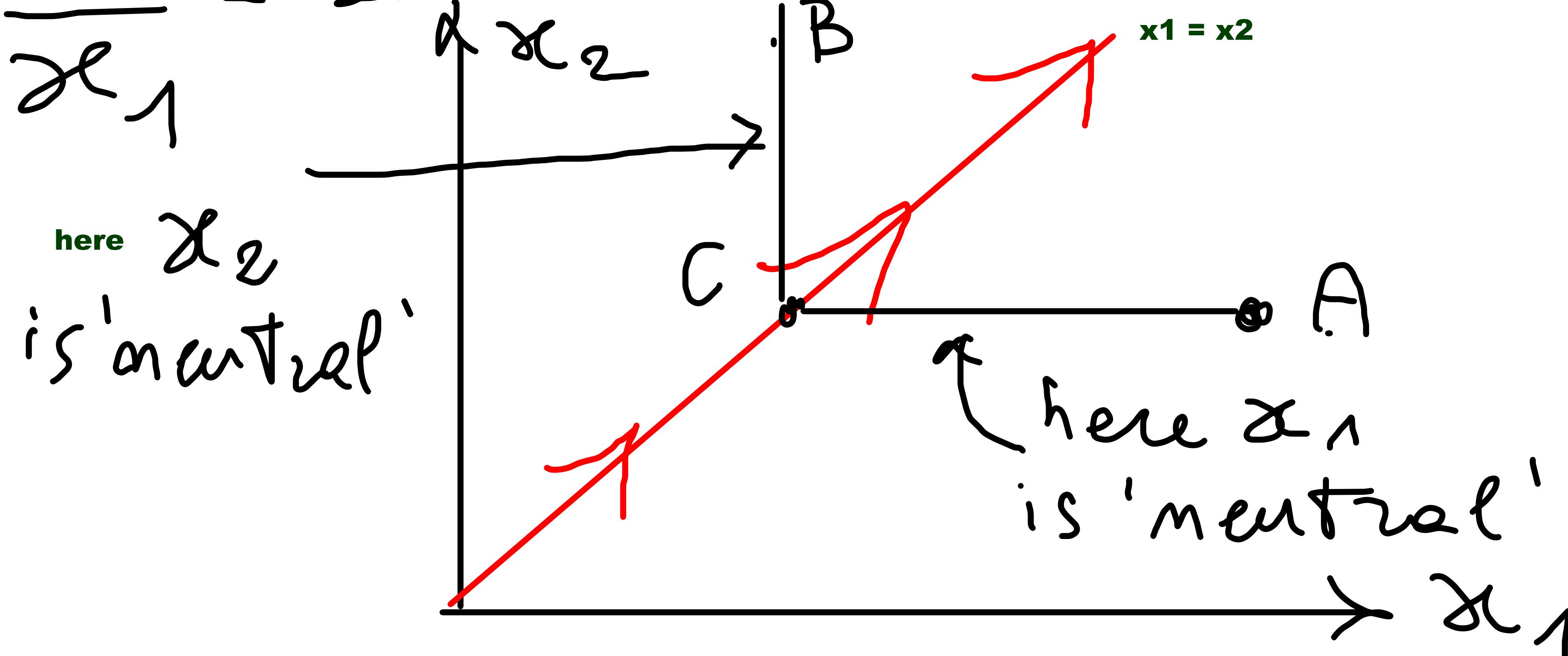


# Perfect complements

$x_1$  = coffee

$x_2$  = sugar

$$\frac{x_2}{x_1} = 1 \quad \text{desired ratio}$$

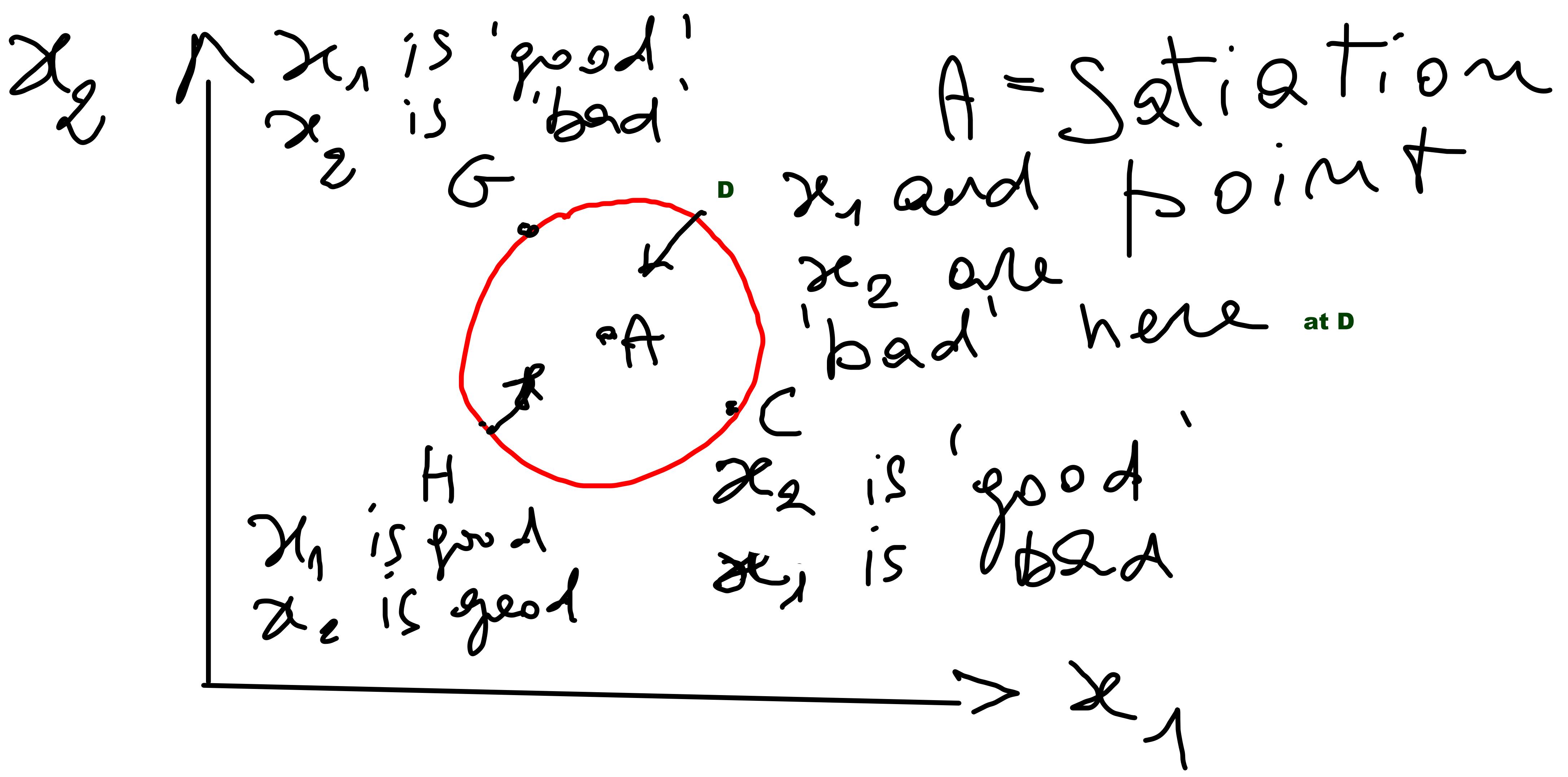


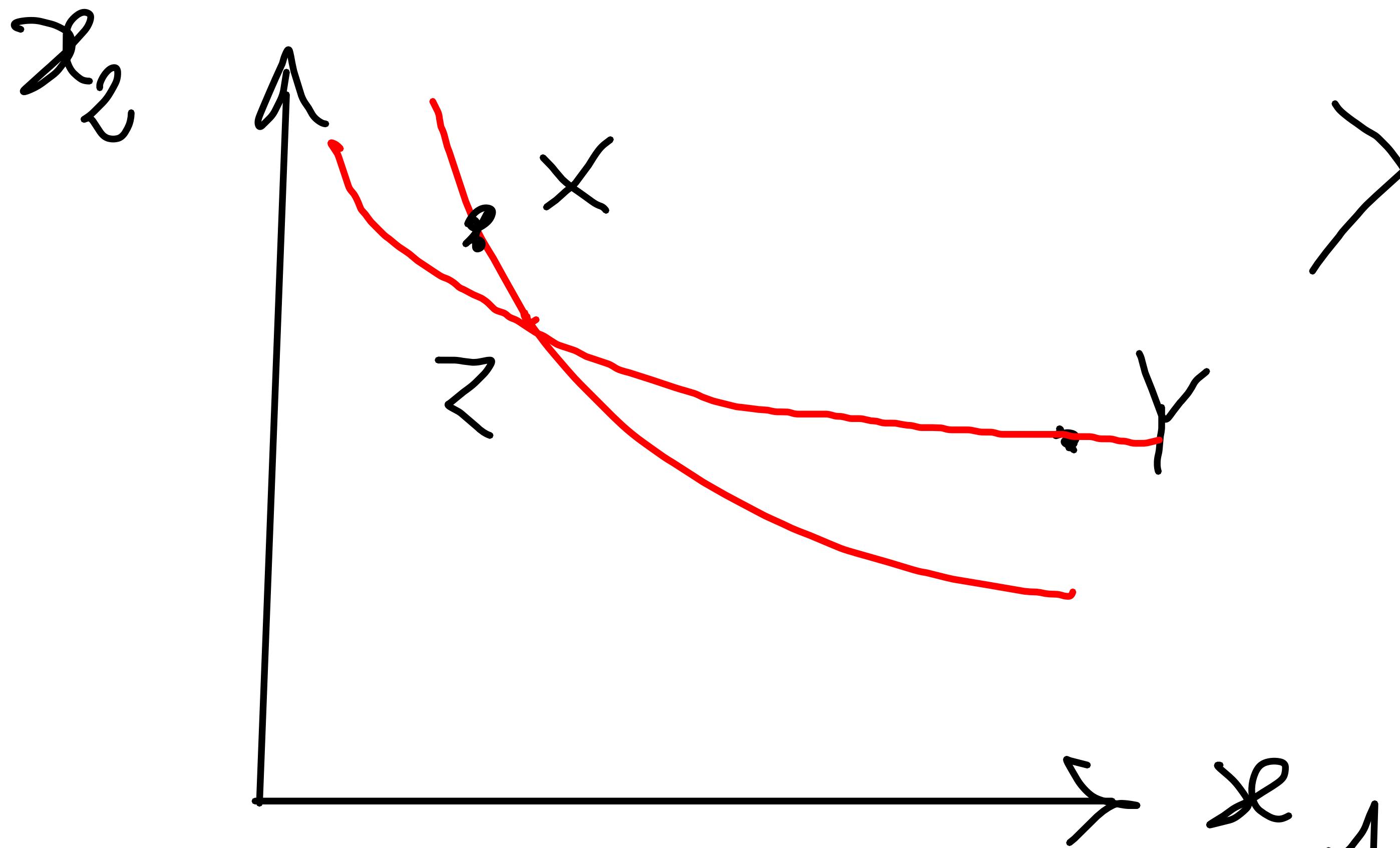
$$A: \frac{x_2}{x_1} < 1$$

$$C \sim A$$

$$B \sim A \sim C$$

indifference curves are L shaped and form a 90° degree angle at the point of intersection with straight line of desired consumption ratio



