Post contractual opportunism in finance

We now remove the assumption that project's type is hidden information. Investors fully disclose the project characteristics to the lender.

We also remove the assumption that the risky and safe projects yield the same expected cash flow. Instead, we assume that the safe project (that is the project with higher success probability) yields a higher expected cash flow, though its cash flow in case of success is lower.

We want to discuss the contractual implications of lender's imperfect monitoring of the actions carried out by the borrower after the contract is signed.

Hidden action: moral hazard

r_L = risk adjusted interest rate fixed by the bank

l = risky project α_l = success probability of l low success probability

h = safe project α_h = success probability of h high success probability

L = *cost of project l and h*

$CF_{h,s}$ = *cash flow if success* $CF_{h,f} = 0$ = *cash flow if failure*

$CF_{l,s}$ = *cash flow if success* $CF_{l,f} = 0$ = *cash flow if failure*

Assume safe project yields lower cash flow, if success, but succeeds with greater frequency and yields higher expected value (expected cash flow)

$$CF_{l,s} > CF_{h,s} \text{ and } \alpha_h > \alpha_l \quad \text{such that} \quad EV_h > EV_l$$

Expected cash flow: $EV_h = \alpha_h\, CF_{h,s} + (1 - \alpha_h)0$ **>** $\alpha_l\, CF_{l,s} + (1 - \alpha_l)0 = EV_l$

Problem statement: the bank wants to avoid that, after signing the contract fixing the risk-adjusted interest rate r_L , the borrower who has promised to carry out the safe project *h* invests money L on the risky *l-project* now the bank has fixed the contractual risk-adjusted rate rL referring to the low-risk investment "h"

Expected profit

project "h" $E\Pi_h = EV_h - \alpha_h(1 + r_L)L = \alpha_h CF_{s,h} - \alpha_h(1 + r_L)L$

project "l" $E\Pi_l = EV_l - \alpha_l(1 + r_L)L = \alpha_l CF_{s,l} - \alpha_l(1 + r_L)L$

Now consider a type *h* project submitted to the bank for the purpose of contracting a money-lone of size L. To avoid post-contractual opportunism the contract must fulfill the

Borrower's incentive compatibility constraint:

$E\Pi_h \geq E\Pi_l$ that is: $EV_h - \alpha_h(1 + r_L)L \geq EV_l - \alpha_l(1 + r_L)L$

Can be written as: $EV_h - EV_l \geq (\alpha_h - \alpha_l)(1 + r_L)L$

There exists a maximum incentive-compatible interest factor $(1 + r_{Lmax})$ such that the borrower prefers safe project h to risky project l

$$(1 + r_{Lmax}) = \frac{EV_h - EV_l}{(\alpha_h - \alpha_l)L} \geq (1 + r_L)$$

That is: $r_L \leq r_{Lmax}$

Lender's participation constraint:

$$\frac{1+r}{\alpha_h} \leq 1 + r_L$$

$$\frac{1+r}{\alpha_h} - 1 \leq r_L$$

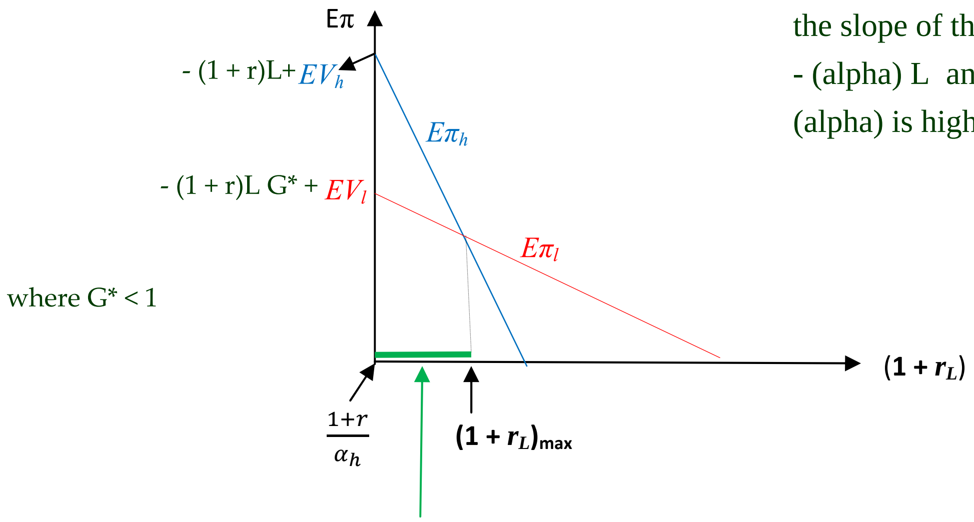
Borrower's incentive compatibility constraint:

$$r_L \leq r_{Lmax}$$

The loan contract is feasible if and only if:

$$\frac{1+r}{\alpha_h} - 1 \leq r_L \leq r_{Lmax}$$

Incentive compatible interest factor



the slope of the blue and red curves is:
- (alpha) L and
(alpha) is higher for "h" than for "l"

incentive compatible range of $(1 + r_L)$ for investment of type "h"

$$E\Pi_h = EV_h - \alpha_h(1 + r_L)L$$

If $(1 + r_L)$ is the minimum risk adjusted interest factor meeting lender's participation constraint:

$$E\Pi_h = EV_h - (1 + r)L$$

$$E\Pi_l = EV_l - \alpha_l(1 + r_L)L$$

If $(1 + r_L)$ is the minimum risk adjusted interest factor meeting lender's participation constraint:

$$E\Pi_l = EV_l - \frac{\alpha_l}{\alpha_h}(1 + r)L = EV_l - G^*(1 + r)L$$

Where $G^* = \frac{\alpha_l}{\alpha_h} < 1$

Effect of a credit constraint D on the incentive-compatible interest rate

Credit constraint:
Suppose the bank is unwilling to finance the whole project cost L.
The borrower is forced to contributing to the project with own funds D

