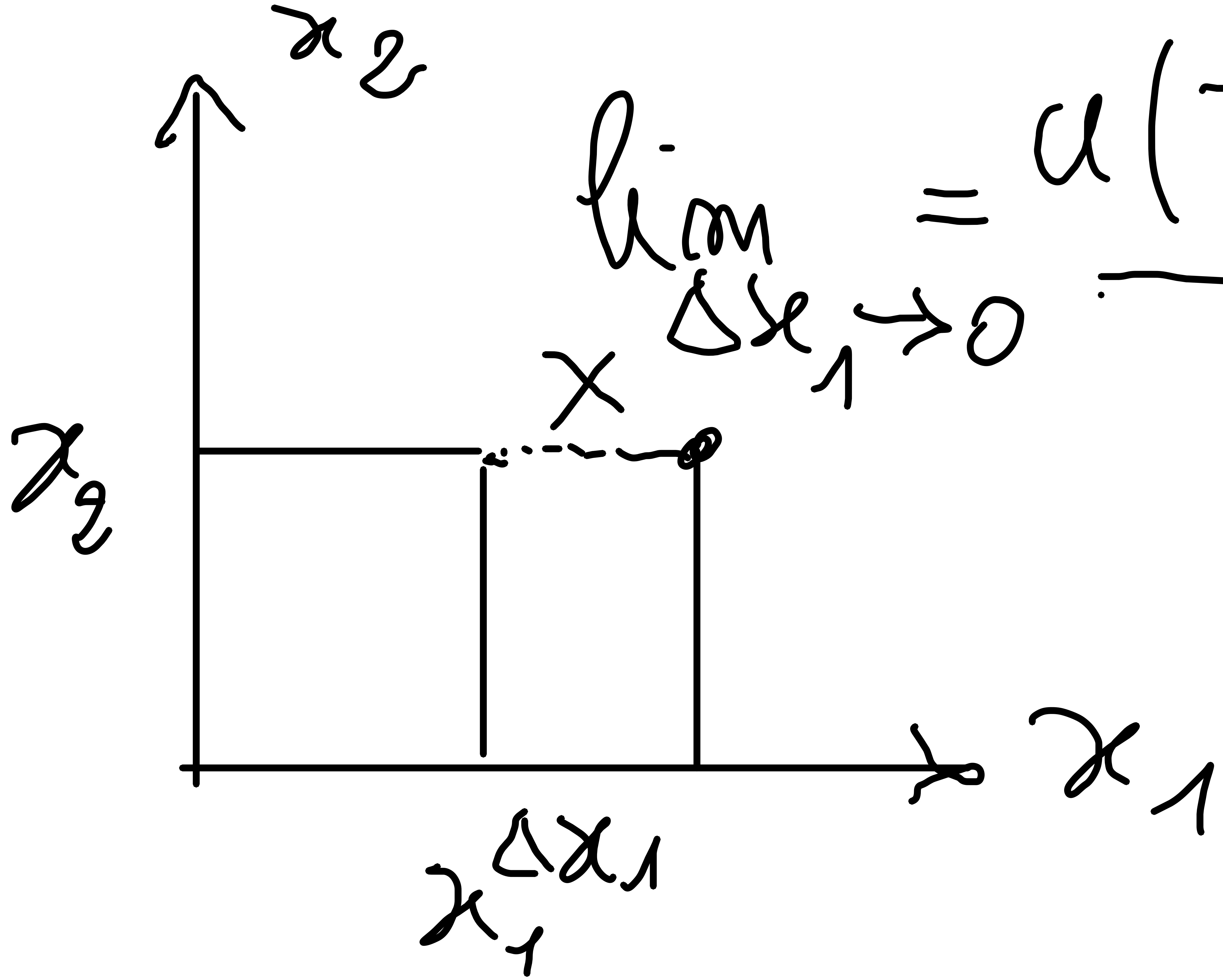


$$\lim_{\Delta x_1 \to 0} = \frac{u(x_1 + \Delta x_1, x_2) - u(x_1, x_2)}{\Delta x_1}$$

Marginal Utility of x_1 at X

$MU_1(x_1, x_2)$ depends on the function $u(x_1, x_2)$ representing preference order

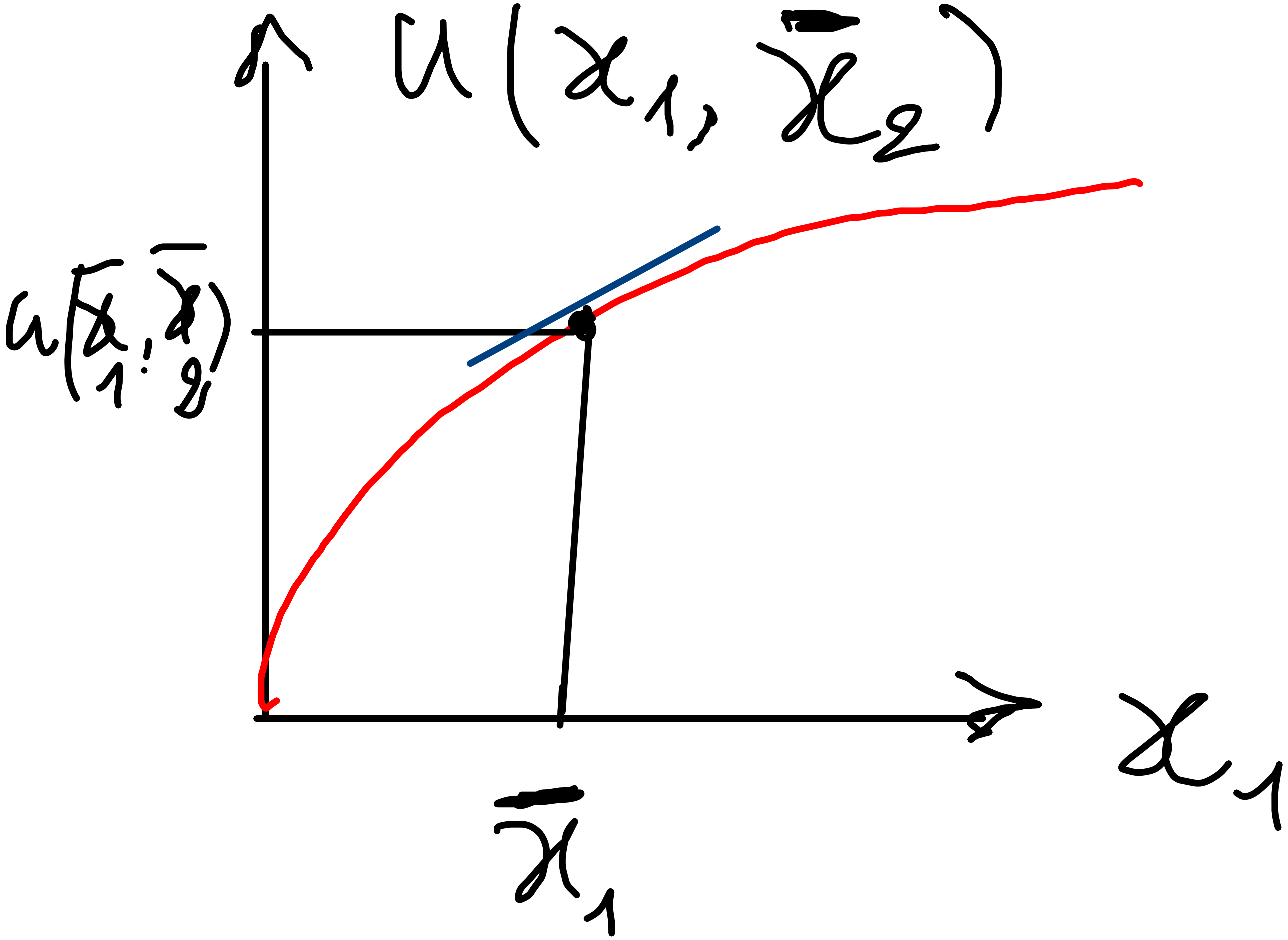
$$u(x_1, x_2) = x_1 + x_2$$

Take positive monotonic transformation of u(x1, x2):