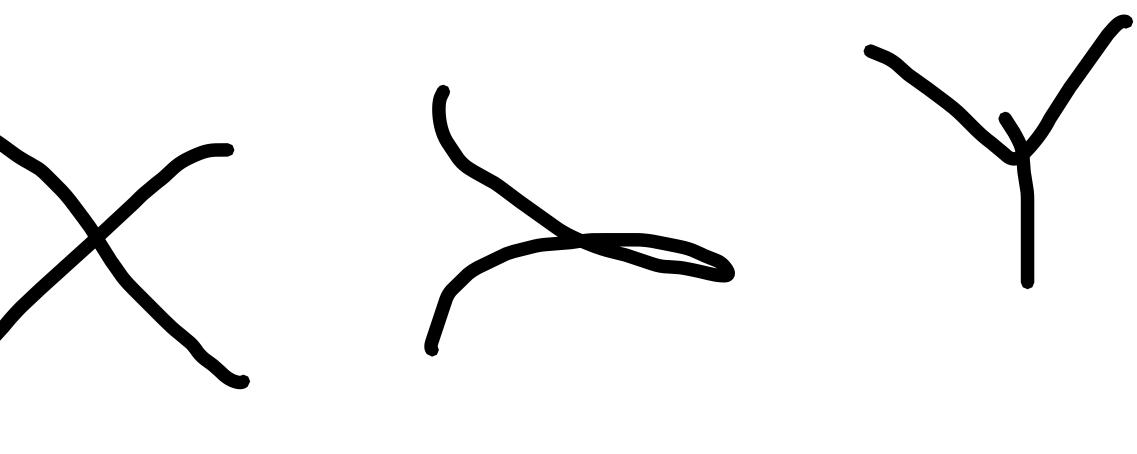


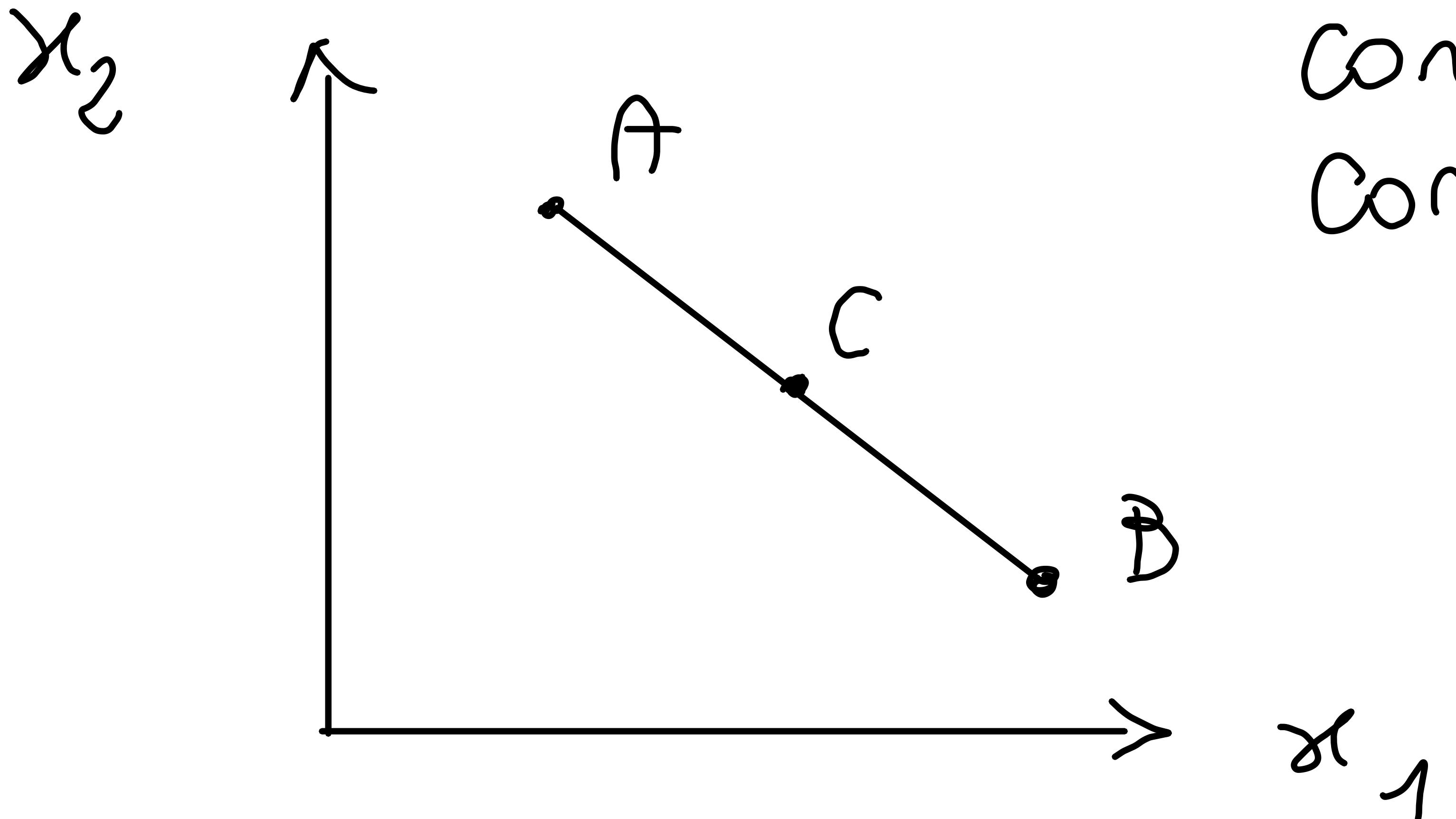
if indifference curves cross there is a

violation of Transitivity

indifference curves do not cross

Pref. are  
monotonic  
continuous  
transitive





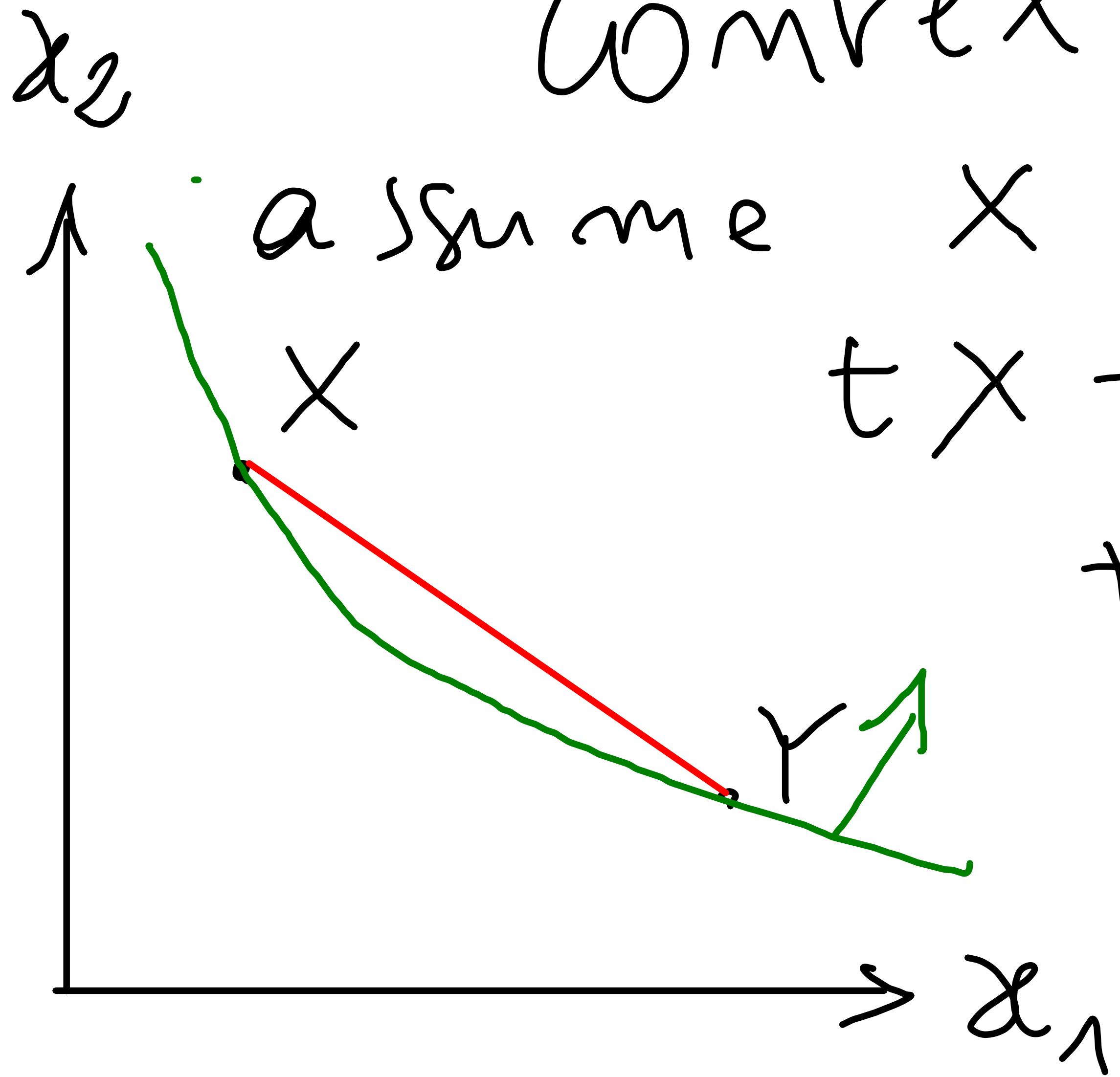
