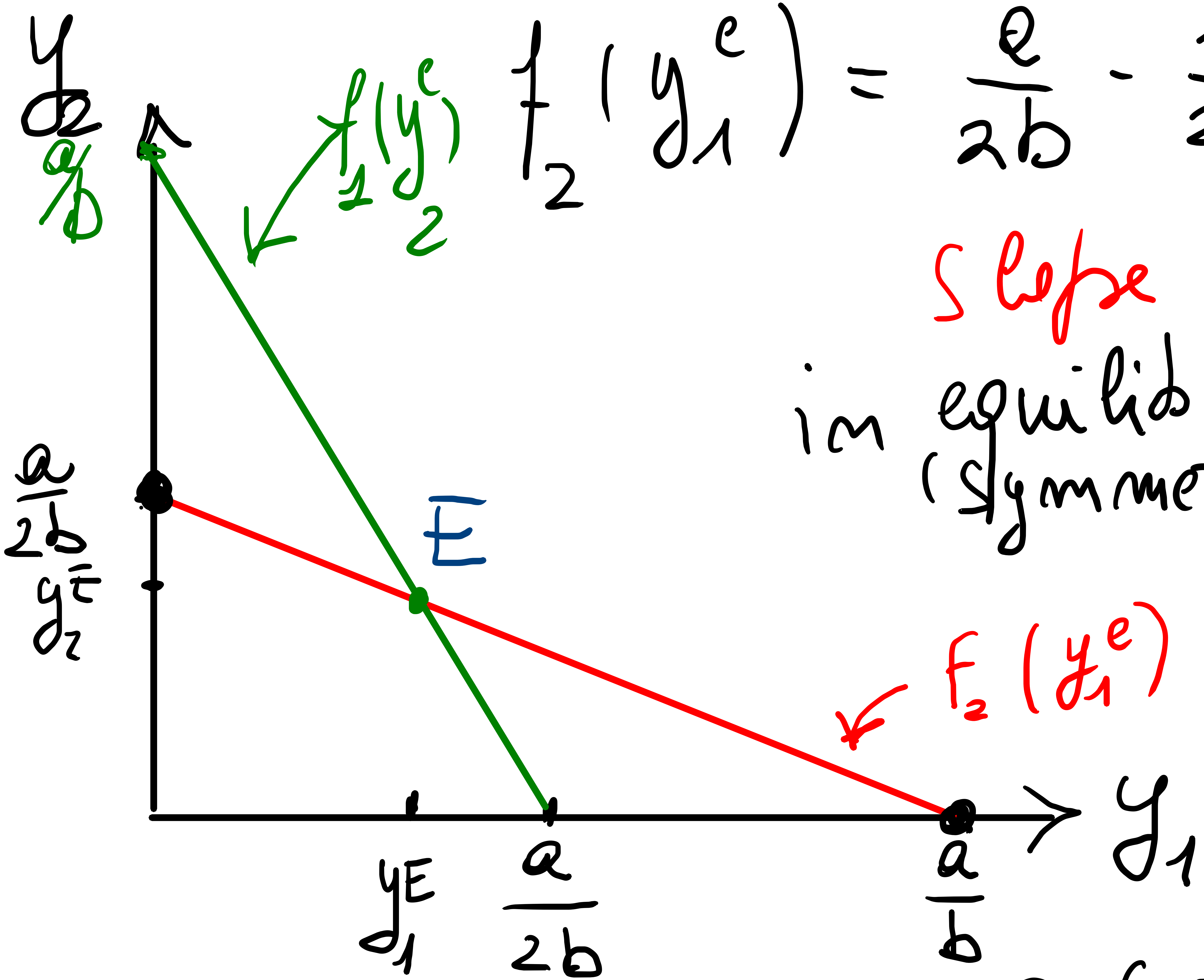


$p' = p^e$ if $y_2 = 0$

$p = a - by$

$$y_2 = \frac{a}{2b} - \frac{1}{2} y_1^e = f(y_1^e)$$

$$y_1 = \frac{a}{2b} - \frac{1}{2} y_2^e = f(y_2^e)$$



$$f_2(y_1^e) = \frac{a}{2b} - \frac{1}{2} y_1^e \qquad f_1(y_2^e) = \frac{a}{2b} - \frac{1}{2} y_2^e$$