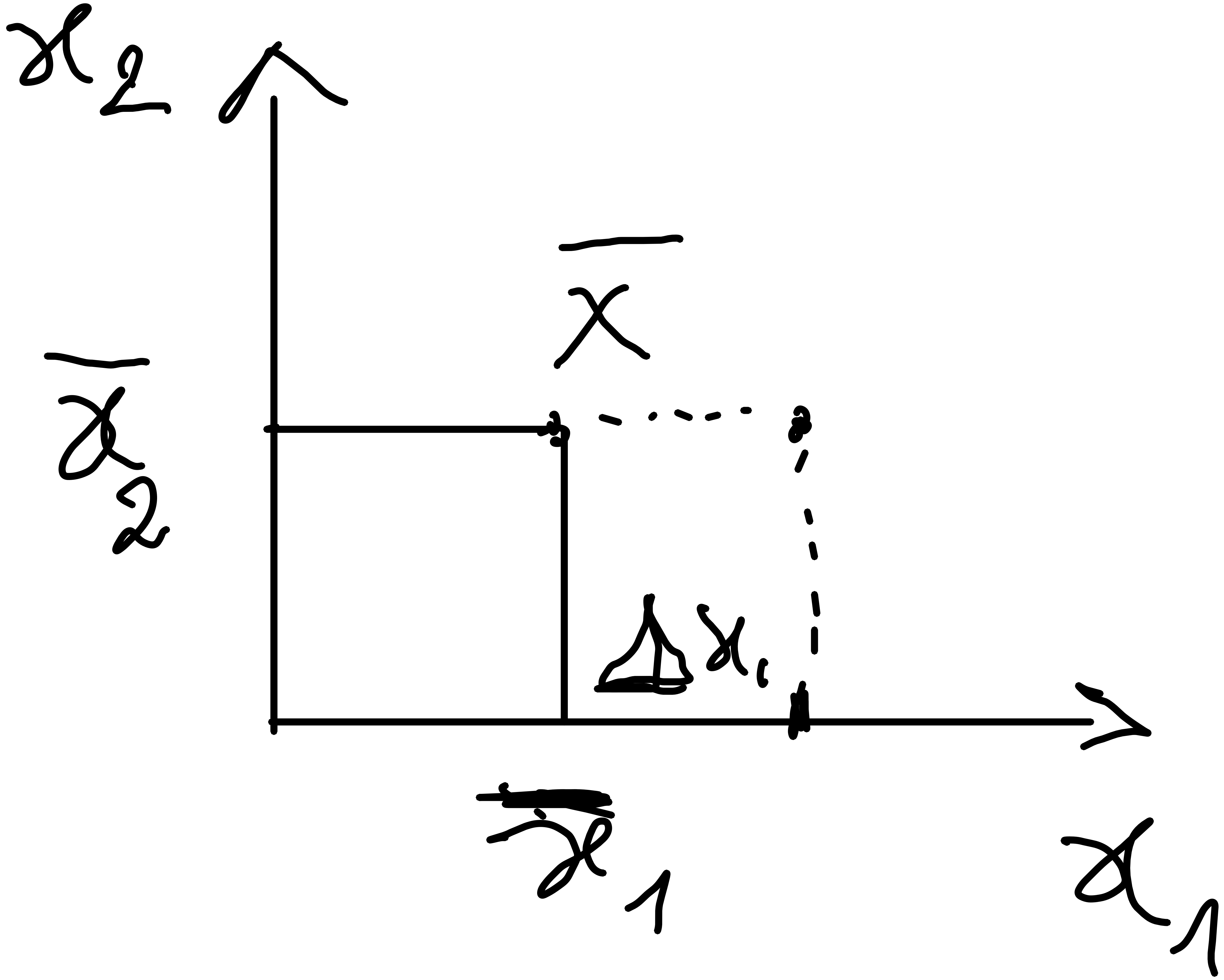
$$v(x_1, x_2) = 2u(x_1, x_2) = 2x_1 + 2x_2$$

$$\frac{\partial u(x_1, x_2)}{\partial x_1} = 1 \qquad \frac{\partial v(x_1, x_2)}{\partial x_1} = 2$$

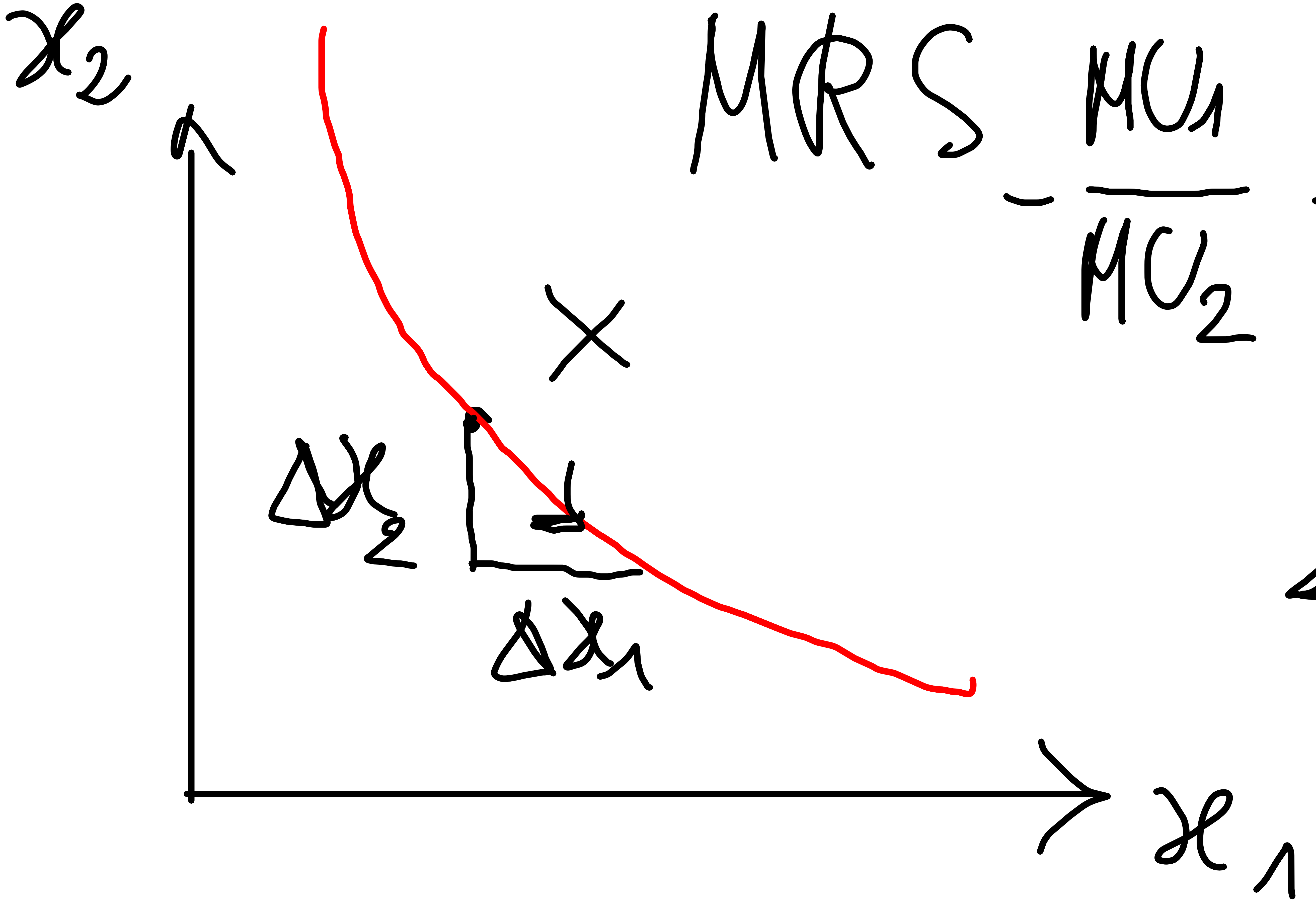
$U(x_1, \bar{x}_2)$



$MU_1(\bar{x}_1, \bar{x}_2)$



We hold the quantity of x2 fixed and we consider
the increase in utility produced by a unit increase of x1



$$MRS \; -\frac{MU_1}{MU_2} = \frac{\Delta x_2}{\Delta x_1} = MRS$$

$$\Delta U(x_1, x_2) = 0$$

if we move on indifference curve

The change in utility produced by the change in x1 and x2 along the indifference curve is:

$$MU_1 \cdot \Delta x_1 + MU_2 \cdot \Delta x_2 = 0 \qquad \frac{\Delta x_2}{\Delta x_1} = -\frac{MU_1}{MU_2}$$