Convex  
Combination

$$C = \frac{1}{2}A + \frac{1}{2}B$$

$$C = \frac{1}{2}A + \left(1 - \frac{1}{2}\right)B$$

**general convex combination**

$$X = tA + (1-t)B \quad 0 \leq t \leq 1$$



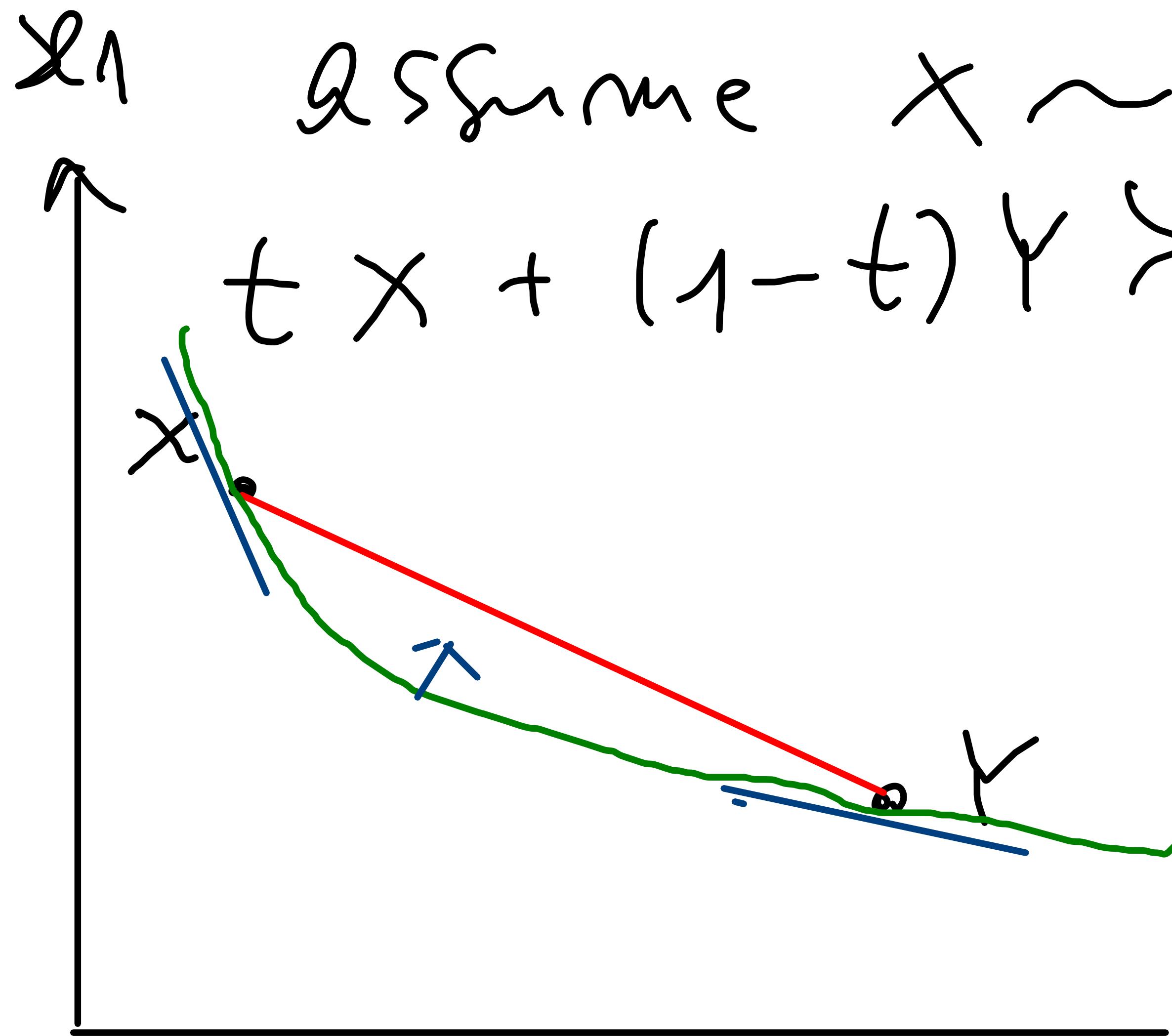
Convex preferences

assume  $x \sim y$  then:

$t x + (1-t)y \sim x$

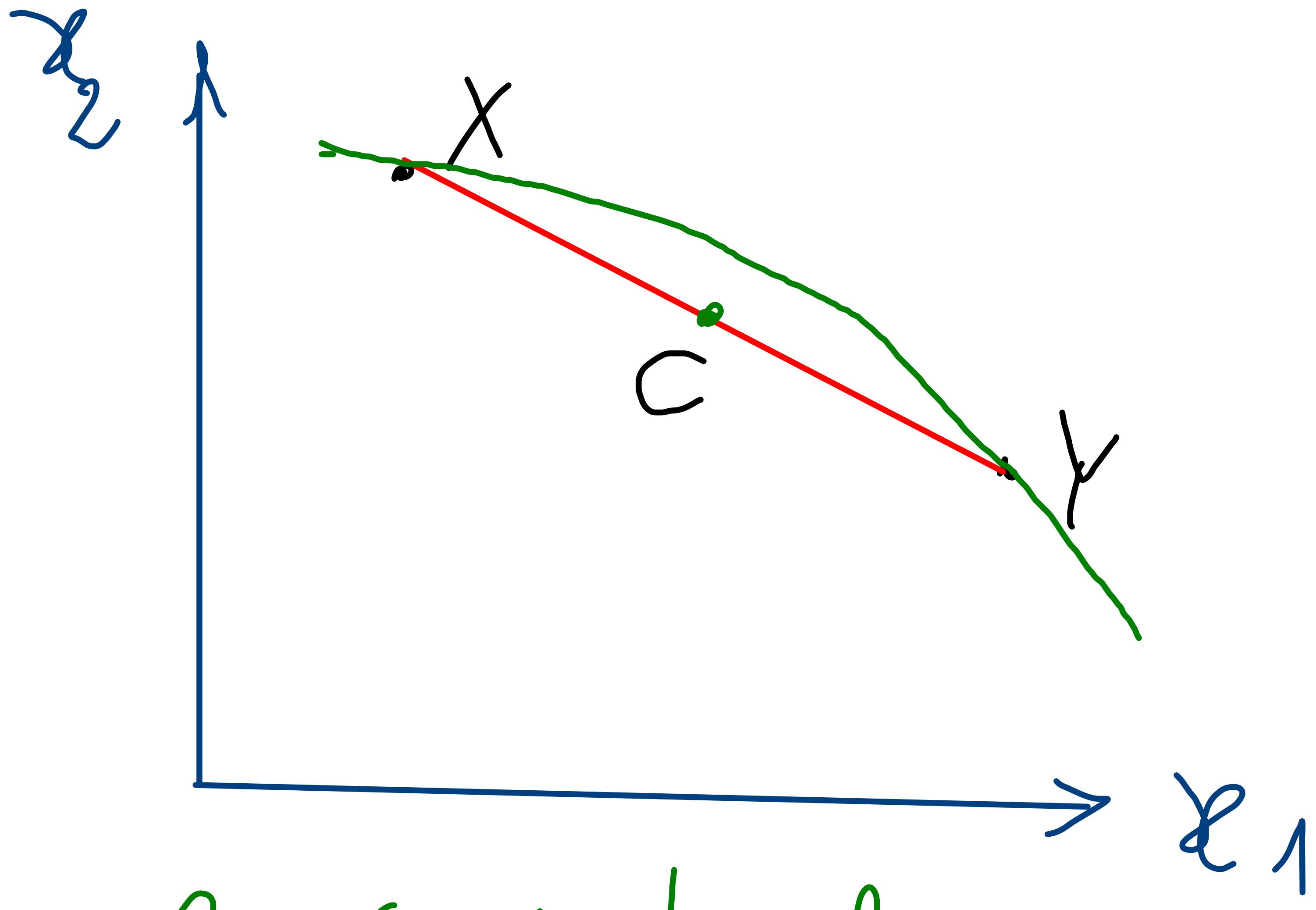
$0 \leq t \leq 1$

# Strict convexity



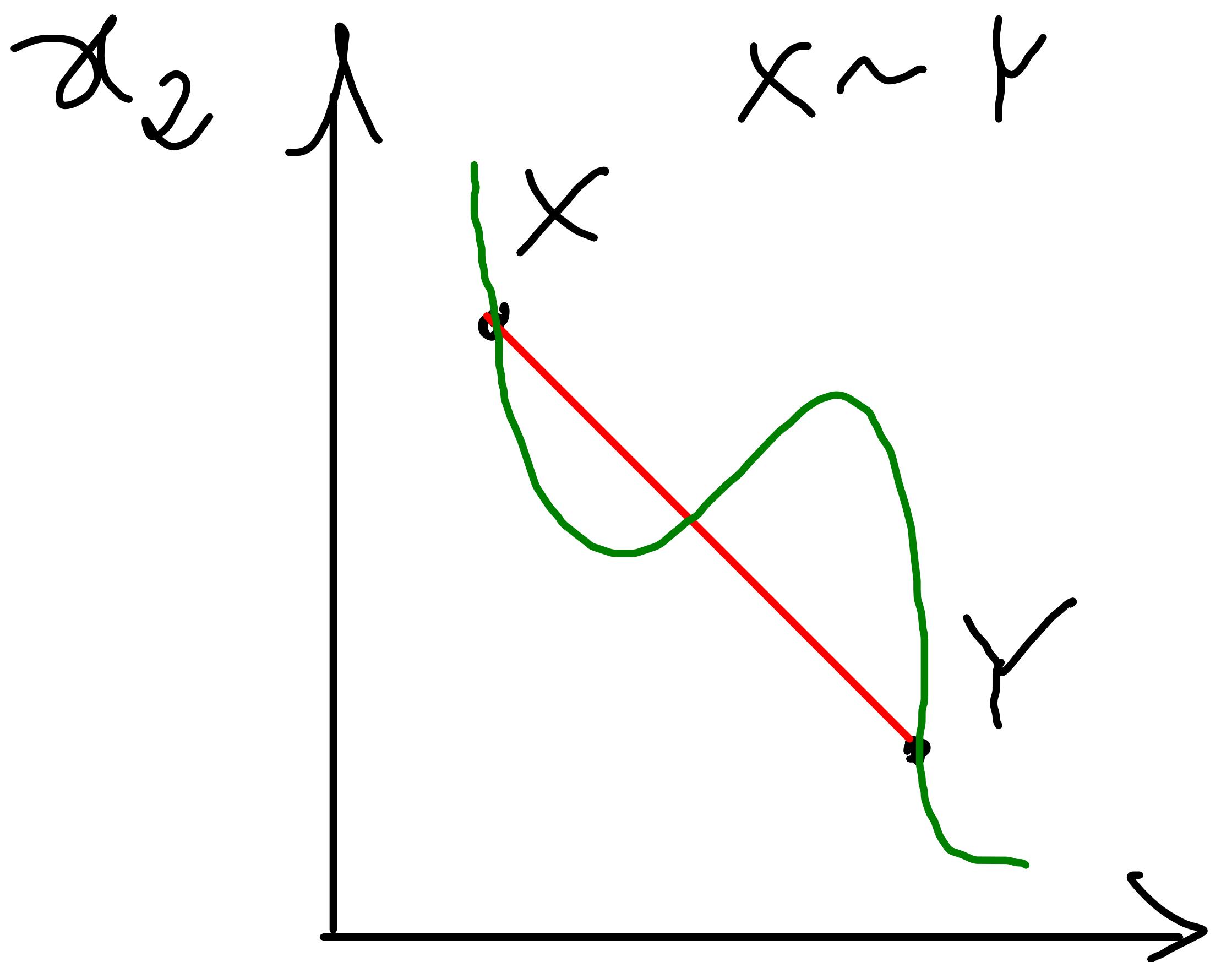
with strict  
convexity  