slope $-\frac{1}{2}$

in equilib. $y_1^e = y_2 = \frac{a}{2b} - \frac{1}{2} y_1$ (symmetric)

$$y_1 = \frac{a}{2b} - \frac{1}{2} y_1$$

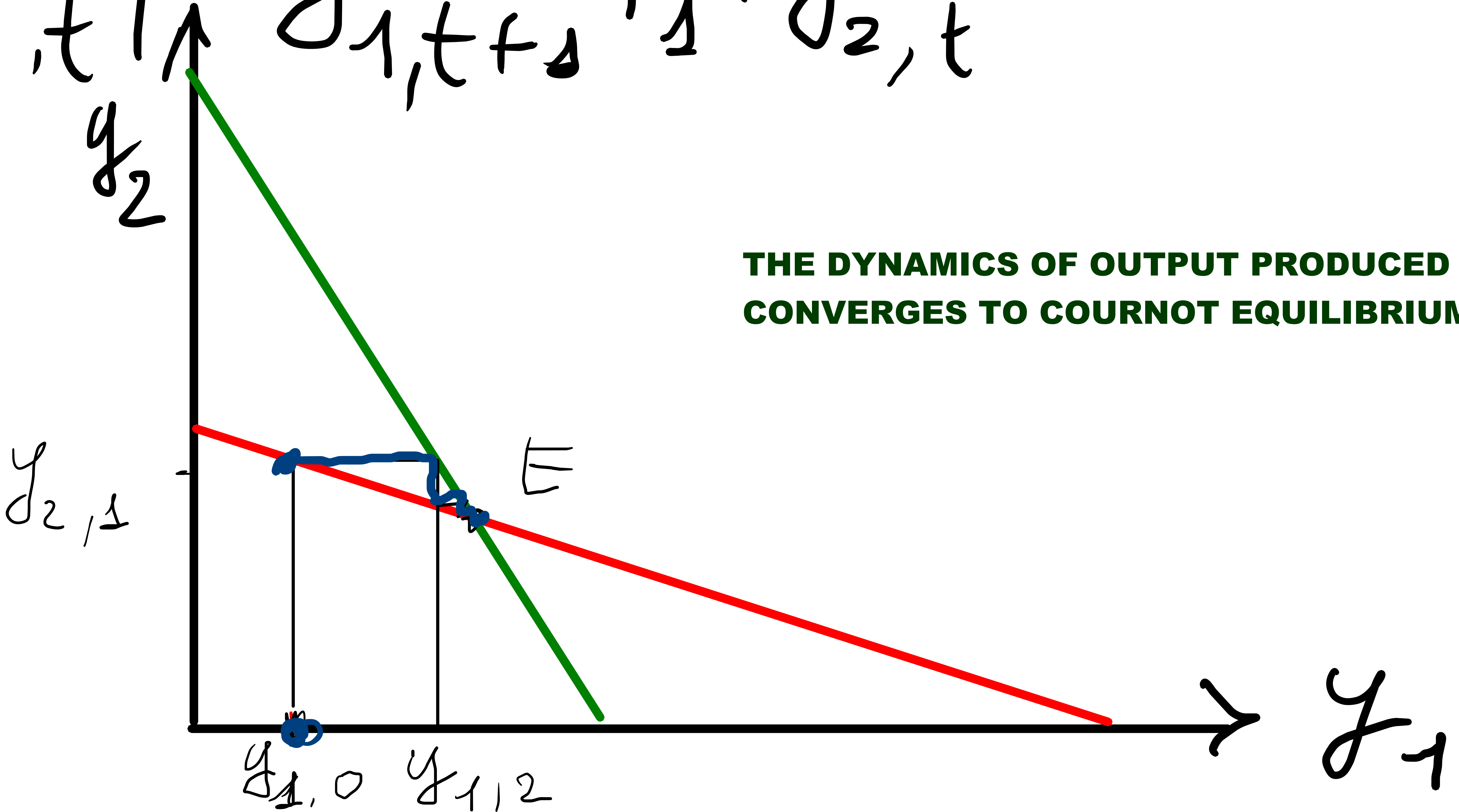
$$y_1^* = \frac{a}{3b} = y_2^*$$

monopoly $y^* = \frac{a}{2b} < \frac{2}{3}\frac{a}{b} =$ Cournot output $y^* = \frac{2}{3} \cdot \frac{a}{b}.$

each firm predicts that the output of the other firm at t + 1 is equal to the output produced by the same firm at time t (which is known)

firm 2: $y^e_{1,t+1} = y_{1,t}$ $\quad y_{2,t+1} = f_2(y^e_{1,t+1}) =$

$y_{2,t+1} = f_2(y_{1,t})$ $\quad y_{1,t+1} = f_1(y_{2,t})$



THE DYNAMICS OF OUTPUT PRODUCED BY FIRM 1 AND 2 CONVERGES TO COURNOT EQUILIBRIUM

$MC = c$ Cournot with n identical firms

$$y_2 = \frac{a-c}{2b} - \frac{1}{2} y_1^e$$

$n-1$ other firms in industry