Expected profit with credit constraint: D > 0

$$E\Pi_h = EV_h - D - \alpha_h(1 + r_L)(L - D)$$

$$E\Pi_l = EV_l - D - \alpha_l(1 + r_L)(L - D)$$

The incentive-compatibility constraint that makes the low-risk project h more attractive than the high-risk project l is

$$E\Pi_h \geq E\Pi_l,$$

that is:

$$EV_h - \alpha_h(1 + r_L)(L - D) \geq EV_l - \alpha_l(1 + r_L)(L - D)$$

$$EV_h - EV_l \geq (\alpha_h - \alpha_l)(1 + r_L)(L - D)$$

This can be written as:

$$D(\alpha_h - \alpha_l) + EV_h - EV_l \geq (\alpha_h - \alpha_l)(1 + r_L)L$$

$$\left(1 + r_{L,max} | D > 0\right) = \frac{EV_h - EV_l}{(\alpha_h - \alpha_l)(L - D)} \geq (1 + r_L)$$

The higher D, the higher the higher the maximum incentive compatible, risk adjusted interest rate $r_{L,max}$

Conclusion:

In spite of risk neutrality, because the credit constraint D is now forcing the entrepreneur to take some of the losses in case of failure, the bank can fix a higher incentive-compatible interest rate r_L.

The upper-bound $r_{L,max}$ to the incentive-compatible risk-adjusted interest rate r_L is an increasing function of D. By imposing a higher credit constraint D, the bank is free to fix a higher incentive compatible interest rate than it would be the case otherwise.

By contributing D to investment finance, the investor partipates to losses in case of failure. This increases the maximum incentive-compatible interest rate rL(max): that is, the critical value of the risk-adjusted interest rate rL beyond which the investor wound find the 'moral-hazard' option attractive after signing the contract.

Exercise: Borrower and lender are risk neutral. Project type is common information, but there are high monitoring costs that cause 'hidden action' after the contract is signed.

Two projects: A, B

Cost: $L_A = L_B = 100$

Cash flow if s: $CF_{s,A} = 600$ $CV_{S,B} = 880$ $\alpha_{A,S} = 0.8$ $\alpha_{B,S} = 0.5$

Cash flow if s: $CF_{f,A} = 0$ $CV_{f,B} = 0$

The 'safe' interest rate is r = 0.05

What is the maximum incentive compatible interest rate r_L?

Is the bank prepared to lend money L at such a rate?

Solution:

$$EV_A = 600 * 0.8 = 480$$

$$EV_B = 880 * 0.5 = 440$$

$r_{L,\,max}$ is the max. value of r_L such that:

$E\Pi_A \geq E\Pi_B$ that is: $EV_A - \alpha_A(1 + r_L)L \geq EV_B - \alpha_B(1 + r_L)L$

Can be written as: $EV_A - EV_B \geq (\alpha_A - \alpha_B)(1 + r_L)L$

This implies $(1 + r_{Lmax}) = \frac{EV_A - EV_B}{(\alpha_A - \alpha_B)L} \geq (1 + r_L)$

$$(1 + r_{Lmax}) = \frac{40}{0.3 * L} = 1.33$$

That is, the **maximum incentive compatible interest rate** is r_{Lmax}= 0.33

But, is the bank prepared to lend at an interest rate $r_L \leq 0.33$?
Too see this, we must check the bank's **participation constraint**:
we call $r_{L,A}$, $r_{L,B}$ the **lowest interest rate at which the bank is prepared to lend money L on projects A, B, respectively.**

Participation constraint:

$\alpha_A(1 + r_{L,A})L = (1 + r)L$ $\mathbf{1 + r_{L,A} = \frac{1+r}{\alpha_A} = \frac{1.05}{0.8} = 1.3125}$

Since at the risk interest rate $r_{L,max}$ borrower's participation constraint is satisfied ($E\Pi_A > 0$), we can conclude:

Answer : The loan contract L financing type A project is feasible if :

$$\mathbf{1.3125 \leq 1 + r_L \leq 1.33}$$

Exercise: Same data as above, except that

The safe interest rate r = 0.1.

Lender's participation constraint is then $\mathbf{1 + r_{L,A} \geq \frac{1+r}{\alpha_A} = \frac{1.1}{0.8} = 1.375}$

Now the loan contract L_A is not feasible (the loan contract L_A = 100 is not signed) because:

Participation-constrained interest $\boldsymbol{r_{L,A}}$ = 0.375 > 0.333 **=** $\boldsymbol{r_{L,max}}$ = max incentive-compatible interest rate.

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The bank will then propose a different loan contract, such that the borrower has to participate with wealth D to project finance, and the bank contributes $L - D$ = size of the loan.

This will raise $\boldsymbol{r_{L,max}}$ according to:

$$\left(1 + r_{L,max}\right) = \frac{EV_A - EV_B}{(\alpha_A - \alpha_B)(L-D)} = \frac{40}{0.3(100-D)}$$

If D = 20, then $r_{L,max}$ = 0.666 > $\boldsymbol{r_{L,A}}$ **and at** $\boldsymbol{r_{L,A}}$ **<** $\boldsymbol{r_L} < r_{L,max}$ **the contract meets the participation and the incentive-compatibility constraints.**