indifference curves

$x_2$  are strictly convex.

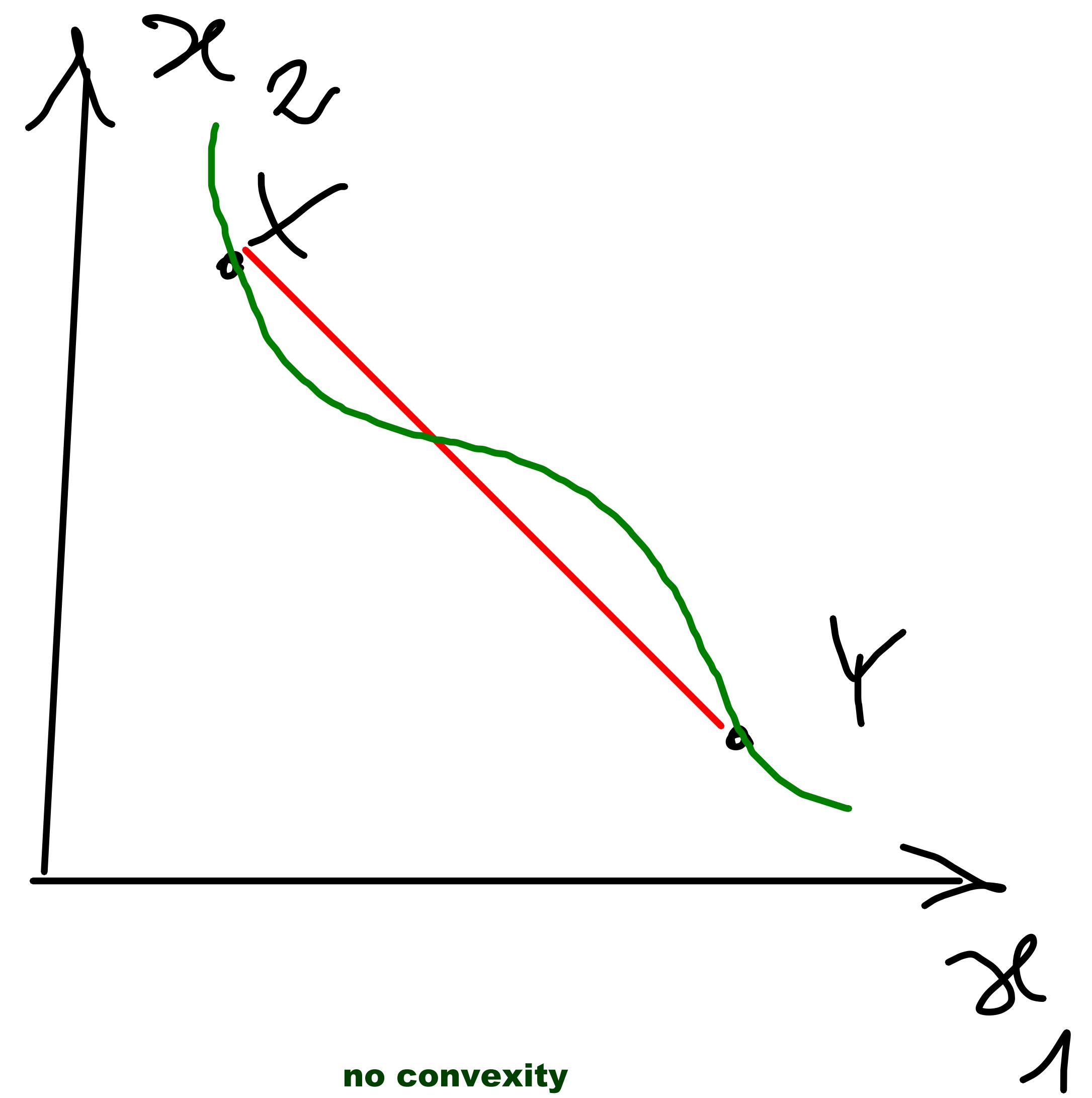


Concave preferences

extreme bundles are preferred to 'averages'