X^e = output produced by the other $n-1$ firms

$$y_2 = \frac{a-c}{2b} - \frac{1}{2} X^e$$

in Cournot equil $X^e = X = (n-1) y_2$

$y_2^* = y_i^* \quad i = 1, \dots, n$

$$y_2^* = \frac{a-c}{2b} - \frac{1}{2}(n-1) y_2^*$$

$$y_2^* = \frac{1}{n+1} \cdot \frac{a-c}{b}$$

industry output $= \dfrac{n}{n+1} \cdot \dfrac{a-c}{b}$

if $n \to \infty$ then

INDUSTRY OUTPUT = $y = n \cdot y_2 = \lim \frac{n}{n+1} \cdot \frac{a-c}{b} = \frac{a-c}{b}$

$p = a - by$

$p = a - a + c$

$y_2 = \frac{1}{n+1} \cdot \frac{a-c}{b}$

$p = c = MC$

$\lim_{n \to \infty} y_2 = 0$

THE PRICE IN COURNOT EQUILIBRIUM CONVERGES TO THE COMPETITIVE PRICE p = MC, IF THE NUMBER OF FIRMS BECOMES ARBITRARILY LARGE. THUS PERFECT COMPETITION MAY BE INTERPRETED AS A LIMIT CASE OF COURNOT COMPETITION WITH A LARGE NUMBER OF FIRMS