1 organized markets

2 price taking consumers

3 money income m

4 preferences are 'well behaved'

a) complete, transitive, continuous

b) monotonic

c) strictly convex

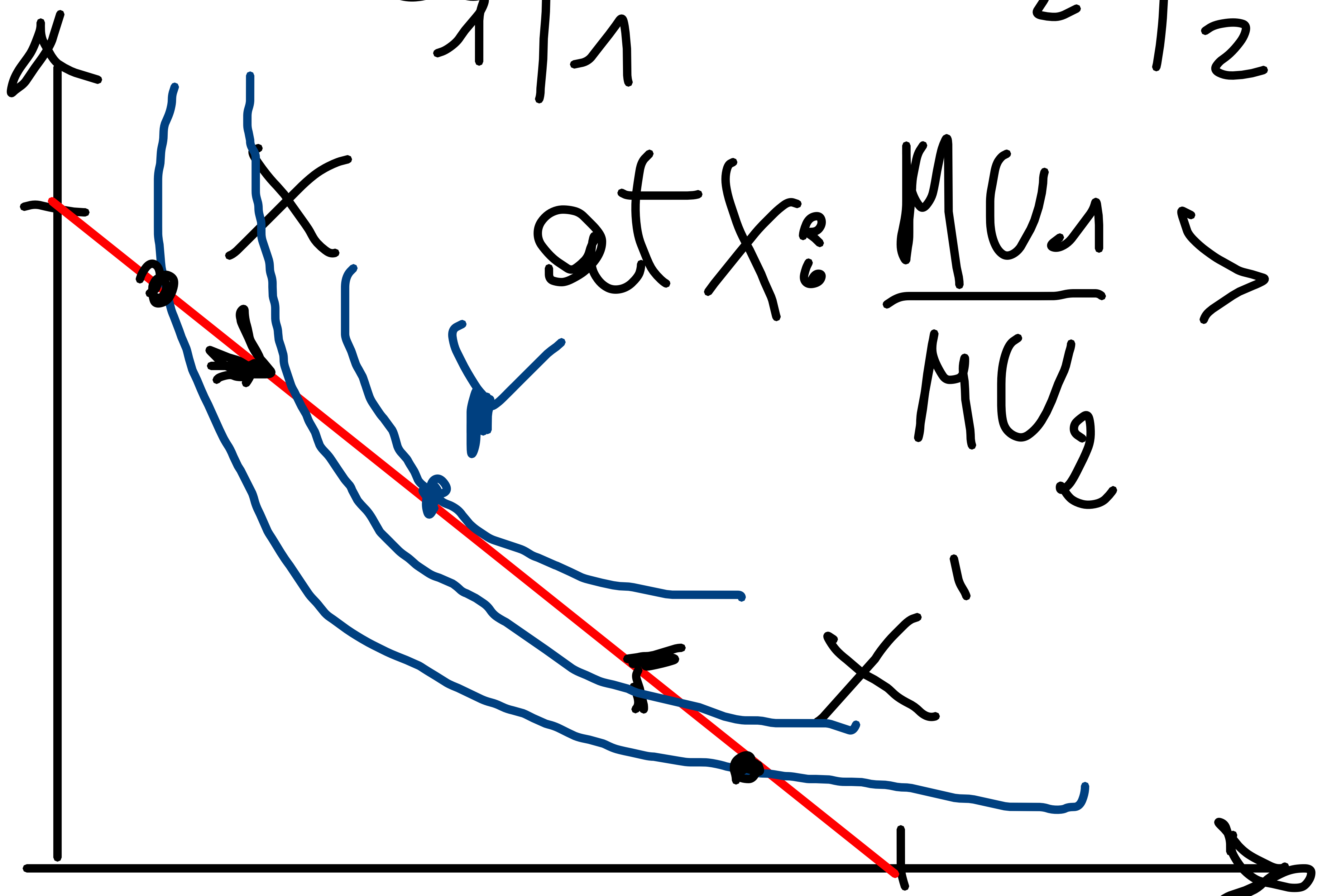
$$\max \; u(x_1, x_2)$$

$$\text{s.t.} \quad x_1 p_1 + x_2 p_2 \leq m$$

If preferences are monotonic the budget constraint is written with strict equality:

$$x_1 p_1 + x_2 p_2 = m$$

monotonic →



at X: $\dfrac{MU_1}{MU_2} > \dfrac{p_1}{p_2}$ $\qquad \dfrac{MU_1}{p_1} > \dfrac{MU_2}{p_2}$

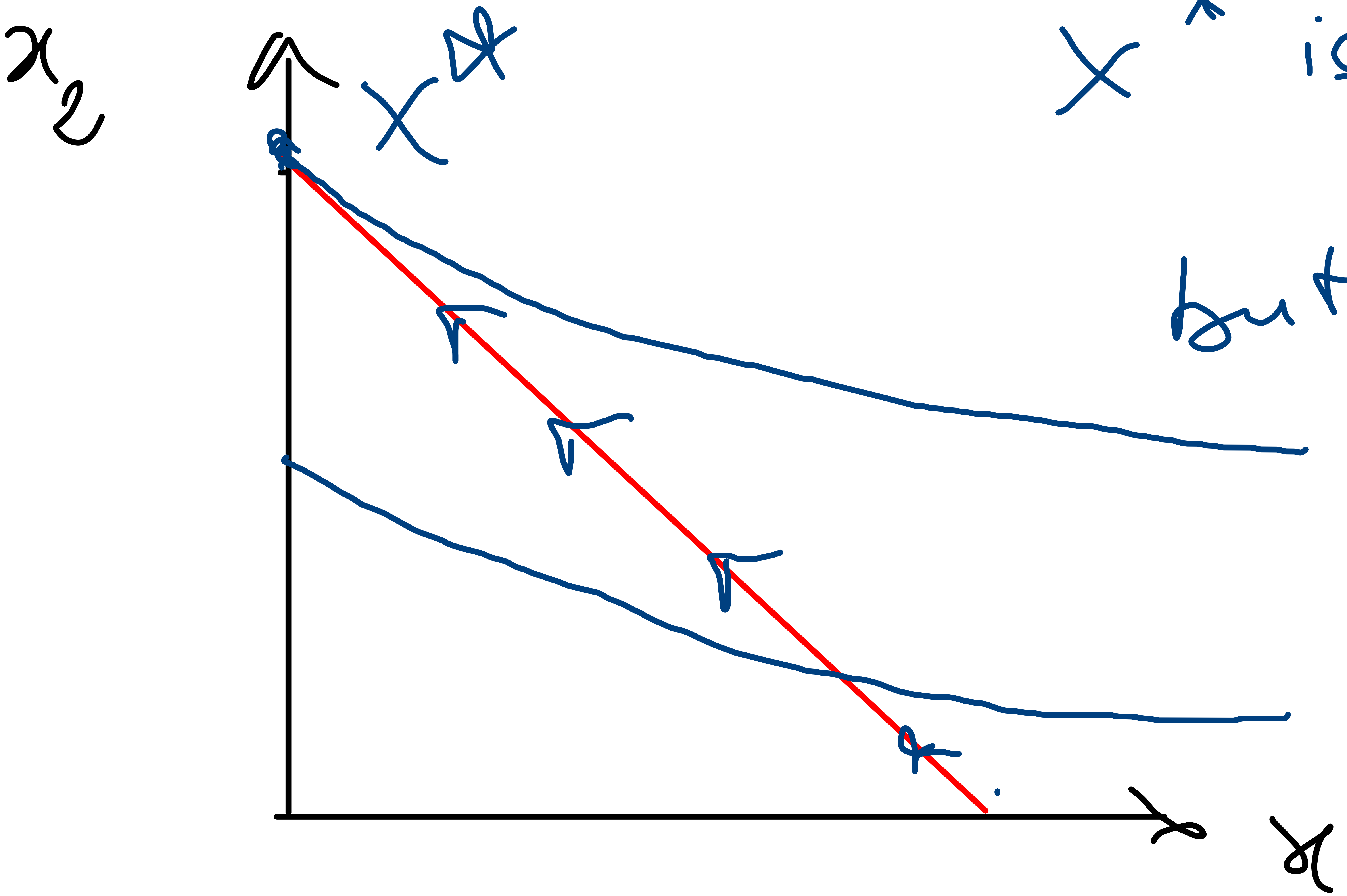
The consumer wants to spend more for x1 and less for x2