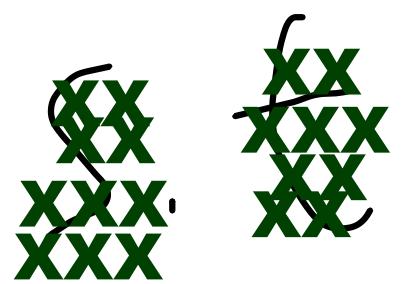
No convexity  
No concavity  
No monotonicity



function  
 $u(x_1, x_2)$

is a

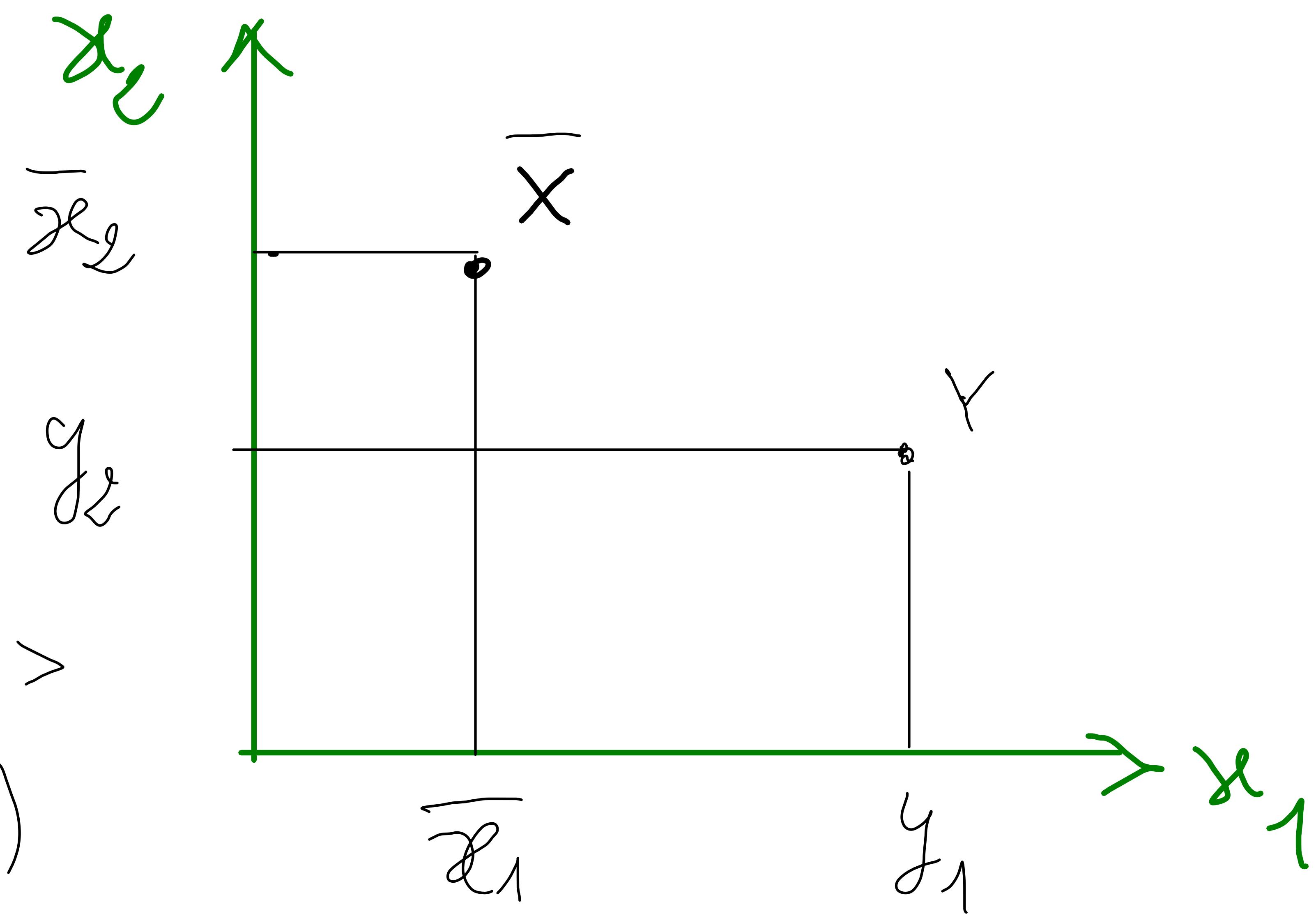
utility function representing a preference order