Stackelberg duopoly

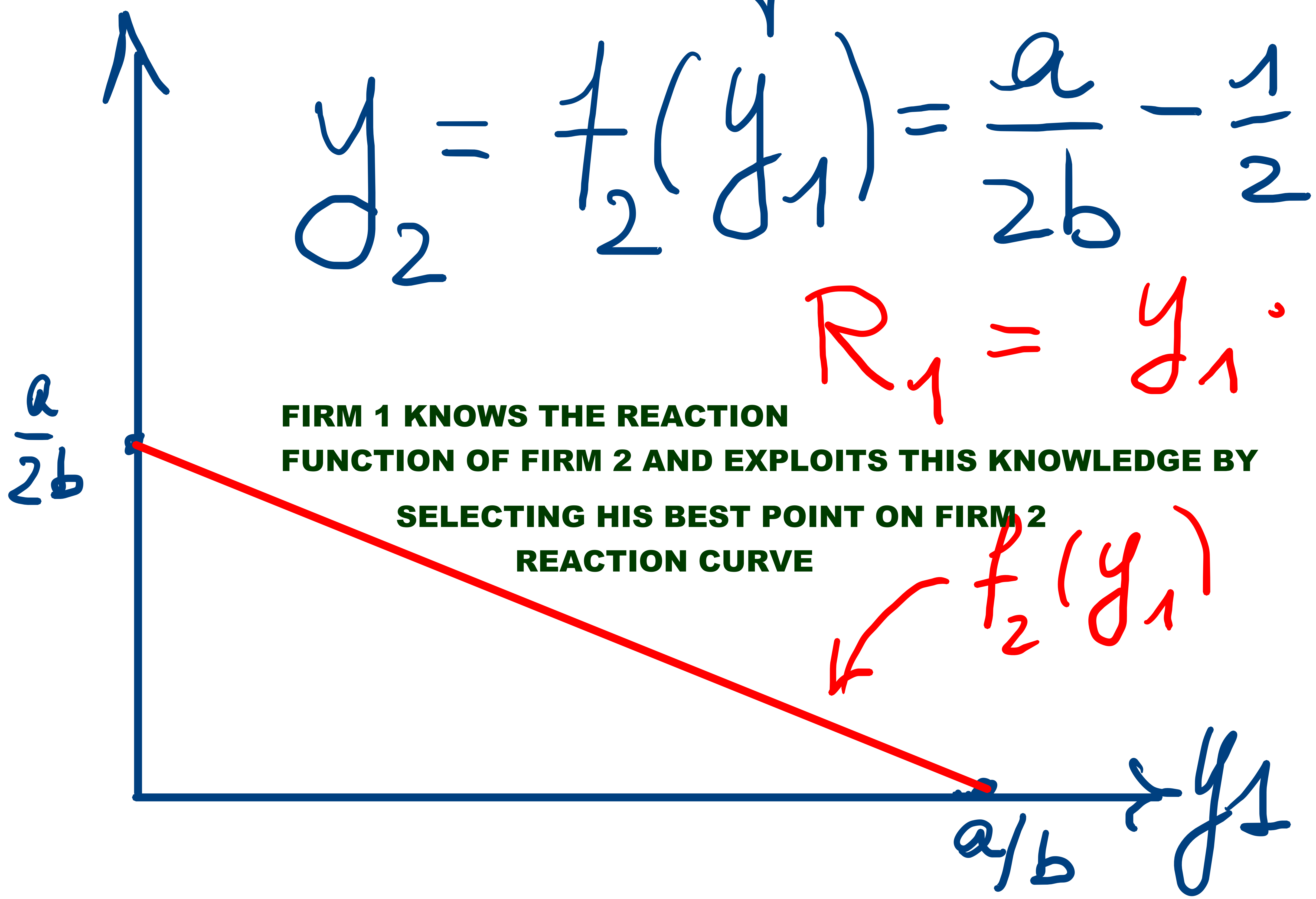
quantity leader 1

" " follower 2

$$MC_1 = MC_2 = 0$$

$$y_2 = f_2(y_1) = \frac{a}{2b} - \frac{1}{2}y_1$$

$$p = a - by = a - by_1 - by_2$$



$$R_1 = y_1 \cdot p = y_1(a - by_1 - by_2)$$

$$= y_1\left(a - by_1 - \frac{a}{2} + \frac{b}{2}y_1\right)$$

$$MR_1 = MC_1 = 0$$

$$R_1 = y_1\frac{a}{2} - \frac{b}{2}y_1^2$$

$$MR_1 = \frac{a}{2} - b y_1 = MC = 0$$

$$y_1^* = \frac{a}{2b} = \text{monopoly output!}$$

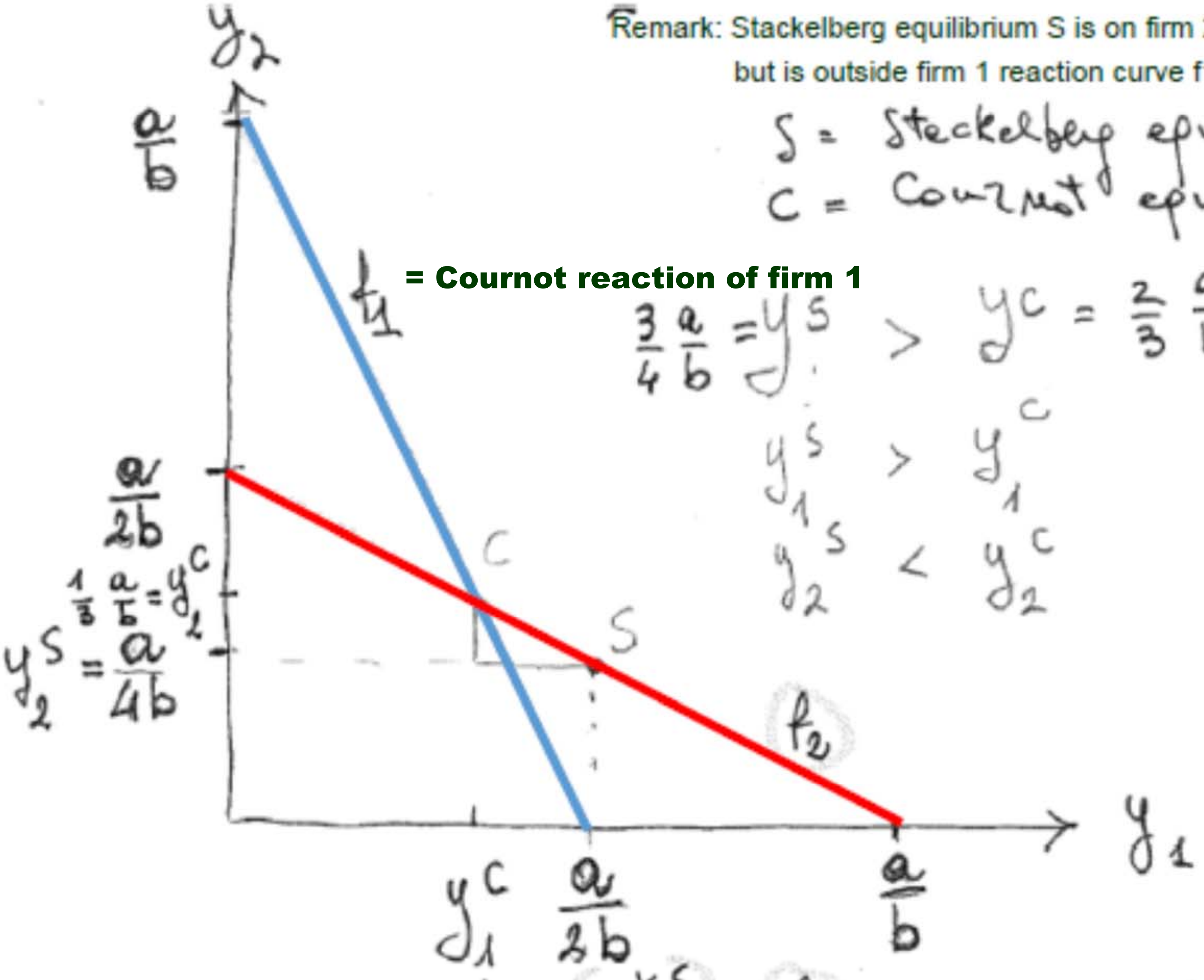
$$y_2^* = \frac{a}{4b} \quad \text{follower}$$

INDUSTRY OUTPUT = $y_{St.}^* = \frac{3a}{4b} > y^*_{Cournot} = \frac{2}{3}\frac{a}{b}$

Remark: Stackelberg equilibrium S is on firm 2 reaction curve f2, but is outside firm 1 reaction curve f1

S = Stackelberg equilibrium

C = Cournot equilibrium



= Cournot reaction of firm 1

$\frac{3}{4}\frac{a}{b} = y^S > y^C = \frac{2}{3}\frac{a}{b}$

$y_1^S > y_1^C$

$y_2^S < y_2^C$

$\frac{1}{3}\frac{a}{b}$ $y_1^{*S} = \frac{a}{2b}$ = monopoly output