at X': $\dfrac{MU_1}{MU_2} < \dfrac{p_1}{p_2}$

The consumer wants to spend more for x2 and less for x1

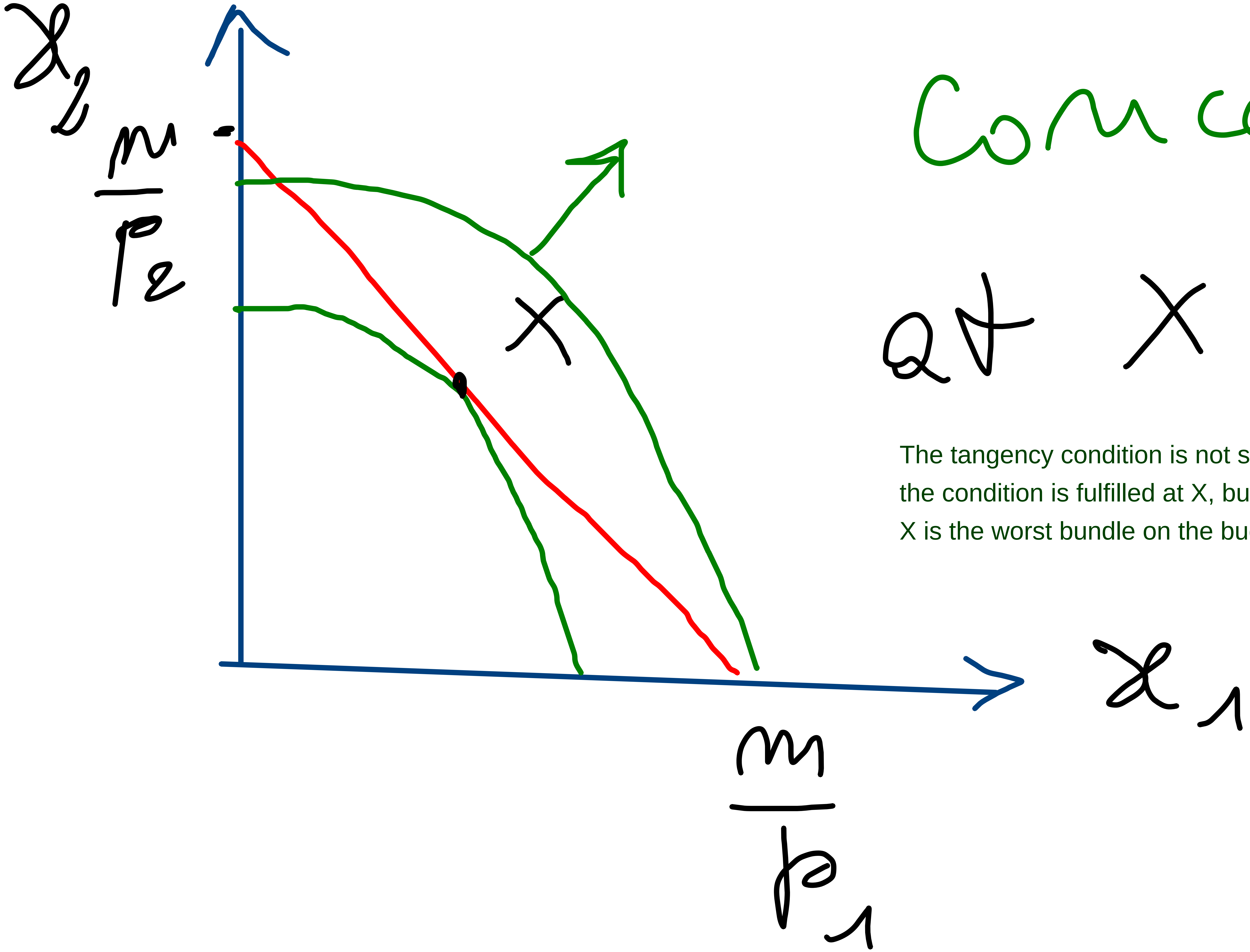
1st order necessary condition for <u>interior</u> optimum X^*

$$\begin{cases} |MRS| = \dfrac{p_1}{p_2} \text{ at } x^* \\ x_1^* p_1 + x_2^* p_2 = m \end{cases}$$



x^* is corner solution

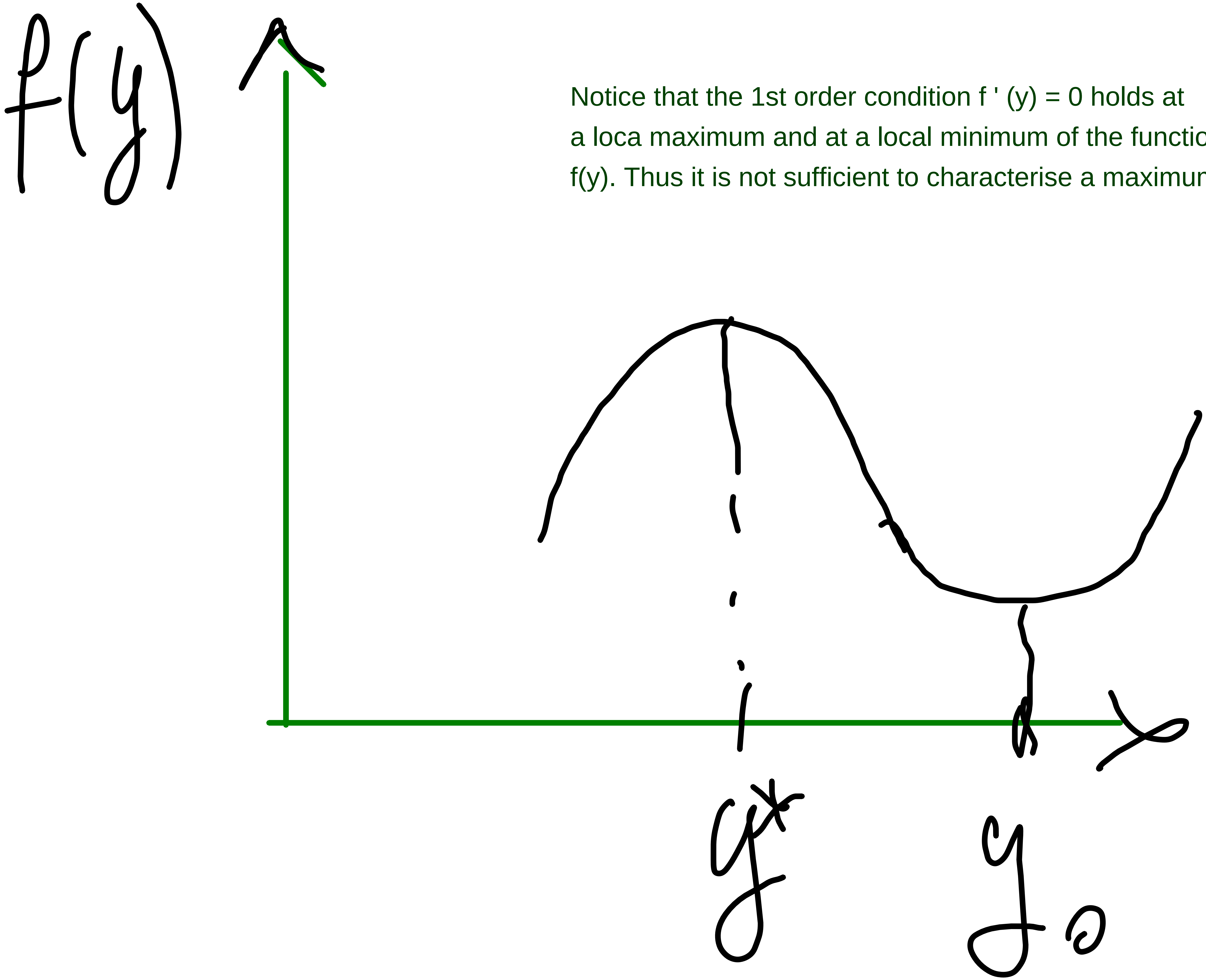
but $\frac{MU_1(x^*)}{MU_2(x^*)} = \frac{p_1}{p_2}$



concave prefs.

at X : $|MRS| = \frac{p_1}{p_2}$

The tangency condition is not sufficient for optimality. Here the condition is fulfilled at X, but with concave preferences X is the worst bundle on the budget line.