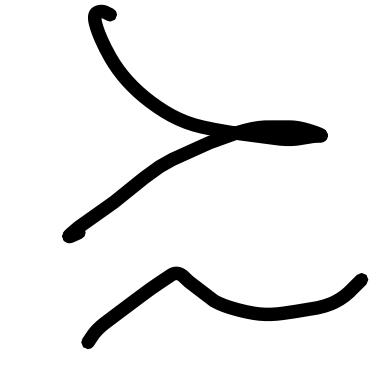


if  
and only if

$x \succ y$   ~~$u(x) > u(y)$~~   $u(\bar{x}_1, \bar{x}_2) > u(y_1, y_2)$

then  $u(x_1, x_2)$  represents the  
preference order



Suppose  
 $u(x_1, x_2)$  represents 

take  $(x_1, x_2)$   $(y_1, y_2)$

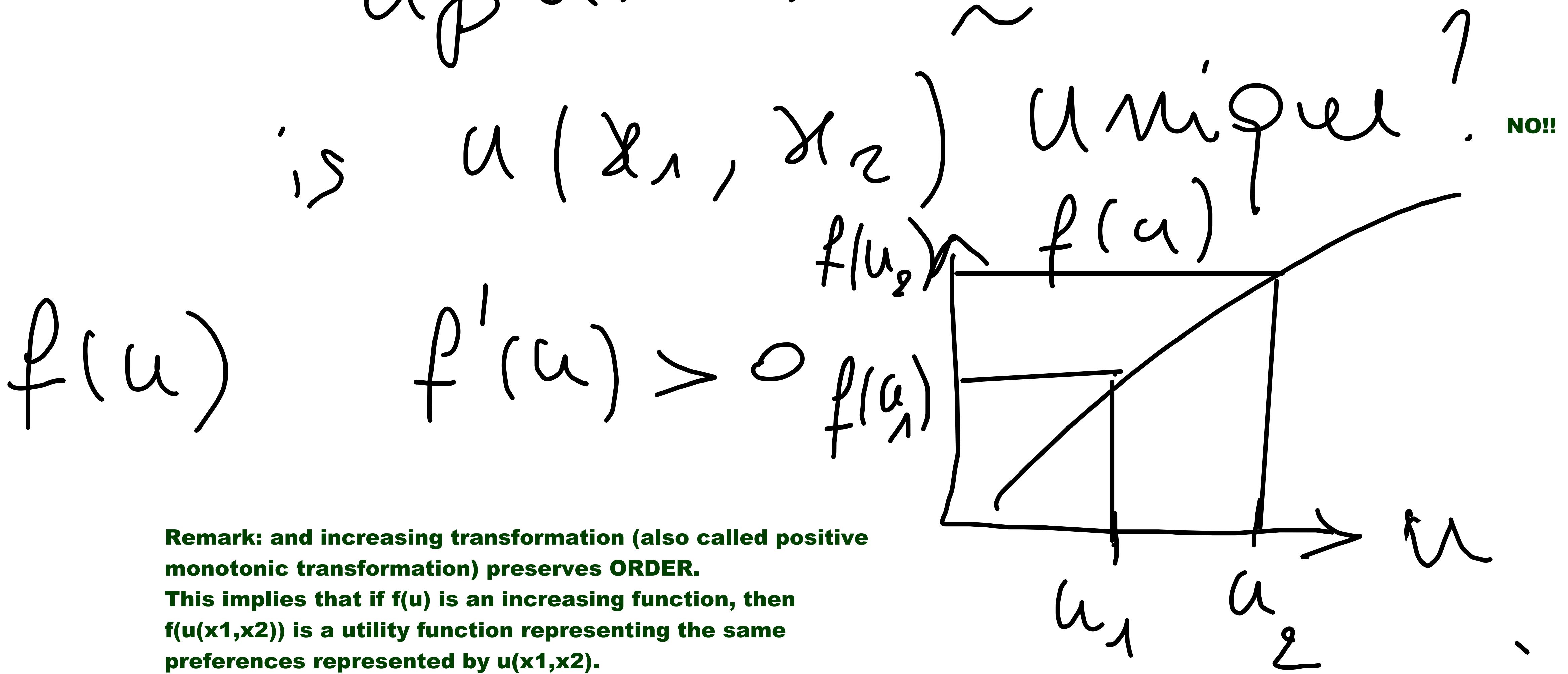
either  $u(x_1, x_2) > u(y_1, y_2)$

or  $u(x_1, x_2) < u(y_1, y_2)$

or  $u(x_1, x_2) = u(y_1, y_2)$

this implies that the preference order is complete. Similar argument applies for transitivity. We can conclude that a preference order can be represented by a utility function only if it is complete and transitive.

Suppose  $u(x_1, x_2)$   
represents

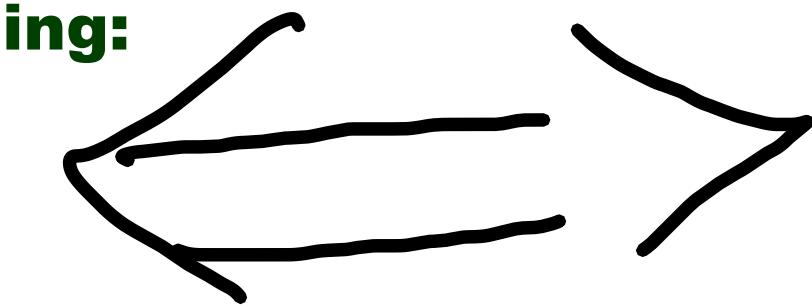


**Remark:** an increasing transformation (also called positive monotonic transformation) preserves ORDER.

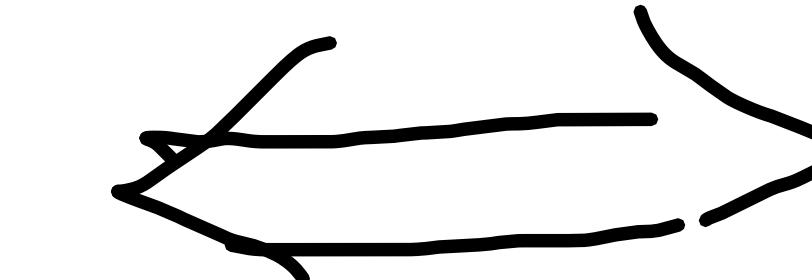
This implies that if  $f(u)$  is an increasing function, then  $f(u(x_1, x_2))$  is a utility function representing the same preferences represented by  $u(x_1, x_2)$ .

**Proof:** Because  $u(x_1, x_2)$  is a utility function representing the given preference order and  $f(u)$  is increasing:

$$(x_1, x_2) \succ (y_1, y_2)$$



$$u(x_1, x_2) > u(y_1, y_2)$$



$$f(u(x_1, x_2)) > f(u(y_1, y_2))$$

$$\checkmark (x_1, x_2) = f(u(x_1, x_2))$$

**$V(x_1, x_2)$  is another utility function representing the same preferences.**