Notice that the 1st order condition f ' (y) = 0 holds at
a loca maximum and at a local minimum of the function
f(y). Thus it is not sufficient to characterise a maximum.

$$\frac{\partial f(y)}{\partial y} =$$

y

$$u(x_1, x_2) = x_1^c \cdot x_2^d$$ COBB DOUGLAS

$c > 0 \quad d > 0$ Pref. are monotone

$$MU_1 = c\, x_1^{c-1} \cdot x_2^d = \frac{c}{x_1} \cdot x_1^c x_2^d$$

$$MU_2 = d\, x_1^c \cdot x_2^{d-1} = \frac{d}{x_2} \cdot x_1^c x_2^d$$

Pref. strict. convex

$$\frac{MU_1}{MU_2} = |MRS| = \frac{c/x_1}{d/x_2} = \frac{c}{d} \cdot \frac{x_2}{x_1}$$

Problem: $\max\ x_1^c \cdot x_2^d$

$\text{s.t. } p_1 x_1 + p_2 x_2 = m$

assume (x_1^*, x_2^*) is interior

$$\left.\begin{aligned} \frac{c}{d} \cdot \frac{x_2}{x_1} &= \frac{p_1}{p_2} \\ x_1 p_1 + x_2 p_2 &= m \end{aligned}\right\}$$

necessary and suff.

$$x_1^* = \frac{c}{c+d} \cdot \frac{m}{p_1}$$

$$x_2^* = \frac{d}{c+d} \cdot \frac{m}{p_2}$$

$$u(x_1, x_2) = x_1^c \cdot x_2^d$$

Take monotone positive TRNANSF.

$$v(x_1, x_2) = \left(x_1^c \cdot x_2^d\right)^{\frac{1}{c+d}} =$$

$$= x_1^{\frac{c}{c+d}} \cdot x_2^{\frac{d}{c+d}}$$

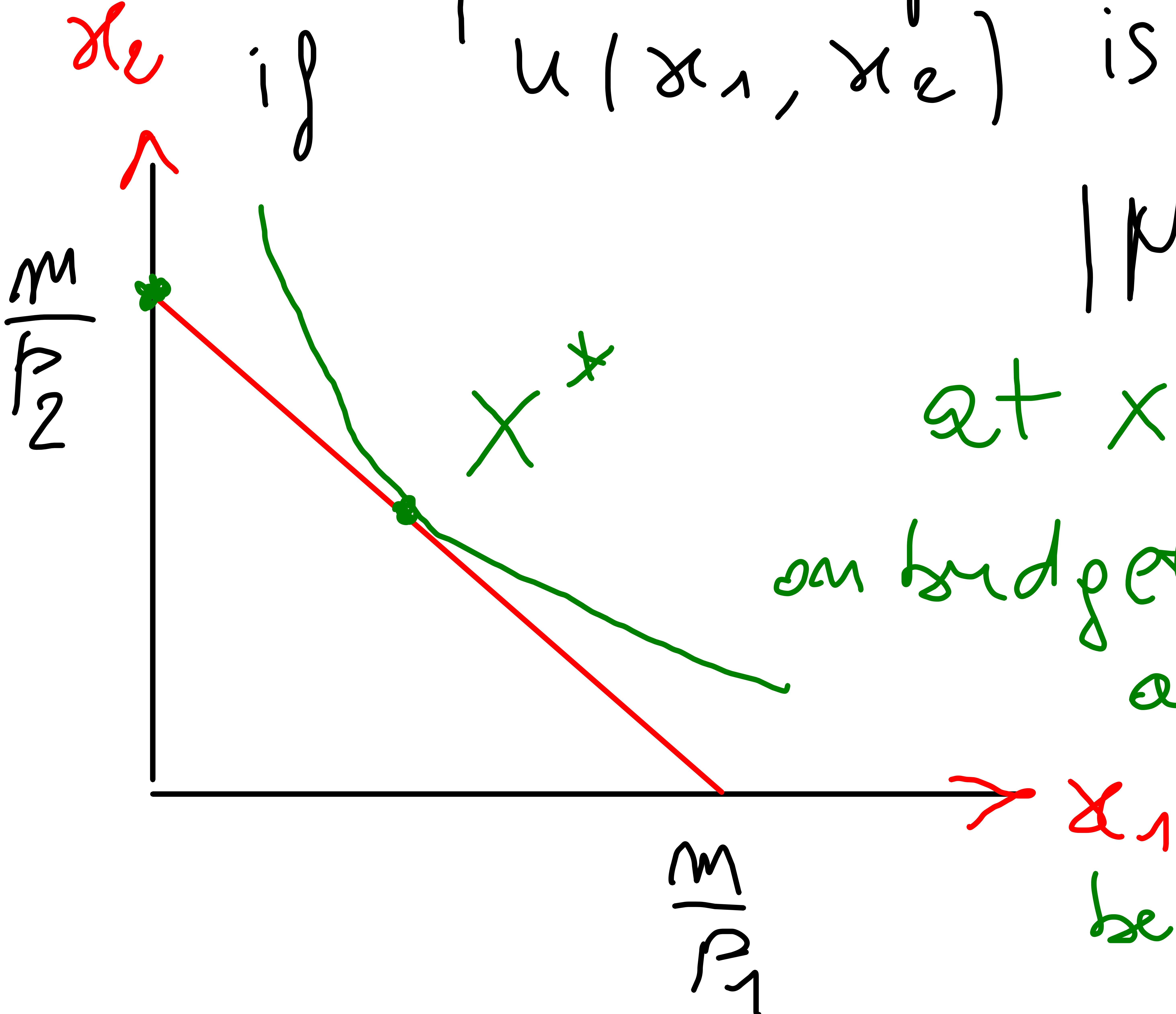
$$\frac{c}{c+d} = a$$

$$= x_1^a \cdot x_2^{1-a}$$

$$\frac{d}{c+d} = 1-a$$

a = fraction of m spent on x1; 1 - a = fraction of m spent on x2

Proof that optimum is unique if $u(x_1, x_2)$ is Cobb-Douglas



$$|MRS| = \frac{c}{d} \cdot \frac{x_2}{x_1}$$

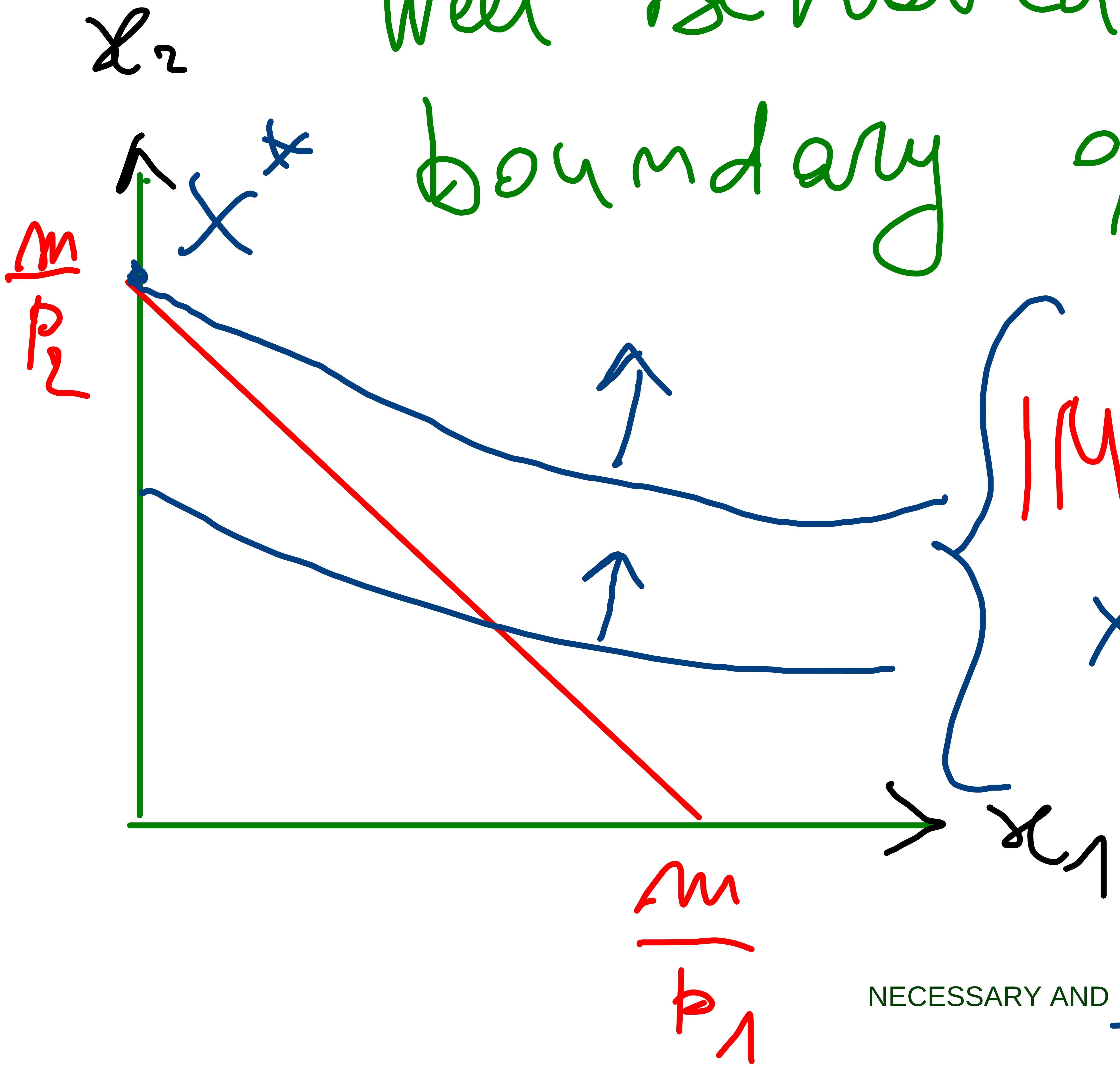
at x^*: $|MRS| = \frac{p_1}{p_2}$

on budget line:

above x^*: $|MRS| > \frac{p_1}{p_2}$

below x^*: $|MRS| < \frac{p_1}{p_2}$.

well behaved preferences and boundary optimum



$m \quad p_1 \quad p_2$

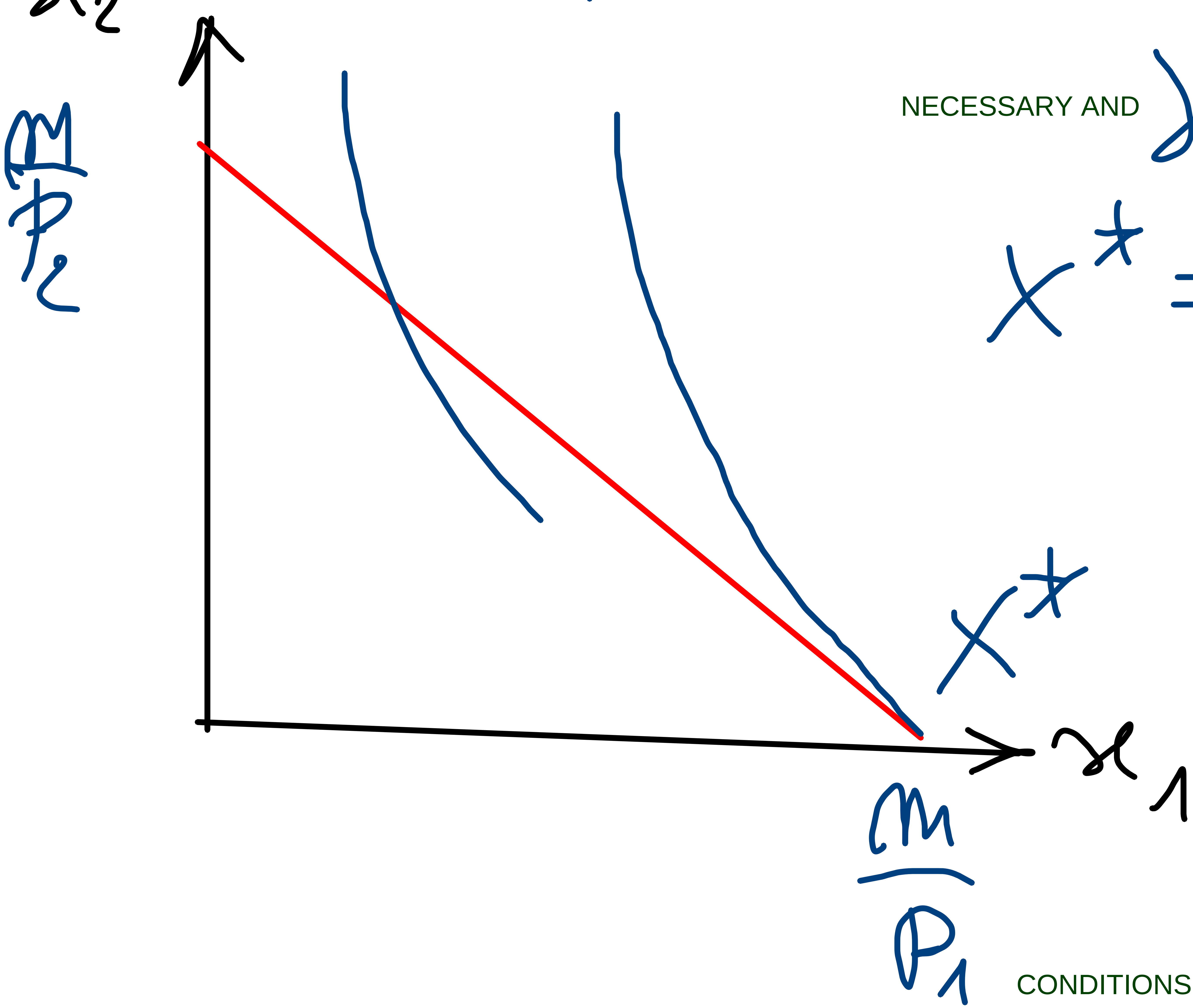
$$|MRS| \leq p_1/p_2$$

$$X^* = \left(0, \frac{m}{p_2}\right)$$

$$x_1^* p_1 + x_2^* p_2 = m$$

NECESSARY AND sufficient for optimality OF X* = (0 , m/p2)

PREF. ARE WELL BEHAVED



NECESSARY AND suff conditions:

$$x^* = \left(\frac{m}{p_1}, 0\right)$$

$$|MRS(x^*)| \geq \frac{p_1}{p_2}$$

$$x_1^* p_1 + x_2^* p_2 = m$$

CONDITIONS are also necessery

BECAUSE THE WEAK INEQUALITY COVERS EVERY POSSIBLE CIRCUMSTANCE UNDER WHICH x* = (m / p1, 0) IS OPTIMUM

WE HAVE PROVED THE

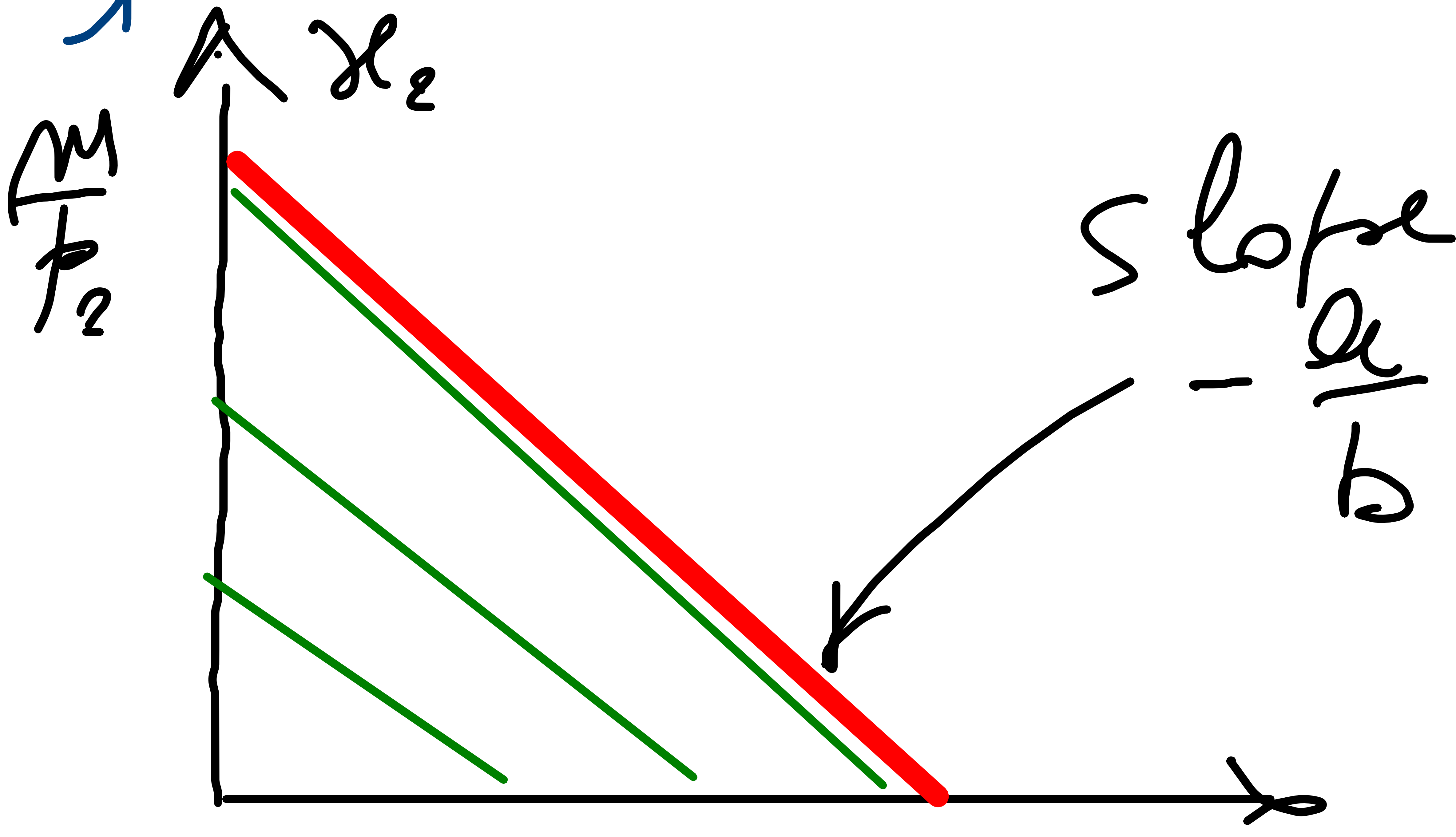
Proposition:

if preferences are strictly convex the optimum choice is unique AND x^* is a global optimum

Perfect substitutes

$$u(x_1, x_2) = a x_1 + b x_2$$

$$|MRS| = \frac{MU_1}{MU_2} = \frac{a}{b}$$



p_1, p_2

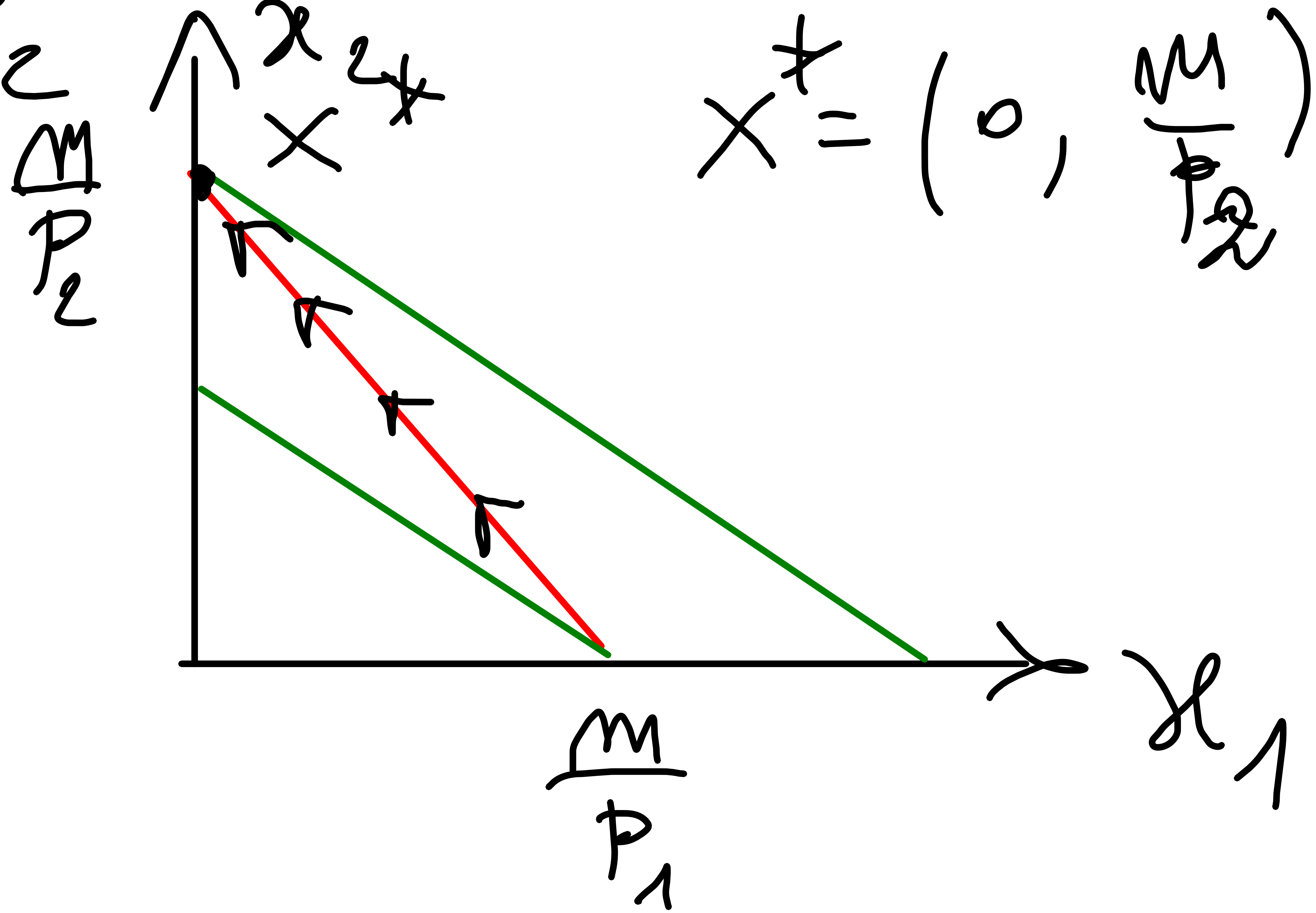
$$\frac{p_1}{p_2} = \frac{a}{b}$$

any X on budget line is optimum

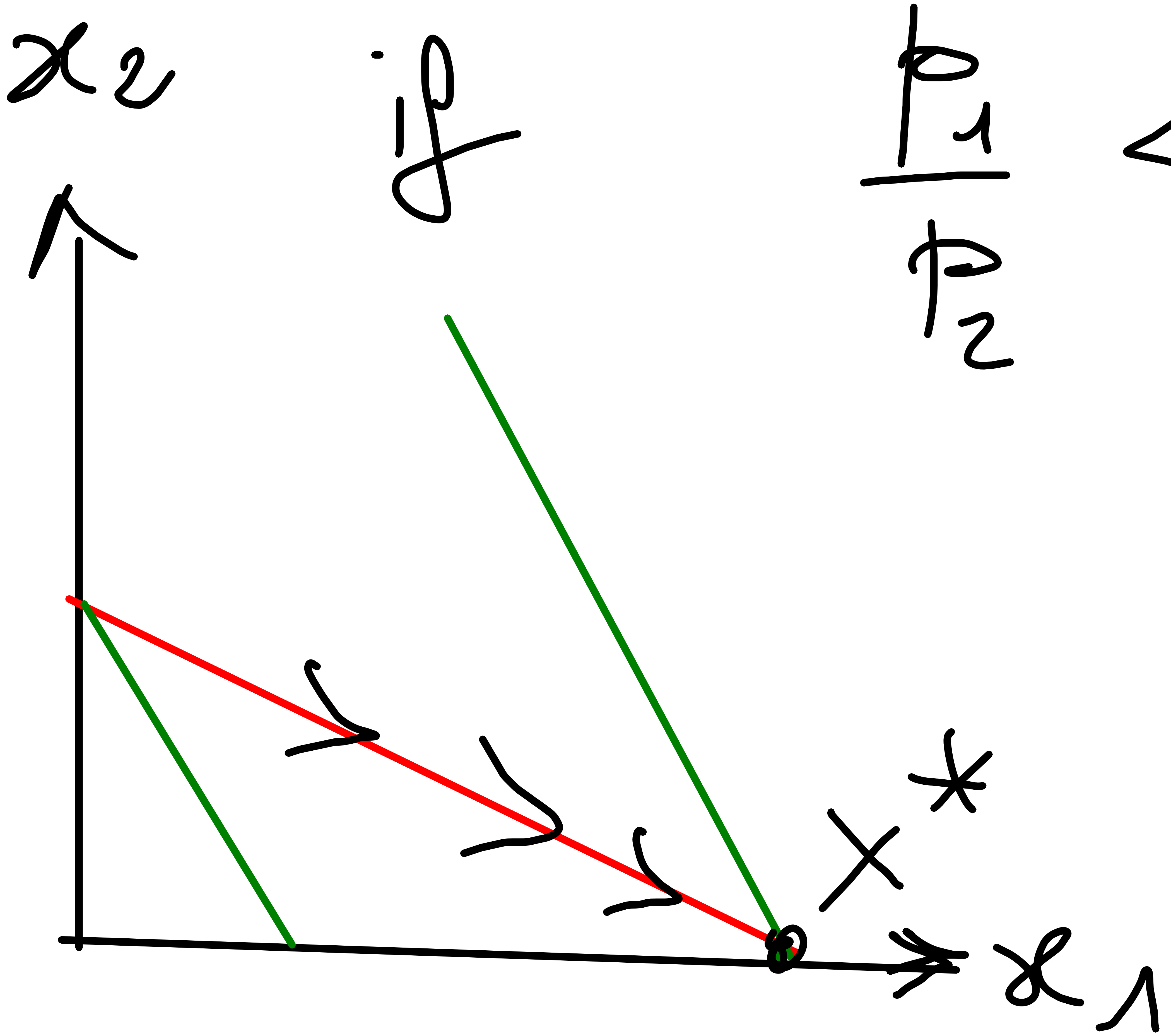
PERFECT SUBSTITUTES:

Suppose $\frac{p_1}{p_2} > \frac{a}{b} = |MRS|$

$$\frac{MU_1}{p_1} < \frac{MU_2}{p_2}$$



$$x^* = \left(0, \frac{m}{p_2}\right)$$



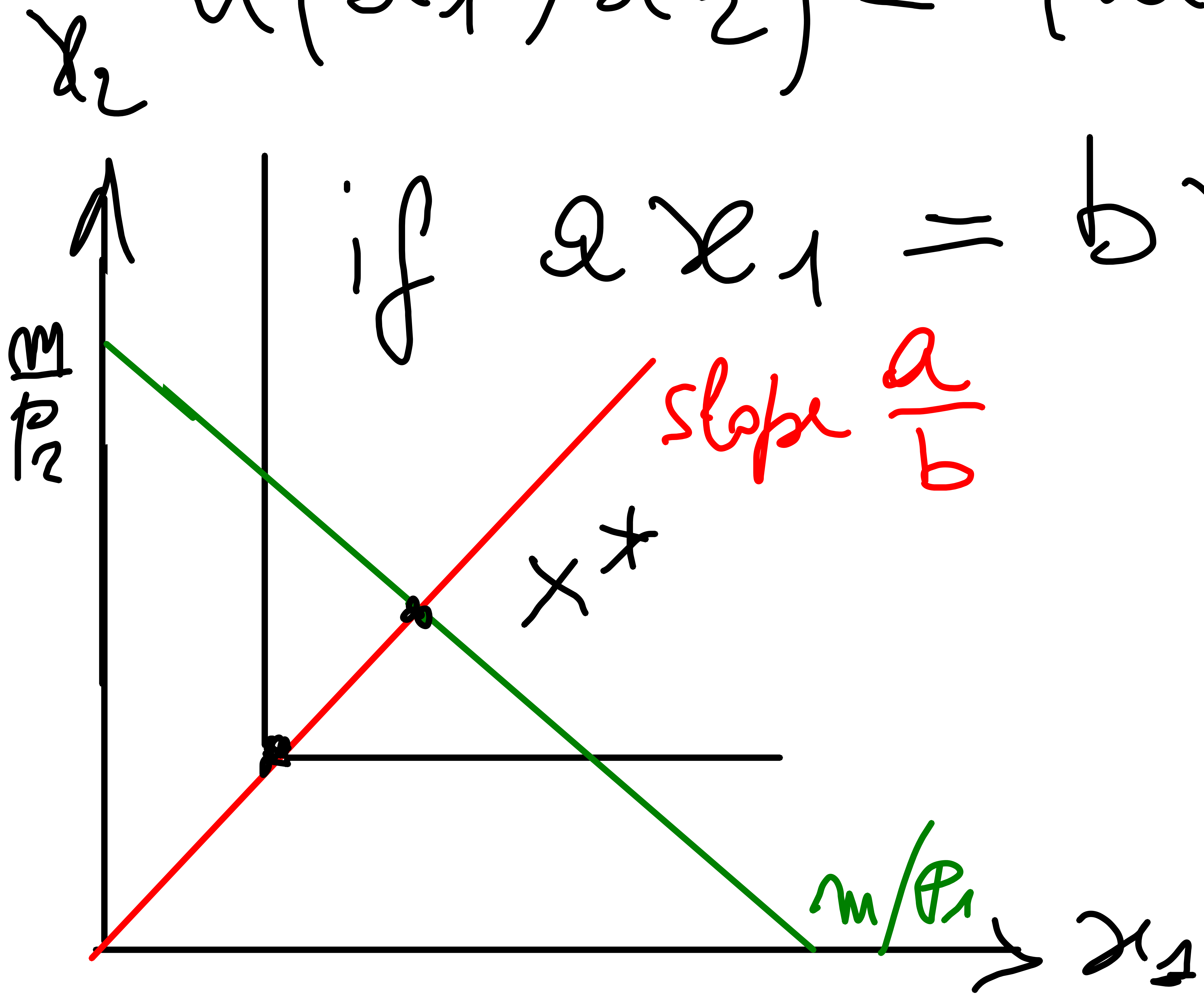
if $\frac{p_1}{p_2} < \frac{MU_1}{MU_2} = \frac{a}{b}$

$x^* = \left(\frac{m}{p_1}, 0\right)$

Perfect substitutes

Perfect complements

$$u(x_1, x_2) = \min(a x_1, b x_2)$$



if $a x_1 = b x_2$ then x_1 and x_2 are in the desired proportion

$$\frac{x_2}{x_1} = \frac{a}{b}$$

$$\begin{cases} \dfrac{x_2}{x_1} = \dfrac{a}{b} \\ x_1 p_1 + x_2 p_2 = m \end{cases}$$

$$x_1 = \frac{m}{p_1 + \frac{a}{b} p_2} \qquad x_2 = \frac{m}{\frac{b}{a} p_1 + p_2}$$

QUASI-LINEAR PREFERENCES:

Well behaved preferences

such that $u(x_1, x_2) = v(x_1) + x_2$

$$|MRS| = \frac{MU_1}{MU_2} = \frac{v'(x_1)}{1}$$ depends only on x_1

$v'(x_1) > 0$ monotoniaity

$v''(x_1) < 0$ strict convexity

$$u(x_1, x_2) = x_1^{1/2} + x_2$$

$$|MRS| = \frac{1}{2} x_1^{-1/2} = \frac{1}{2} \cdot \frac{1}{\sqrt{x_1}}$$

$$u(x_1, x_2) = \log x_1 + x_2$$

$$|MRS| = \frac{1}{x_1} = \frac{p_1}{p_2}$$

1st order cond. for optimum

DEMAND OF x1 DEPENDS ONLY ON THE PRICE RATIO p1 / p2 IF p1 x1 > = m